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*Review Article*

# Fuzzy Meir-Keeler's Contraction and Characterization

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In this study, a fuzzy Meir-Keeler's contraction theorem for complete FMS based on George and Veeramani idea is established. Then, we characterize fuzzy Meir-Keeler's contractions as contractive types induced by functions called fuzzy $L$-function. Moreover, we show that the converse of it is true. Finally, we bring some examples and corollaries certify our results and new improvement.

## 1. Introduction

Fixed point theory and related topics are an active research field with a wide range of applications in mathematics, engineering, chemistry, physics, economics, and computer sciences. Many authors have been studied this theory in hyperstructure spaces alongside the classical metric spaces and normed spaces. Among them, it could be cited probabilistic (and fuzzy) metric spaces. The phrase of fuzzy metric space (FMS), introduced by Kramosil and Michalek [1], then George and Veeramani [2], modified this idea which has applications in quantum particle physics [3] and in the two-slit experiment [4, 5]. Also, the theory of FMS is, in this framework, very disparate from the usual theory of metric best approximation and completion, e.g., see [6] and [7–9], respectively. Grabiec [10] developed and extended fixed point theory to probabilistic metric space. Later on, several authors have participated to the growth of this theory (see [10–17]).

In 2006, Lim [18] introduced $L$-functions (LF) and characterized Meir-Keeler's contractive as a self-map $T$ on $M$ that satisfies $d(T(p), T(q)) < \varphi(d(p, q)), p \neq q \in X$ for some LF $\varphi$ (see also [19]). This characterization prepares it easy to compare such maps with those satisfying Boyd-Wong's condition (see [20]). Then, Meir-Keeler [21] developed Boyd-Wong's result as follows

$$\forall \varepsilon > 0; \exists \delta > 0, \varepsilon \leq d(p, q) < \varepsilon + \delta \Rightarrow d(T(p), T(q)) < \varepsilon. \quad (1)$$

In 2005, Razani ([22], Theorem 2.2) introduced a contraction theorem in FMS. Our main result in this paper is to extend this Theorem to fuzzy Meir-Keeler's contraction. We assert that if $(X, M, *)$ is a FMS and $T$ on $X$ be a fuzzy Meir-Keeler's contractive self-mapping, then, $T$ has a unique fixed point in $X$. Our works are an extension of some recent results that we notice them. Then, we characterize fuzzy Meir-Keeler's contractive map as a map so that

$$\frac{1}{M(T(p), T(q), t)} - 1 < \varphi\left(\frac{1}{M(p, q, t)} - 1\right), \forall p, q \in X, p \neq q, t > 0, \quad (2)$$

where $\varphi$ is a fuzzy LF and $*$ is the minimum $t$-norm, i.e., $r * s = \min\{r, s\}, r, s \in I := [0, 1]$.

## 2. Preliminaries

In what follows, we mention some reported results, definitions, and examples related to the theory of FMS which are needed. More details and explanations can be followed in [2, 6–9, 23, 24].

*Definition 1* (see [24]). A $t$-norm is a function $* : I \times I \longrightarrow I$ such that $*$ is continuous, commutative, associative, $s * 1 = s$, $s \in I$, and $p * q \leq t * s$, where $p \leq t$ and $q \leq s$, and $p, q, t, s \in I$.

In the study of probabilistic and FM spaces, the presentation of $t$-norms was raised by the requirement to assign the triangle inequality (condition (iv) below) in the setting of metric spaces to that of fuzzy metric spaces. Numerous examples of this concept have been cited by various researchers, for instance, one may be found in [14, 15].

*Definition 2* (George and Veeramani [2]). $(X, M, *)$ is said FMS where $X \neq \varnothing$ is a set, $*$ is a continuous $t$-norm, and $M$ on $X^2 \times ]0,+\infty[ \longrightarrow I$ is a function with

(1) $M(p, q, t) > 0$, for all $t > 0$

(2) $M(p, q, t) = 1$, for all $t > 0$ iff $p = q$

(3) $M(p, q, t) = M(q, p, t)$

(4) $M(p, q, t) * M(q, r, s) \leq M(p, r, t + s)$,

(5) $M(p, q, \cdot)$: $]0,+\infty[ \longrightarrow I$ be a continuous fuzzy set-where $p, q, r \in X$ and $t, s > 0$.

Now, we bring two theorems and definitions that play as key roles in this paper, and we continue the next sections based on these concepts to reach our aims.

**Theorem 3** (see [2]). *In a FMS $(X, M, *)$, $p_n \longrightarrow p$ (i.e., $p_n$ converges to $p$) iff,$\forall t > 0, M(p_n, p, t) \longrightarrow 1$ as $n \longrightarrow \infty$.*

*Definition 4* (see [2]). $(p_n)_{n\geq 1}$ is said a Cauchy sequence in $(X, M, *)$ if for all $t > 0$ and $\varepsilon \in (0, 1)$, $\exists n_0 = n_0(\varepsilon, t) \in \mathbb{N}$ so that $M(p_n, p_m, t) > 1 - \varepsilon$ whenever $n, m \geq n_0$. Also, we call it complete if every Cauchy sequence is convergent.

**Theorem 5** (see [9, 17, 23]). *In a FMS $(X, M, *)$, $M : X \times X \times (0,+\infty) \longrightarrow I$ is a continuous function.*

*Definition 6* (see [22]). In a FMS $(X, M, *)$, $T$ on $X$ is said a fuzzy contractive self-mapping (FCM), if

$$\frac{1}{M(T(p), T(q), t)} - 1 < \frac{1}{M(p, q, t)} - 1, \forall p, q \in X, p \neq q, t > 0. \tag{3}$$

## 3. Main Results

In this section, we discuss concerning fuzzy Meir-Keeler's contractive self-mapping. We give a proof of Meir-Keeler's fixed point theorem in FMS. Here, we consider fuzzy Meir-Keeler's contraction (FMK) to state our main results.

*Definition 7.* In a FMS $(X, M, *)$, $T$ on $X$ is a (FMK), if for all $\varepsilon \in (0, 1)$, $\exists \delta > 0$ such that for all $p \neq q \in X, t > 0$,

$$Mp, q, t) > \frac{1}{\varepsilon + \delta + 1} \quad \text{implies} \quad M(T(p), T(q), t) > \frac{1}{\varepsilon + 1}. \tag{4}$$

*Remark 8.* Each FMK is a FCM but the inverse is not necessarily true. To prove this claim, we bring the flowing example.

*Example 1.* It is precise that FMK implies fuzzy contraction but note the inverse is not true because assume that

$$T(p) = \begin{cases} 3 & |p| > 1, \\ 4 & p = 0. \end{cases} \tag{5}$$

and $M_d$ defined as Example 2.9 of [2] with $* = \min$. If $|p| > 1$ and $q = 0$ then $T(p) = 3$ and $T(q) = 4$. Thus, $|T(p) - T(q)| = 1 < |p| = |p - q|$. Therefore, we get $|T(p) - T(q)| < |p - q|$, for all $p, q, p \neq q$, i.e., $T$ is a FCM. But if $|p| > 1$, $q = 0$, and $\varepsilon = 1/2$, then, there exists $\delta > 0$ so that

$$\begin{aligned} M(p, q, t) &= \frac{t}{t+|p|} \\ &> \frac{1}{1 + 1/2 + \delta}, \end{aligned} \tag{6}$$

holds. Hence, if $T$ be a FMK, we easily have that

$$\begin{aligned} M(T(p), T(q), t) &= \frac{t}{t+|T(p) - T(q)|} \\ &= \frac{t}{t + 1} \\ &> \frac{1}{1 + 1/2}, \end{aligned} \tag{7}$$

for all $p \neq q \in X$. It means that $t > 2$, which is a contradiction. Thus, $T$ does not satisfy FMK, and this proves our claim.

**Theorem 9.** *If $(X, M, *)$ is a complete FMS, where t-norm is defined as $* = \min$. Suppose self-mapping $T$ on $X$ is a FMK. Then, $T$ has a unique fixed point.*

*Proof.* For all $p \neq q \in X$ and $t > 0$, we get $M(T(p), T(q), t) > M(p, q, t)$. Since if there are $p_0, q_0 \in X$ and $t_0 > 0$ so that $M(T(p_0), T(q_0), t_0) \leq M(p_0, q_0, t_0)$. For simplicity, we put $M_2 = M(T(p_0), T(q_0), t_0)$ and $M_1 = M(p_0, q_0, t_0)$. Hence, for each $\delta > 0$

$$\begin{aligned} M_1 \geq M_2 &\geq \frac{M_2}{1 + \delta M_2} \\ &= \frac{1}{(1/M_2 - 1) + \delta + 1}. \end{aligned} \tag{8}$$

Then, by (4), $M_1 > M_2$, which is a contradiction. Let $p_0 \in X$. Set $p_{n+1} = T(p_n)$, $n \in \mathbb{N}$ and suppose that $p_{n+1} \neq p_n$, $\forall n \in \mathbb{N}$, since otherwise $T$ has a fixed point. Assume that $t > 0$ be arbitrary and fixed after choosing. Now, let $c_n = 1/M(p_{n+1}, p_n, t) - 1$. $(c_n)$ is a nonincreasing sequence. So, $(c_n)$converges to $\eta$. Thus, $\exists N \in \mathbb{N}$ such that, $|c_n - \eta| < r$, for all $n \geq N$. We claim that $\eta = 0$. If $0 < \eta$, there is $d > 0$ such that, for all $p, q \in X$ and $t > 0$, where $M(p, q, t) > 1/1 + \eta + d$, we have $M(T(p), T(q), t) > 1/1 + \eta$. Select $r > 0$ such that $(d/2 - r, d/2 + r) \subset [0,+\infty)$. We have

$$|d/2 - (c_n - \eta) - d/2| < r, \tag{9}$$

thus

$$\frac{d}{2} - (c_n - \eta) \in \left(\frac{d}{2} - r, \frac{d}{2} + r\right), \tag{10}$$

so $1/M(p_{n+1}, p_n, t) - 1 - \eta < d$. It means that $M(p_{n+1}, p_n, t) > 1/1 + \eta + d$. Hence by 2, $M(p_{n+2}, p_{n+1}, t) > 1/1 + \eta$, which is a contradiction, since $c_j = 1/M(p_{j+1}, p_j, t) - 1 \geq \eta$, for all $j \in \mathbb{N}$.

Now, we prove that $(p_n)$ is a Cauchy sequence. If $(p_n)$ is not, then there is $0 < \varepsilon_0 < 1$, so that, for any $m \in \mathbb{N}$, there are $i_m, j_m > m$ such that $M(p_{i_m}, p_{j_m}, t) \leq 1 - \varepsilon_0$. For each $0 < e < \varepsilon_0$, there is $d > 0$ such that for any $p, q \in X$ and $t > 0$, if $M(p, q, t) > 1/1 + e + d$ then $M(T(p), T(q), t) > 1/1 + e$. There exists $M > 0$ such that for all $i \geq M$ and $t > 0$, $M(p_i, p_{i+1}, t) \geq 1 - e + d/1 + e + d$. Take $j_M > i_M \geq M$ such that

$$M\left(p_{j_M}, p_{i_M}, t\right) \leq 1 - \varepsilon_0. \tag{11}$$

Then, we get

$$\begin{aligned} M\left(p_{i_{M-1}}, p_{i_{M+1}}, t\right) &> M\left(p_{i_{M-1}}, p_{i_M}, \frac{t}{2}\right) * M\left(p_{i_M}, p_{i_{M+1}}, \frac{t}{2}\right) \\ &\geq \left(1 - \frac{e+d}{1+e+d}\right) * \left(1 - \frac{e+d}{1+e+d}\right) \\ &= \frac{1}{1+e+d}. \end{aligned} \tag{12}$$

It presents that $1/M(p_{i_{M-1}}, p_{i_{M+1}}, t) - 1 < e + d$. Thus, by (4), $1/M(p_{i_M}, p_{i_{M+2}}, t) - 1 < e$. Also,

$$\begin{aligned} M\left(p_{i_{M-1}}, p_{i_{M+2}}, t\right) &> M\left(p_{i_{M-1}}, p_{i_M}, \frac{t}{2}\right) * M\left(p_{i_M}, p_{i_{M+2}}, \frac{t}{2}\right) \\ &\geq \left(1 - \frac{e+d}{1+e+d}\right) * \left(\frac{1}{1+e}\right) \\ &= \frac{1}{1+e+d}. \end{aligned} \tag{13}$$

In other words, $M(p_{i_{M-1}}, p_{i_{M+2}}, t) > 1/1 + e + d$. Thus, by (4), $M(p_{i_M}, p_{i_{M+3}}, t) > 1/1 + e$. By induction, $M(p_{i_M}, p_{j_M}, t) > 1/1 + \varepsilon_0$. By (11), $1 - \varepsilon_0 > 1/1 + \varepsilon_0$, i.e., $1 - \varepsilon_0^2 > 1$, which is a contradiction. Thus, $(p_n)$ is convergent to $p^* \in X$. We get

$$M(p_{n+1}, Tp^*, t) = M(Tp_n, Tp^*, t) > M(p_n, p^*, t) \longrightarrow 1. \tag{14}$$

Then, $M(p_{n+1}, Tp^*, t) \longrightarrow 1$. So, $Tp^* = p^*$.

For proving the uniqueness of $p^*$, suppose there is a $q^* \neq p^*$ with $T(q^*) = q^*$, it follows that

$$M(p^*, q^*, t) = M(T(p^*), T(q^*), t) > M(p^*, q^*, t). \tag{15}$$

This is a contradiction, then, the uniqueness is proved.

Note that Theorem 9 is a generalization of ([22], Theorem 2.2); when we consider $t$-norm $* = \min$, this shows one of the most reason of improvement of [22]. □

## 4. Characterization of Fuzzy Meir-Keeler's Contractions

In this part of the paper, we characterize FMK maps. In Theorem 11, we provided a sufficient and necessary condition for FMK maps by tools of fuzzy $L$-function. Also, this generalizes Theorem 1 of [21]. More precisely, we show that if a self-mapping $T$ on $X$ be a FMK then there is a fuzzy $L$-function $\phi$ from $(0, +\infty)$ into itself such that, for all $t > 0, p \neq q \in X$, $1/M(T(p), T(q), t) - 1 < \phi(1/M(p, q, t) - 1)$. Also, we show that the converse of it is true. In some sense, our work is very close to Suzuki [19], and Lim [18].

*Definition 10* (see [18]). Function $\varphi$ from $[0, +\infty)$ into itself is said to be a fuzzy LF if $\varphi^{-1}(0) = 0$ and $\forall s \in (0,+\infty)$, $\exists \delta > 0$ so that $\varphi(t) \leq s, \forall t \in [s, s + \delta]$.

In the following, we bring a theorem to show the condition of reaching a self-map $T$ on $X$ in a FMS to FMK.

**Theorem 11.** *Suppose $(X, M, *)$ is a FMS, and the t-norm is defined as $* = \min$. Then, a self-map $T$ on $X$ is FMK iff there exists a (nondecreasing) fuzzy LF $\varphi$ as (2) is satisfied.*

*Proof.* Assume that $T$ is a FMK. By Definition 7, let a function $\eta$ from $(0, 1)$ into $(0, +\infty)$ is defined such that

$$\frac{1}{M(p, q, t)} - 1 < \varepsilon + 2\eta(\varepsilon) \text{implies} \frac{1}{M(T(p), T(q), t)} - 1 < \varepsilon, \tag{16}$$

for $\varepsilon \in (0, 1)$. With such $\eta$, let present a nondecreasing function $\kappa$ from $(0, 1)$ into $[0, +\infty)$ by

$$\kappa(t) = \inf \{\varepsilon > 0 : t \leq \eta(\varepsilon) + \varepsilon\}, \tag{17}$$

for any $t \in (0, 1), p, q \in X, p \neq q$. Since $t \leq t + \eta(t)$, we get $\kappa(t) \leq t$ for $t \in (0,+\infty)$. Suppose that the function $\bar{\varphi} : [0,+\infty) \longrightarrow [0,+\infty)$ is defined by

$$\bar{\varphi}(t) = \begin{cases} \kappa(t) & \text{if } \min \{\varepsilon > 0 : t \leq \eta(\varepsilon) + \varepsilon\} \text{exists, where} t > 0, \\ 0 & \text{if} t = 0, \\ \dfrac{t + \kappa(t)}{2} & \text{otherwise.} \end{cases} \tag{18}$$

It is obvious that $\bar{\varphi}(0) = 0$. $0 < \bar{\varphi}(s) \leq s$ for $s \in (0,+\infty)$. Let $s \in (0,+\infty)$ be fixed. If $\bar{\varphi}(t) \leq s$, where $t \in (s, s + \eta(s)]$, one can put $\delta = \kappa(s)$. Otherwise, $\exists\sigma \in (s, s + \eta(s)]$ with $s < \bar{\varphi}(\sigma)$. But $\sigma \le s + \eta(s)$, then, we get $\kappa(\sigma) \le s$, and if $\kappa(\sigma) = s$ then

$$\bar{\varphi}(\sigma) = \kappa(\sigma) = s. \tag{19}$$

Thus, we get $s = \bar{\varphi}(\sigma)$. This is a contradiction. Thus, it is concluded that

$$\kappa(\sigma) < s < \bar{\varphi}(\sigma) = \frac{\sigma + \kappa(\sigma)}{2}. \tag{20}$$

Now, we shall select $u \in (\kappa(\sigma), s)$ with $\sigma \le u + \eta(u)$, and let $\delta = s - u > 0$. Consider $t \in [s, s + \delta]$. Since

$$u + \eta(u) \ge \sigma = 2\frac{\sigma + \kappa(\sigma)}{2} - \kappa(\sigma) > 2s - u = \delta + s \ge t, \tag{21}$$

we obtain $\kappa(t) \le u$. Therefore

$$s = \frac{u + s + \delta}{2} \ge \frac{\kappa(t) + t}{2} \ge \bar{\varphi}(t). \tag{22}$$

Hence, $\bar{\varphi}$ is a fuzzy LF. If we consider $p, q \in X$ with $p \ne q$ and fixed. The definition of $\bar{\varphi}$ implies that $\forall t > 0$ there is $\varepsilon \in (0, \bar{\varphi}(t)]$ in which $t \le \eta(\varepsilon) + \varepsilon$. Thus, $\exists\varepsilon \in (0, \varphi(1/M(p, q, l) - 1))$ where $1/M(p, q, l) - 1 \le \varepsilon + \eta(\varepsilon)$. Therefore,

$$\frac{1}{M(T(p), T(q), l)} - 1 \le \varepsilon \le \bar{\varphi}\left(\frac{1}{M(p, q, l)} - 1\right), \tag{23}$$

holds. Therefore, $\bar{\varphi}$ satisfies (2). We define function $\bar{\bar{\varphi}}$ as

$$\bar{\bar{\varphi}}(t) = \sup\{\bar{\varphi}(s) \colon s \le t\}, \tag{24}$$

for any $t \in (0, 1)$, we get

$$0 < \bar{\varphi}(t) \le \bar{\bar{\varphi}}(t) \le t, \forall t \in (0, 1). \tag{25}$$

Hence, $\bar{\bar{\varphi}}$ also satisfies (2). Easily can be verified that $\bar{\bar{\varphi}}$ is a nondecreasing fuzzy LF. This completes the proof. □

Considering the following example, we briefly explain the Theorems 9 and 11.

*Example 2.* Let $X = I \cup \{3n, 3n + 1\}_{n\in\mathbb{N}}$ with usual distance on $\mathbb{R}$ and $T$ on $X$ be defined as follows:

$$T(p) = \begin{cases} \dfrac{p}{2} & 0 \le p \le 1, \\ 1 - \dfrac{1}{n + 2} & p = 3n + 1, \\ 0 & p = 3n. \end{cases} \tag{26}$$

Let we set $M$ as Example 2.9 of [2] and $* = \min$ then,

$$\delta(\varepsilon) = \begin{cases} 2\varepsilon & 0 < \varepsilon \le \dfrac{1}{2}, \\ 1 & \dfrac{1}{2} < \varepsilon < 1. \end{cases} \tag{27}$$

Therefore, $T$ satisfies the conditions of Theorem 9. Hence, $T$ has a (unique) fixed point in $X$. Also, if we consider

$$\varphi(t) = \begin{cases} \dfrac{2}{3}t & 0 \le t \le \dfrac{3}{4}, \\ 2t - 1 & \dfrac{3}{4} < t \le 1, \\ 1 & 1 \le t < \infty. \end{cases} \tag{28}$$

Thus, $\varphi$ is a fuzzy LF and

$$\begin{aligned} \frac{1}{M(T(p), T(q), t)} - 1 &= \frac{|T(p) - T(q)|}{t}, \\ &\le \varphi\left(\frac{|p - q|}{t}\right), \\ &= \varphi\left(\frac{1}{M(p, q, t)} - 1\right). \end{aligned} \tag{29}$$

In this step, we can easily reach to the following corollaries.

**Corollary 12.** *Suppose $(X, M, *)$ is a FMS, where $* = \min$ and $T$ are a self-mapping on $X$. If there exists a fuzzy L-function $\varphi : (0, +\infty) \longrightarrow (0, +\infty)$ where (2) is satisfied, then, $T$ has a unique fixed point.*

*Proof.* Let $\varepsilon > 0$ has given. By Definition 10 there is $\delta > 0$ such that for all $t \in [\varepsilon, \varepsilon + \delta]$, $\varphi(t) \le \varepsilon$. It means that if

$$\varepsilon \le \frac{1}{M(p, q, t)} - 1 < \varepsilon + \delta, \tag{30}$$

then by (2)

$$\frac{1}{M(T(p), T(q), t)} - 1 < \varphi\left(\frac{1}{M(p, q, t)} - 1\right) < \varepsilon. \tag{31}$$

It means that (4) holds. Thus, $T$ has a (unique) fixed point. □

Based on theorems in Section 3 and 4, we have the following result:

**Corollary 13.** *Suppose that $(X, M, *)$ be a FMS where $* = \min$ and $T$ is a self-mapping on $X$. The followings statements are equivalent:*

*(1) $T$ is a FMK*

*(2) There is a fuzzy LF $\varphi$ such that (2) satisfies*

*Proof.* By Theorem 11 and Corollary 12, we easily obtain the desired results. □

## 5. Conclusion

In this paper motivated by the results of Razani [22], a new class of FMK contractions in a complete FMS was introduced by reducing the contractive condition of the so-called Meir-Keeler's contractive maps. In Theorems 9, we established a fixed point theorem; and in Theorem 11, we provided a sufficient and necessary condition for fuzzy FMK maps. Our work generalizes Theorem 1.1 of [22] and Theorem 1 of [21].

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no competing interests.

## Authors' Contributions

All authors contributed equally. All authors read and approved the final manuscript.

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