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AN ANALYSIS TO DESIGN MERGED DELAY TRANSFORMED IIR DOWN SAMPLER FILTER: AN EFFICIENT DOWN SAMPLER FILTER ARCHITECTURE

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Abstract

An efficient architecture of Down Sampler Filter using Merged Delay Transformed Technique in terms of cost reduction and increase throughput has been proposed. The stability analysis of Merged Delay Transformed filters, finite word length effect on Transformed filters as well as conventional filters and peak error between the output from transformed IIR filters and from conventional filters have been investigated on the basis of commercially available software and mathematical calculations. The Butterworth, Chebyshev-1, Chebyshev-2 and Elliptic filters have been considered in this paper for the sake of comparison. The simulation results show that 84.59%, 73.56% and 50.83% & 78.71%, 73.85% and 51.89% reduction in cost can be achieved with different specifications by using the proposed scheme when it is compared with conventional FIR filter, conventional IIR filter and conventional polyphase IIR filter, respectively. It has been elaborated that the stability of transformed IIR Decimation filters will not disturb as the Decimation Factor M increases. It is also shown that the transformed Elliptic filter is more stable and the transformed Chebyshev–2 filter is a marginally stable filter. The simulation results show that the peak error is in the range of 10^{-10} to 10^{-17} which increases with an increase in order of the filter. On the basis of simulation results and mathematical calculations, it is shown that the decomposition of higher order system into 1st order and 2nd order systems using parallel implementation architecture with Merged Delay Transform increases the processing rate of the filter and a reduction in the cost of implementation as well as peak error.

Keywords:

Finite Word Length Effect, IIR Down Sampler Filter, Merged Delay Transformation, Stability

1. INTRODUCTION

Digital filters play an important role in Multi rate DSP and are applicable in the vast areas of electronics industries. Digital filters are widely used in data analysis and digital equipment for processing audio signals [1], data records [2], images and signals from biomedical instrumentation [3] and radar/sonar and other sensing systems. The study of digital filter entails a mixture of analytical concepts, simple hand calculations and employment of powerful computer tools [4].

Digital filters are classified in two major categories; Infinite Impulse Response (IIR) filter or Recursive filter and Finite Impulse Response (FIR) Filter or Non-recursive Filter [5]. By changing the weight and coefficient quantity, the frequency response of the filter can be realized. IIR or recursive filters play an important role in digital world because of their capability to de-noise the digital images and use in Satellite Image Processing [6], Digital Mammography [7] and in the field of scientific applications [8].

In order to accommodate various applications of filters, different types of IIR filters have already been designed such as Adaptive filters [9], SDM based IIR filter [10], a Constrained Causal Stable IIR filter [11] and Pipelined Lattice IIR digital filter [12], etc.

There are number of techniques to realize the multirate filters like: Frequency Response Masking (FRM) [13], Half Band Filter (HBF) [14], Polyphase Decomposition (PPD) [15] and Cascade Integrator Comb (CIC) [16], etc.

In this paper, a technique to realize a computationally efficient architecture of a recursive filter named Merged Delay Transformation (MDT) is explored where filtering and down sampling is obtained in a single stage instead of separate stages. Merged Delay Transformation is used to transform the conventional IIR filter structure into an efficient multirate structure. With the help of Merged Delay Transformation, various recursive filters can be designed such as Butterworth filter, Chebyshev-1 filter, Chebyshev-2 filter and Elliptic filter. This technique gives the advantage of power consumption and reduces the complexity as well as computational cost because it works at lower data rate and requires less multiplier. MDT is applicable in communication and wireless applications due to low power consumption and stability. In the proposed work, Transfer Function $H(z)$ of 1st order recursive filter is converted into $H(z^M)$ which has Down Sampler in cascade form. By using Noble Identity, filter structure is rearranged in such a way that outputs of the filter are not thrown away. Thus, an efficient structure of 1st order recursive filter can be achieved. In order to obtain 2nd order system, a pair of 1st order system is combined in parallel form with complex conjugate coefficients.

For the implementation, since the multipliers are more expensive than the adders therefore the MDT reduces the computational cost of the filter in terms of number of multipliers per output sample. The cost reduction of MDT filter is comparatively higher than FIR, conventional IIR and polyphase IIR filters and this reduction will increase with an increase in the Decimation Factor; M. Due to implementation in parallel form, MDT filter requires less hardware.

In this paper, the cost of the MDT IIR Down Sampler filter has been compared with FIR, conventional IIR and polyphase IIR filters with different specifications.

Before the implementation of digital filter, it is necessary to test the stability of the filter. The stability of MDT filter is estimated on the basis of pole - zero location in z- plane. In MDT, additional $M - 1$ poles and equal number of zeros are introduced in unit circle of z-plane at concurrent positions, where M is a Decimation Factor. MDT is less sensitive towards Coefficient Quantization and it provides good results at 8- bit implementation.

In this paper, stability of Merged Delay Transformed Infinite Impulse Response Decimation filters or IIR Down Sampler filters have been determined and compared with stability of original IIR Decimation filters without Merged Delay Transform. The peak error between the output from the proposed transformed filters and from the original filters is also investigated for order N = 1 to 6 at Decimation Factor M = 15. The Finite Word Length effect on transformed filter as well as original filter is also investigated.

The remaining part of the paper is organized as follows: In section 2, the details related to proposed schemes like recursive filter, decimation process and Merged Delay Transformation have been described. The calculations and study of the cost reduction factor in terms of multipliers per output sample using various realization techniques such as FIR, conventional IIR and polyphase IIR filter have been discussed in section 3. In section 4, stability analysis of the IIR down sampler filter with and without MDT is explored. In section 5, description of finite word length effect using coefficient quantization and effect on the position of pole-zero pairs due to coefficient quantization on IIR decimation filter with and without MDT is discussed. Section 6 shows the simulation results dealing with the full comparative study of frequency response, stability, and finite word length effect and peak error of conventional filters as well as transformed filters. Section 7 concludes the paper followed by the references at the end.

2. CONCEPTS RELATED TO PROPOSED WORK

2.1 RECURSIVE OR IIR FILTER AND DECIMATION PROCESS

The impulse response of recursive filter is always continuous [17]. The Transfer Function of IIR Filter is given by,

$$H(z)=\frac{\sum_{k=0}^{N} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}. \tag{1}$$

IIR filters are widely used in such applications where linear phase is not required like high speed and low power communication trans-receiver system. IIR filter can achieve the equal level of specifications with lower order than FIR filter. The efficiency of IIR filter is higher than FIR filter because it has sharp cutoff, minimum processing time and requires less memory and hardware [18].

The basic building block of decimation process is shown in Fig.1. In decimation, Down Sampler follows an anti-aliasing digital filter [20].

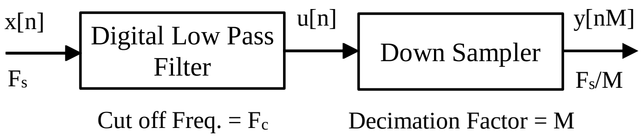


Fig.1. Decimation Process

The anti-aliasing filter is usually low pass filter or band pass filter. Filter is used to remove spectral components of the signal that may occur due to aliasing at lower sampling rate. The decimation is given by,

$$y[n]=y[nM]=\sum_{k=-\infty}^{\infty} h[k]*x[nM-k]. \tag{2}$$

2.2 MERGED DELAY TRANSFORMATION

A noble realization of the IIR decimation filter has been proposed in [21] which is named as Merged Delay Transformation (MDT). This transformation process is obtained analytically and it can be implemented directly to 1st order and 2nd order IIR filters. The computational efficiency of the system is increased because the present output can be directly computed from M^{th} old output by dropping (M – 1) intermediate outputs. A sample data rate is decreased by the Decimation Factor M due to merging (M – 1) delay elements in recursive path of filter. In this transformation, output of the filter $y[n]$ is feedback after passing it by M number of unit delay elements; therefore, sampling rate is reduced by M. This transformation can be applied to obtain a higher-order IIR filter structure [22]. An N^{th} order IIR Down Sampler filter can be decomposed into N parallel sections of 1st order filter. It provides stability for Coefficient Quantization and reduces number of multiplications.

A general difference equation of MDT IIR Down Sampler filter is given by,

$$y[n]=a^M y[n-M]+\sum_{k=0}^{M-1} a^k bx[n-k] \tag{3}$$

where, a and b are the filter coefficients. Thus, the system function of 1st order Merged Delay Transformed filter is given by,

$$H(z)=\frac{\sum_{k=0}^{M-1} a^k bz^{-k}}{1-a^M z^{-M}}. \tag{4}$$

System function of 2nd order MDT is also expressed as,

$$H_{1R}(z)=\frac{\sum_{k=0}^{M-1} H_k\left(z^M\right)z^{-k}}{1-P_F z^{-M}-Q_F z^{-2M}}. \tag{5}$$

Here the following substitutions are used,

$$P_F = 2P_{MR} \tag{6}$$

$$Q_F = -\left(P_{MR}^2+P_{MI}^2\right) \tag{7}$$

and

$$H_k(z_M)=Q_{kR}-(P_{MR}Q_{kR}+P_{MI}Q_{kI})z^{-M}. \tag{8}$$

By using Eq.(4), Eq.(5), an efficient architecture of 1st order and 2nd order Merged Delay Transformed IIR Down Sampler filter can be obtained. These architectures are obtained by using Commutator Switch Model. Thus, an architecture of 1st order and 2nd order Merged Delay Transformed IIR Down Sampler Filter is shown in Fig.2 and Fig.3. In Merged Delay Transformation, higher order transfer function is decomposed into parallel 1st order and 2nd order transfer function.

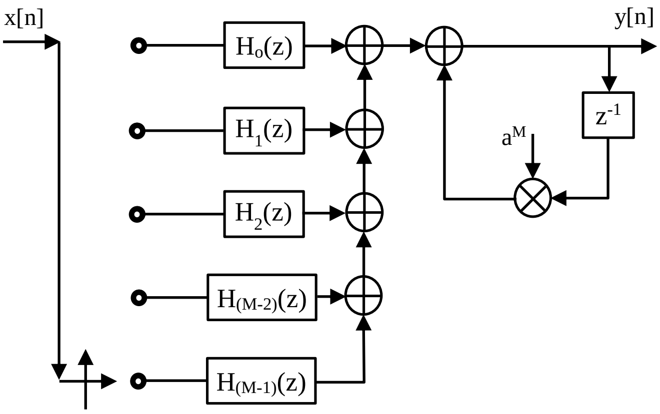


Fig.2. An Efficient Architecture of *1st* Order Merged Delay Transformed Filter using Commutator Switch Model

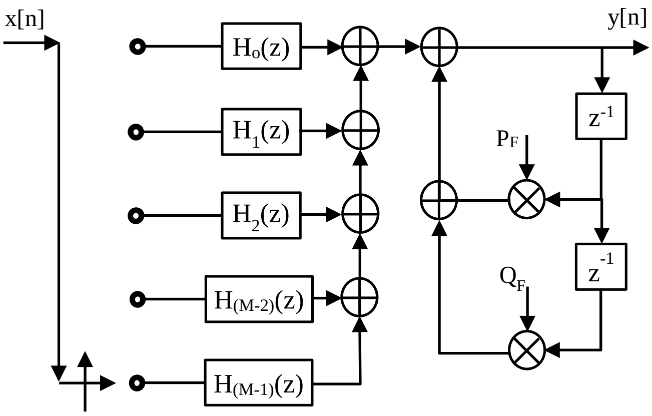


Fig.3. An Efficient Architecture of 2nd Order Merged Delay Transformed Filter using Commutator Switch Model

3. COST REDUCTION TECHNIQUE FOR IMPLEMENTATION OF MERGED DELAY TRANSFORMED IIR DOWN SAMPLER FILTER

A Decimation filters can be designed using FIR, conventional IIR, polyphase IIR and MDT filters. Their pass and stop band frequencies are obtained for various value of Decimation Factor; *M*. The minimum required filter order *N* can be calculated using these frequencies. Therefore, in this section, computational cost of MDT Recursive Decimated filter has been compared with different implementation techniques of decimation filter in terms of multipliers per output sample.

3.1 DETERMINATION OF MINIMUM REQUIRED FILTER ORDER

A Conventional IIR filter can be designed by using following specifications such as cut off frequency (ω_c), upper cut off frequency (f_{uc}), lower cut off frequency (f_{lc}), pass band ripple (α_p) and stop band ripple (α_s). The minimum order of decimation filter required for implementation can be calculated either by using theoretical calculations of filter or by using MATLAB functions.

Therefore, minimum required order of filter by theoretical calculations and their respective MATLAB functions for all types of Infinite Impulse Response filter such as Butterworth filter, Chebyshev-1 filter, Chebyshev-2 filter and Elliptic filter have been tabulated in Table.1 using [7].

In Table.1, various parameters are given to calculate the minimum required order of Recursive filter such as pass band frequency (Ω_p), stop band frequency (Ω_s), pass band ripple (α_p) and stop band ripple (α_s), which are,

$$\Omega_p = 2\pi f_{PB} \tag{9}$$

$$\Omega_s = 2\pi f_{SB} \tag{10}$$

$$\alpha_p = \frac{1}{\sqrt{1+\varepsilon^2}} \tag{11}$$

and,

$$\alpha_s = \frac{1}{\sqrt{1+\lambda^2}}. \tag{12}$$

3.2 CALCULATION OF THE COMPUTATIONAL COST TO DESIGN VARIOUS FILTER STRUCTURES

Delay, adder and multiplier are essential elements to implement any digital system. In the case of multiplier, if multiplication factor is greater than unity then signal gets amplified or it will be attenuated if multiplication factor is less than unity. Since a lot of complications have to be faced on applying multiplications and moreover multipliers provides highest computational cost therefore the main objective of any design technique is to reduce the number of multipliers per output sample. The computational cost of various filters such as FIR, conventional IIR, polyphase IIR and MDT filters are given in Table.2.

Table.1. Filter orders and their respective MATLAB functions for various IIR Filters

S. No.	IIR Filter Types	Formula for Filter Order *N*	MATLAB Functions for Minimum Required Filter Order
1	Butterworth Filter	$N \geq \dfrac{\log\sqrt{\dfrac{\lambda}{\varepsilon}}}{\log\dfrac{\Omega_s}{\Omega_p}}$	buttord
2	Chebyshev-1 Filter	$N \geq \dfrac{\cosh^{-1}\left(\dfrac{\lambda}{\varepsilon}\right)}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$	cheb1ord
3	Chebyshev-2 Filter	$N \geq \dfrac{\cosh^{-1}\left(\dfrac{\lambda}{\varepsilon}\right)}{\cosh^{-1}\left(\dfrac{1}{\Omega_p}\right)}$	cheb2ord
4	Elliptical Filter	$N \geq \left(\dfrac{\log_{16}\left(\dfrac{\lambda}{\varepsilon}\right)}{\log\dfrac{1}{q}}\right)$	ellipord

Table.2. Requirement of multipliers per output sample for different filter structures with filter order *N* and Decimation Factor *M*

S. No.	Filter Structures	Required Multipliers
1	FIR	N
2	Conventional IIR	$(2N + 1)M$
3	Polyphase IIR	$(MN + M + 1)$
4	Merged Delay Transformed IIR Decimated Filter	$(N + MN/2)$

3.3 SIMULATION EXAMPLES TO DESIGN DECIMATED IIR FILTER AND COMPARISON WITH VARIOUS FILTER STRUCTURES IN TERMS OF NUMBER OF MULTIPLIERS PER OUTPUT SAMPLE

Example 1: *All types of IIR Decimated filters such as Butterworth, Chebyshev-1, Chebyshev-2 and Elliptic have been designed using following specifications [23] as given in Table.3, to find the minimum order of the filter.*

In the decimation process, a Down Sampler follows an anti-aliasing filter. Therefore, filter frequency must fulfill the following conditions:

$$f_{PB} = \frac{35 f_s}{M} \tag{13}$$

and

$$f_{SB} = \frac{50 f_s}{M} \tag{14}$$

Table.3. Specifications of decimated IIR filter for Example 1

Specifications	Values
Sampling Frequency (ω_s)	100 samples/sec
Lower Cut-off Frequency (f_{lc})	35 Hz
Upper Cut-off Frequency (f_{uc})	50 Hz
Pass-Band Ripple (α_p)	0.01dB
Stop-Band Ripple (α_s)	50dB

All types of the decimated filters such as Equiripple FIR, Butterworth, Chebyshev-1, Chebyshev-2 and Elliptic filters have been simulated in MATLAB and required minimum order of the filter for mentioned cases are calculated for various Decimation Factor; *M*. and the calculation is tabulated in Table.4.

As given in Table.4 the Elliptic filter achieves the lowest order of the filter among all other filters such as Equiripple FIR, Butterworth, Chebyshev-1 and Chebyshev-2 filter for the same Decimation Factor; *M*. Therefore, the design of Elliptic filter is used where the computational cost is concerned. In Fig.4, the variation of order of the filter for various filters has been plotted with respect to Decimation Factor, *M*.

The computational cost for FIR, conventional IIR, polyphase IIR and MDT filters are given in Table.5 and the computational cost of various filters can be evaluated by using Table.2.

Table.4. Minimum filter order for various filters with different Decimation Factor *M* for Example 1

M \ Filter	Equiripple FIR	Butterworth Filter	Chebyshev-1 Filter	Chebyshev-2 Filter	Elliptic Filter
2	30	14	8	8	6
4	62	17	9	9	6
6	93	17	9	9	6
8	124	17	9	9	6
10	155	18	9	9	6
12	186	18	9	9	6
14	217	18	9	9	6
15	233	18	9	9	6

Table.5. Comparison of Computational Costs between various filter structures for Example 1

	Order of		Computational Cost of				% Reduction in Cost of MDT when Compare with		
M	**FIR**	**Elliptic IIR**	**FIR**	**Conventional IIR**	**PPD IIR**	**Transform IIR**	**FIR**	**Conventional IIR**	**Polyphase IIR**
2	30	6	30	26	15	12	60	53.85	20
4	62	6	62	52	29	18	70.97	65.38	37.93
6	93	6	93	78	43	24	74.19	69	44.19
8	124	6	124	104	57	30	75.81	71.15	47.37
10	155	6	155	130	71	36	76.77	72.31	49.30
12	186	6	186	156	85	42	77.42	73.08	50.59
14	217	6	217	182	99	48	77.88	73.63	51.52
15	233	6	233	195	106	51	78.11	73.85	51.89

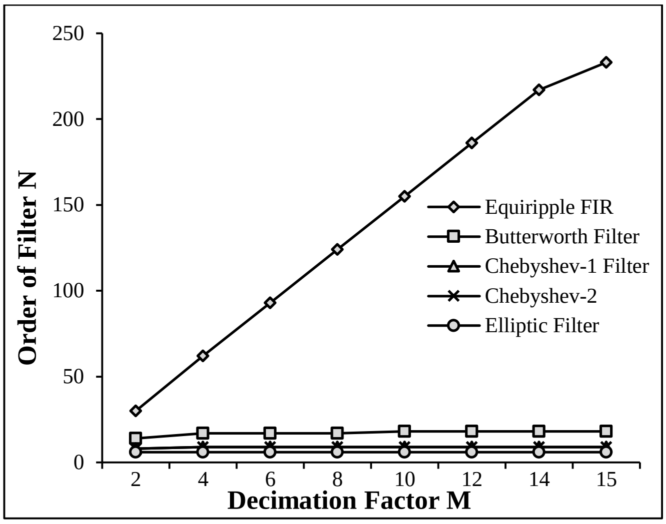


Fig.4. Filter Order for various Filters with Different Decimation Factor for Example 1

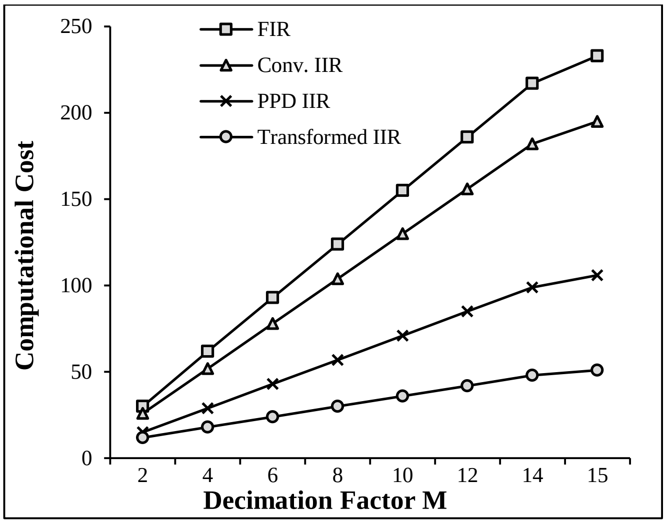


Fig.5. Computational Cost Comparison Plot for various Filter Structures for Example 1

It can be seen in Table.5, that MDT achieves 78.71%, 73.85% and 51.89% reduction in cost with above specifications (Table.3), when MDT IIR Down Sampler is compared with FIR, conventional IIR and polyphase IIR filters, respectively. The variation of computational costs of all the filters with respect to Decimation Factor has been shown in Fig.5.

Example 2: *The specifications of filter for Example 2 have been given in Table.6.*

Table.6. Specifications of Decimated IIR Filter for Example 2

Specifications	Values
Sampling Frequency (f_s)	100 samples/sec
Lower Cut-off Frequency	45 Hz
Upper Cut-off Frequency	55 Hz
Pass-Band Ripple (α_p)	0.01 dB
Stop-Band Ripple (α_s)	80 dB

In order to obtain the output of filter, filter frequency must fulfill the following conditions:

$$f_{PB} = \frac{45 f_s}{M} \tag{15}$$

and,

$$f_{SB} = \frac{55 f_s}{M} \tag{16}$$

The Table.7 shows that in this case also, the Elliptic filter achieves the lowest filter order among all other filters for the same Decimation Factor; *M*. So, from Simulation Examples 1 and 2, it is clear that the Elliptic filter design is used where the computational cost is more concerned.

The computational cost for FIR, conventional IIR, polyphase IIR and MDT filters using Table.2 is given in Table.8.

The variation of computational costs for various filters with Decimation Factor; *M* have been shown in Fig.7 for Example 2, where the MDT filter has lowest cost in terms of multipliers per output sample, when it is compared with other structures such as FIR, conventional IIR and polyphase IIR filters.

From Table.8, it can be seen that MDT filter achieves 87.56%, 73.56% and 50.83% of reduction in cost with above specifications (Table.6), when it is compared with FIR, conventional IIR and polyphase IIR filters structures, respectively. This study shows that the obtained cost reduction can be increased while increasing the value of Decimation Factor; *M*.

Table.7. Minimum filter order for various filters with different Decimation Factor *M* using MATLAB for Example 2

Filter / M	Equiripple FIR	Butterworth Filter	Chebyshev-1 Filter	Chebyshev-2 Filter	Elliptic Filter
2	48	14	8	8	6
4	97	17	9	9	7
6	145	17	9	9	7
8	193	17	9	9	7
10	242	18	9	9	7
12	338	18	9	9	7
14	434	18	9	9	7
15	482	18	9	9	7

Table.8. Comparison of Computational Costs between various filter structures for Example 2

M	Order of		Computational Cost of				%Reduction in Cost of MDT when compared with		
	FIR	Elliptic IIR	FIR	Conventional IIR	PPD IIR	Transform IIR	FIR	Conventional IIR	PPD IIR
2	48	6	48	26	15	12	75	53.85	20
4	97	7	97	60	33	21	78.35	65	36.36
6	145	7	145	90	49	28	80.69	68.89	42.86
8	193	7	192	120	65	35	81.87	70.83	46.15
10	242	7	242	150	81	42	82.64	72.00	48.15
12	338	7	338	180	97	49	85.50	72.78	49.48
14	434	7	434	210	113	56	87.10	73.33	50.44
15	482	7	486	225	121	59.5	87.66	73.56	50.83

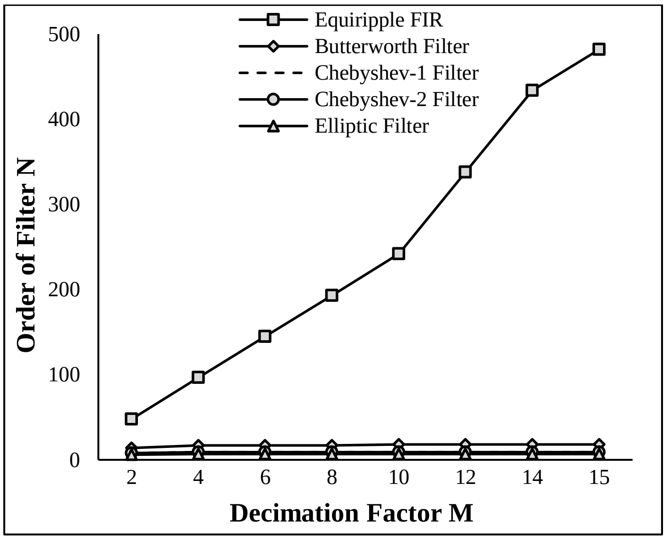


Fig.6. Filter Order for various Filters with Different Decimation Factor for Example 2

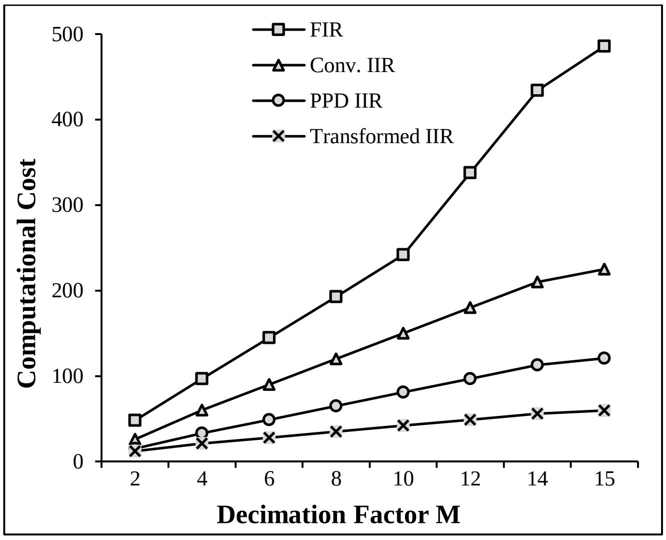


Fig.7. Filter Order for various Filters with Different Decimation Factor for Example 2

4. EFFECT ON STABILITY DUE TO MDT-IIR DOWN SAMPLER FILTER

Stability testing is an essential part for designing any system before its practical use. Stability of the system is affected by operating conditions and environments. An important condition for stability of system is Bounded Input Bounded Output (BIBO) [25]. While considering this condition, if a small input is given to the system, the output of the system will be a constant value or zero or at least bounded in any finite limit.

Let a system has bounded input $x[n]$, then following terminologies are essential for stability of the system:

- A system must be followed by BIBO condition.
- System Impulse Response $h[n]$ must be absolutely summable such as;

$$\sum_{k=-\infty}^{\infty}|h(k)|<\infty \tag{17}$$

- Coverage region of $H(z)$ must include $|z| < 1$
- All the poles of the system Transfer Function must have negative real coefficients.

As given in Eq.(17), sum of all output impulse responses of the system is always less than infinite or have bounded value, if applied input also have bounded value. Therefore, it is named as Bounded Input Bounded Output [25]. In most of the systems, it is not possible to calculate the infinite sum. Therefore, the sum S_k for the stability can be calculated by,

$$S_k=\sum_{k=0}^{N-1}|h(k)|<\infty. \tag{18}$$

Since in IIR filter, if the magnitude response and phase response are in consideration, therefore stability is an important issue to design a filter because the output impulse response of IIR filter always extends towards infinite so that a Bounded Input Bounded Output condition is not applicable for IIR filters. Due to this, an alternative method is used to find the stability of IIR filter. In this method, stability is estimated on the basis of the location of poles in z-plane.

For N^{th} order stable Infinite Impulse Response System Function, all the poles must lie inside the unit circle of z-plane [26]. The stability region in the z-plane is shown in Fig.4.

4.1 STABILITY OF 1st ORDER MDT-IIR FILTER

The Eq.(4) can be rewritten as:

$$H(z)=\frac{1}{1-a^M z^{-M}}\left[b+abz^{-1}+ab^2z^{-2}\cdots ab^{M-1}z^{-(M-1)}\right] \tag{19}$$

In Eq.(19), the polynomial has $(M-1)$ number of zeros and poles and all the poles lie within a circle of radius a, which is centered at origin. The values of pole-zero for 1st order MDT-IIR filter with $M = 6$ are given in Table.9.

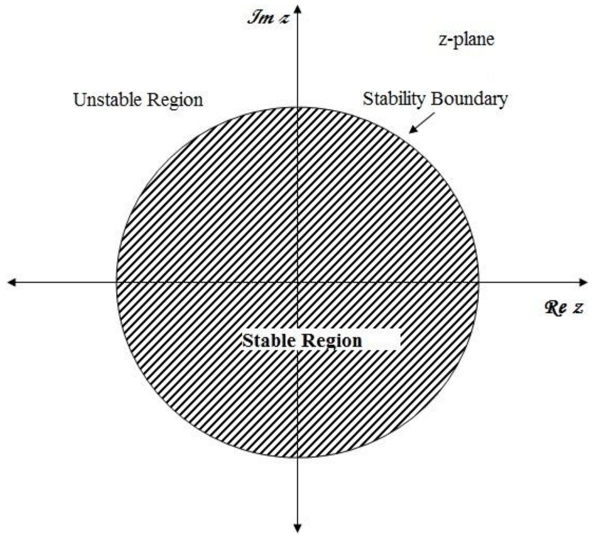


Fig.8. Stability Region in z-plane

Table.9. Values of zeros and poles with and without Merged Delay Transformation for $N = 1$ and $M = 6$

Filters	Zeros	Poles
Conventional Filter	-1	0.5774
MDT Filter	0	-0.5774
	-0.2887 + j0.5000	-0.2887 + j0.5000
	-0.2887 - j0.5000	-0.2887 - j0.5000
	-0.5774	0.5774
	0.2887 + j0.5000	0.2887 + j0.5000
	0.2887 - j0.5000	0.2887 - j0.5000

The stability of the 1st order filter can be given by the roots of the system and from Eq.(19), since all the poles of the system are always less than unity ($|a| < 1$). Therefore, Transfer Function of 1st order MDT filter always exhibits stability.

4.2 STABILITY OF 2nd ORDER MDT-IIR FILTER

The Eq.(5) can be written as:

$$H_{1R}(z)=\frac{Numerator}{1-2P_{MR}z^{-M}+\left(P_{MR}^2+P_{MI}^2\right)z^{-2M}}. \tag{20}$$

To ensure stability of 2nd order IIR Filter Transfer Function, the necessary conditions are as follows;

$$\left|\left(P_{MR}^2+P_{MI}^2\right)\right|<1 \tag{21}$$

and,

$$2P_{MR}<1+\left(P_{MR}^2+P_{MI}^2\right) \tag{22}$$

Since the coefficient a_1 and a_2 are the complex conjugate in nature therefore,

$$a_1 = a_{1R} + jb_{1I} \text{ and } a_2 = a_{2R} - jb_{2I}$$

$$|a_1|=|a_2|=\sqrt{a_{1R}^2+b_{1I}^2}<1.$$

Since, higher power of a_1 and a_2 are further away from unit circle [20], therefore, both the conditions for stability of the filter are justified. A higher order Transformed filter can be decomposed into 1st order and 2nd order sections so stability conditions can be satisfied for any order of MDT Recursive filter.

5. FINITE WORD LENGHT EFFECT ON STABILITY OF MDT-IIR DOWN SAMPLER FILTER USING COEFFICIENT QUANTIZATION

There are various methods to design a digital filter and it can be implemented by various structures. Since all the structures provide equal response, yet some of them are more sensitive towards Coefficient Quantization and suffer from the problem of Finite Word Length. Due to Finite Word Length Effect, actual coefficient values are differed from obtained values in the designing process.

The Coefficients as well as the Frequency Response of the system are changed due to the limitations of word length. Due to this fact, the output of digital filter deviates from the designed response. In Finite Impulse Response Filter, it affects only the Frequency Response but in case of Infinite Impulse Response filter, poles and zeros of Transfer Function are displaced from their original position. If the poles of the filter move outside of the unit circle in z-plane then it may lead the filter towards instability.

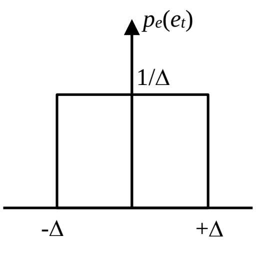
5.1 COEFFICIENT QUANTIZATION EFFECT

Generally, Quantization takes place in feedback path due to the multiplication of coefficients. When a number is added to the another number in a feedback loop then the output value of the system will increase rapidly until it would exceed the Fixed Word Length of the system. Thus, Quantization Process is used to prevent this problem.

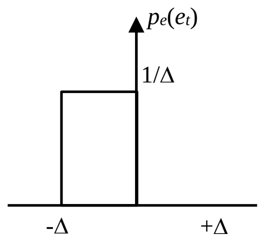
Fig.9. Quantization Process

The process of Quantization is shown in Fig.5. If x is an exact value in design process, the rounded value will be denoted by $Q_r(x)$, the truncated value is represented by $Q_t(x)$ [27] and the magnitude truncated value is $Q_{mt}(x)$ and if the length of word is fixed as $(b + 1)$ bits, then the quantized step size is given by:

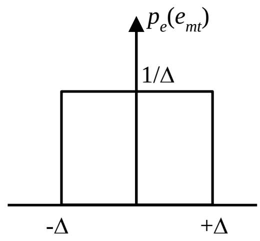
$$\Delta = 2^{-b}. \tag{23}$$



(a)



(b)



(c)

Fig.10. Probability Density Function for (a) Rounding Error (b) Truncation Error and (c) Magnitude Truncation Error

The rounding selects the value which is far away from the original value by the factor of ±Δ/2 so, the quantized value is always near to the un-quantized value. Therefore, the rounding error e_r. can be represented as:

$$e_r = Q_r(x) - x \tag{24}$$

then

$$+\frac{\Delta}{2} < e_r < -\frac{\Delta}{2}. \tag{25}$$

In Truncation, lower order bits will be discarded. The quantized value from truncation is always less than and equal to the original value. So the truncation error e_t lies between

$$-\Delta < e_t < 0. \tag{26}$$

The magnitude truncation selects the value which has Magnitude Response less than and equal to original value. So the magnitude truncation error e_{mt}. can be expressed as follows:

$$-\Delta < e_{mt} < \Delta. \tag{27}$$

By using Eq.(25) to Eq.(27), the graphical representation of Quantization Errors is shown in Fig.6. [28]

The noise power of the system is given by,

$$\sigma^2 = \frac{\Delta^2}{12}. \tag{28}$$

In MDT technique, to implement the higher order filter, the Transfer Function of the filter is decomposed into 1st order and 2nd order sections with complex conjugate poles. Since, the memory has Finite Word Length to store data so some number of bits is either rounded or truncated but in MDT, due to complex conjugate pairs of poles, the error in one pole is independent to the other poles of the filter.

6. SIMULATION RESULTS AND DISCUSSIONS

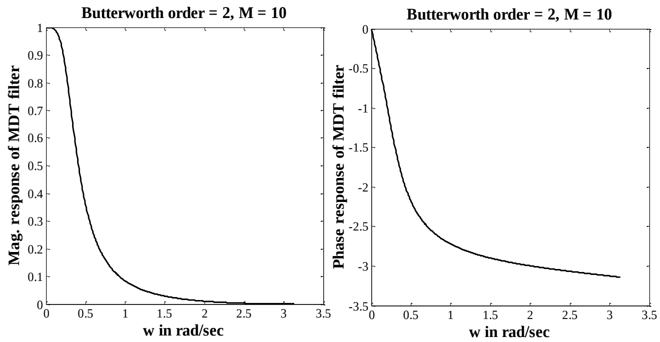
The proposed scheme is simulated with and without MDT using MATLAB for all types of convectional filters such as

Butterworth, Chebyshev-1, Chebyshev-2 and Elliptic IIR filters. The following results have been observed:

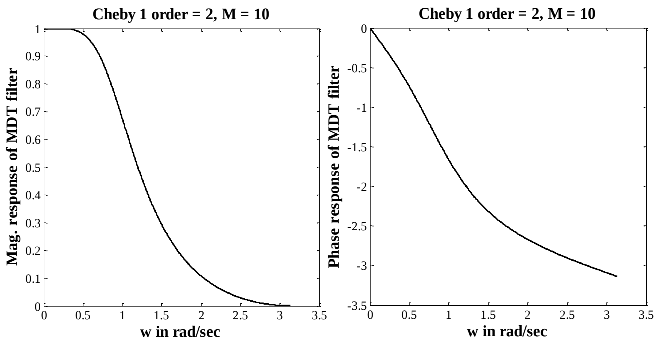
6.1 FREQUENCY RESPONSE OF MDT DOWN SAMPLER FILTERS AT FIXED DECIMATION FACTOR *M* AND FIXED FILTER ORDER *N*

By using a random input sample, the Magnitude Response and Phase Response of the 2nd order MDT- IIR Down Sampler filter is obtained. To design a filter, following specifications have been used: *Normalized cutoff frequency* = $1/M$, *Decimation Factor* $M = 10$, *filter order* $N = 2$, *pass-band ripple* = 0.01 *dB and stop-band attenuation* = 50 *dB* [23].

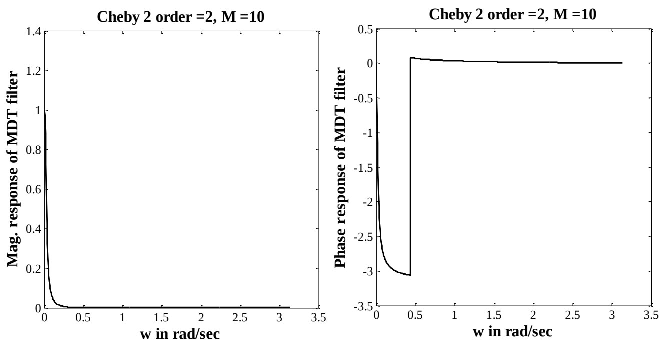
Magnitude Response and Phase Response of 2nd *order* Infinite Impulse Response Down Sampler filter with and without MDT for all types of filters are shown in Fig.11.



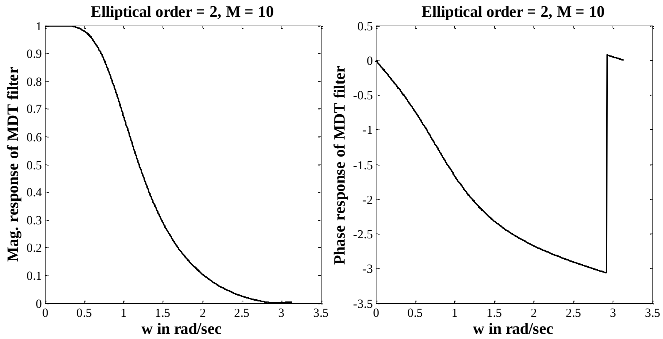
(a)



(b)



(c)



(d)

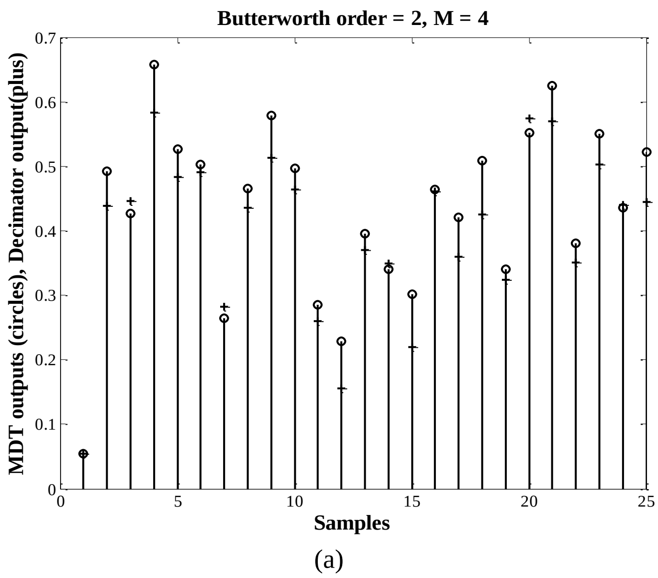
Fig.11. Magnitude and Phase Response of *2nd* Order Merged Delay Transformed IIR Down Sampler Filter with Decimation Factor $M = 10$ (a) Butterworth Filter (b) Chebyshev-1 Filter (c) Chebyshev-2 Filter and (d) Elliptic Filter

6.2 COMPARISON OF OUTPUT SAMPLES FROM MERGED DELAY TRANSFORMED IIR DECIMATED FILTER AND CONVENTIONAL IIR DECIMATED FILTER

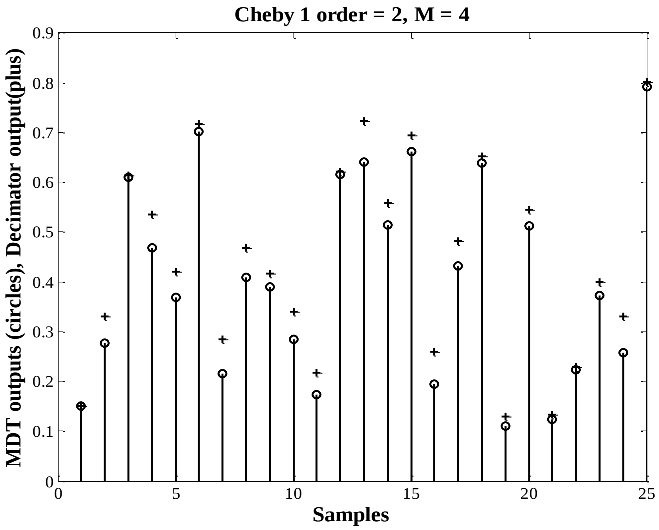
In the MDT, a 2nd order MDT filter is obtained by combining two 1st order parallel sections which have complex conjugate coefficients. Due to complex conjugate property, real parts of output have equal magnitude whereas imaginary parts have equal magnitude but opposite in sign.

In order to find the correctness of the MDT filters, all types of filter are simulated with $N = 2$ and $M = 8$ and output samples of filter with and without MDT are shown within a single plot where a sign of circle represents the output sampled value of MDT decimated filter and a sign of plus represents a output sampled value of conventional decimated IIR filter. Output samples of *2nd* order IIR filters are shown in Fig.12.

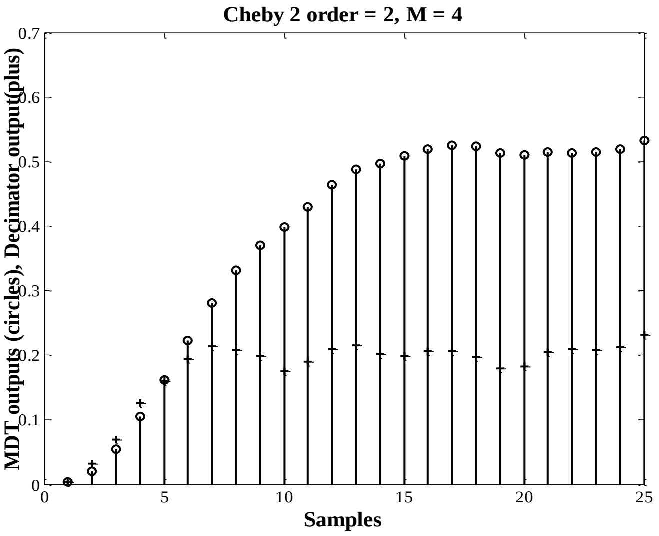
From Fig.12, it is observed that the output samples of Transformed filter for all types of Filters are almost having similar value with conventional IIR decimated filter.



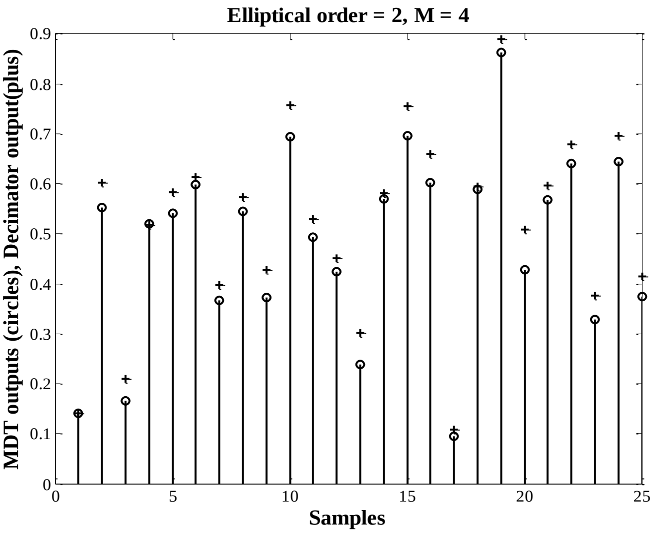
(a)



(b)



(c)

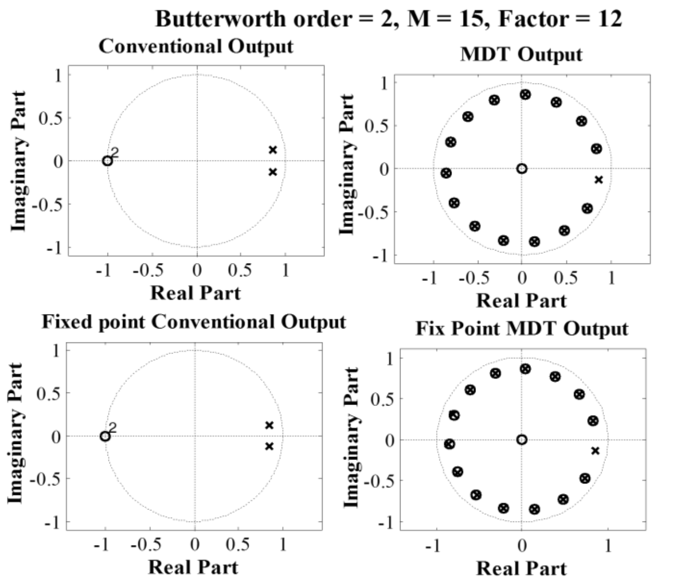


(d)

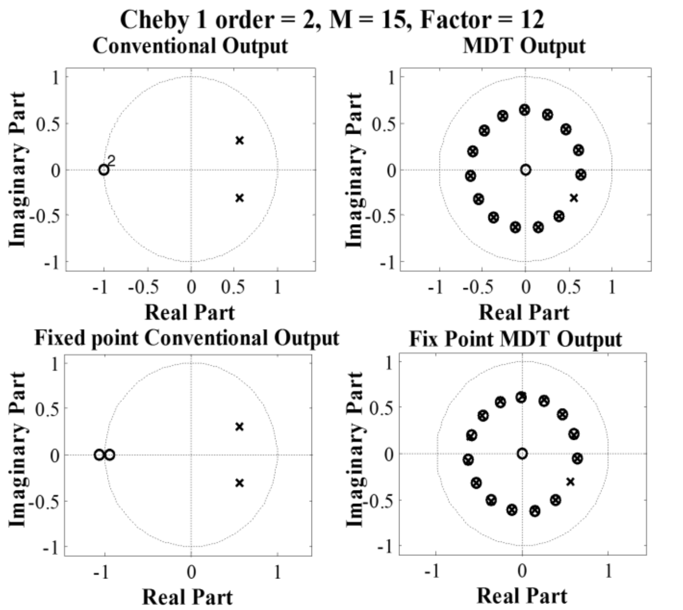
Fig.12. Sampled Output of Merged Delay Transformed Filter and Original Decimated Fitter at $N = 2$ and $M = 4$ (a) Butterworth Filter (b) Chebyshev-1 Filter (c) Chebyshev-2 Filter and (d) Elliptic Filter

6.3 COMPARISON OF STABILITY OF CONVENTIONAL IIR FILTER AND MDT-IIR DECIMATED FILTER

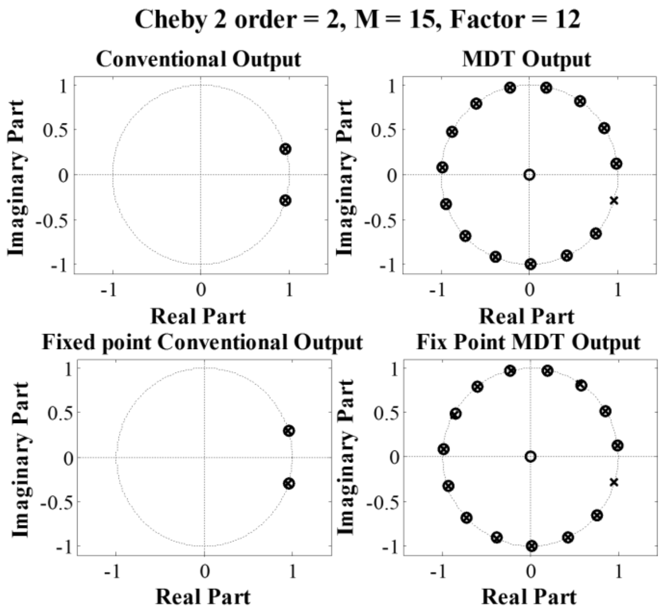
With $N = 2$ and $M = 10$, pole-zero plots of all filters have been simulated and shown in Fig.13. Pole-zero plots of conventional filters are shown in left part of Fig.13, whereas, plots of Merged Delay Transformed filters are shown in right part of Fig.13.



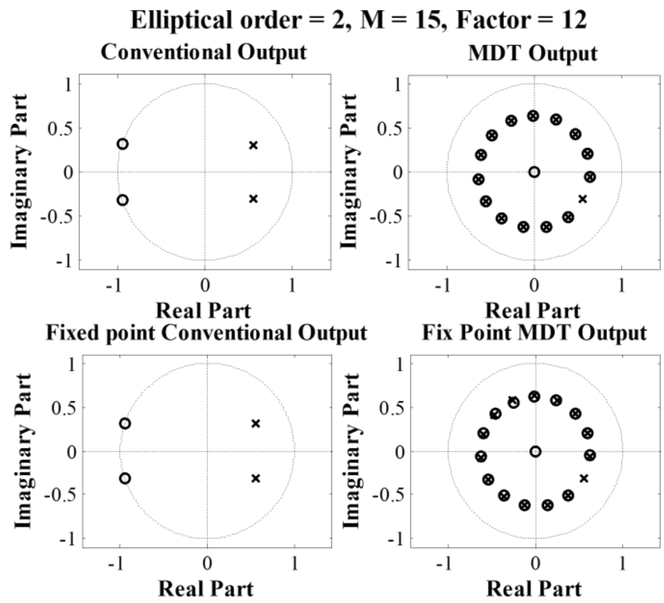
(a)



(b)



(c)



(d)

Fig.13. Pole-Zero Plots for 2nd Order Conventional and Merged Delay Transformed Filter at M = 15 (a) Butterworth Filter (b) Chebyshev-1 Filter (c) Chebyshev-2 Filter and (d) Elliptical Filter

From the simulation results, it can be observed that the Transformed Elliptic filter is more stable because the poles and zeros of the filter are very near to the origin of the circle and the Transformed Chebyshev-2 filter is marginally stable filter because poles and zeros lie on the unit circle.

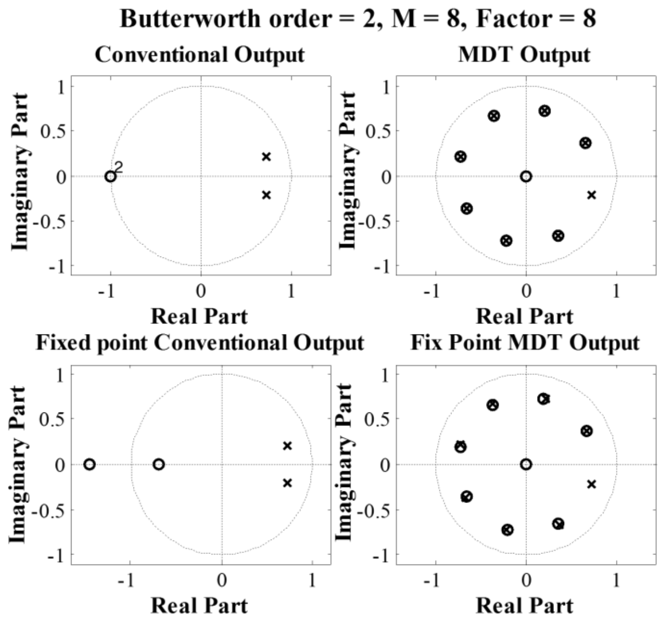
From Fig.13, it can be seen that the order of the stability of the Transformed filters in descending order is Elliptic filter > Chebyshev-1 filter > Butterworth filter > Chebyshev-2 filter. It has also been observed that the MDT introduces (M – 1) number of additional poles and equal numbers of zeros at concurrent positions within the unit circle.

6.4 COMPARISON OF FINITE WORD LENGTH AND QUANTIZATION ON THE STABILITY OF MDT-IIR DOWN SAMPLER FILTER

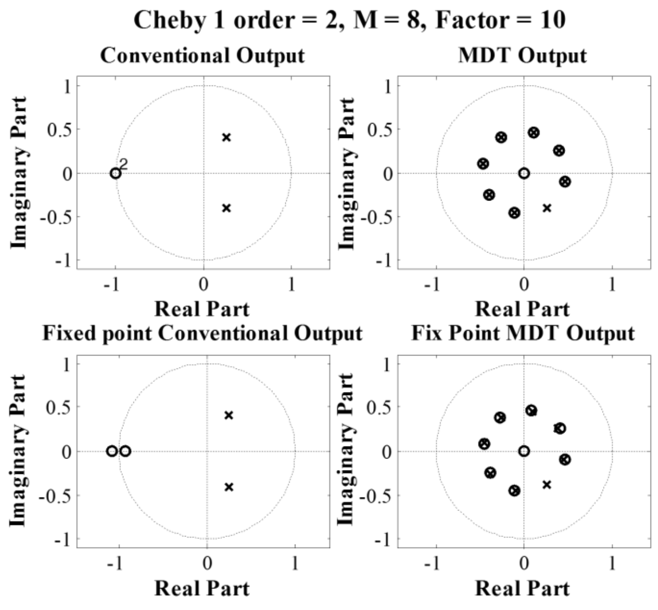
With N = 2 and M = 15, all the filters have been simulated for various bit length as shown in Fig.14. The results of [29] observed from pole-zero plots have been tabulated in Table.10.

Table.10. Fixed Word Length for filters

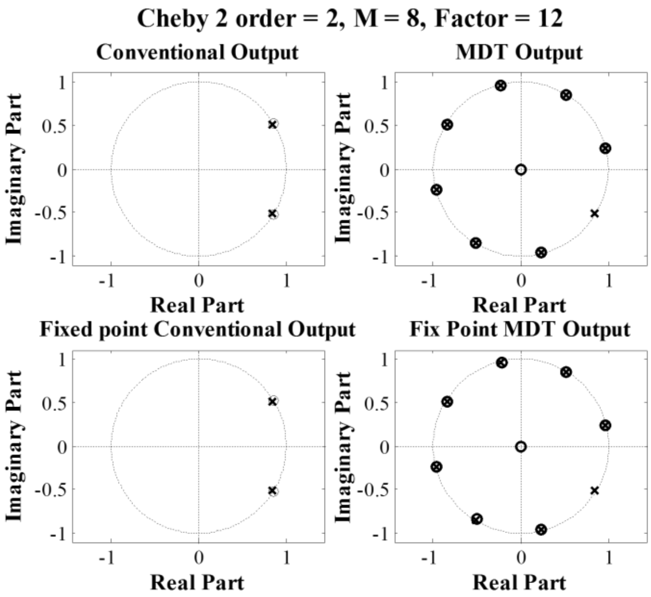
Filter	Minimum Number of Bit Length Required
Butterworth Filter	8
Chebyshev-1 Filter	10
Chebyshev-2 Filter	12
Elliptic Filter	14



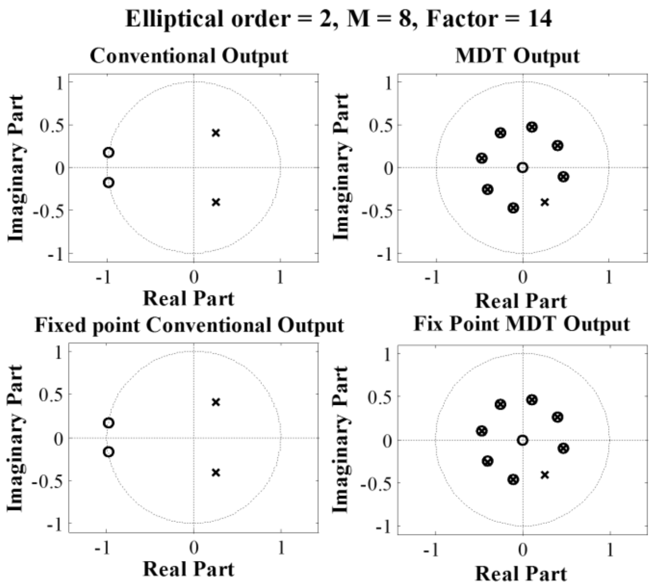
(a)



(b)



(c)



(d)

Fig.14. Pole-Zero plots with Fixed Word Length for N = 2 and M = 8 (a) Butterworth Filter (b) Chebyshev-1 Filter (c) Chebyshev-2 Filter and (d) Elliptical Filter

In Fig.13, results are observed from pole-zero plots of above mentioned filters and the observations show that there is a variation in minimum numbers of required bit length for above filters for providing the satisfactory results. From Table.10, it is observed that the number of minimum bits required to avoid truncation or overflow is minimum in case of MDT Butterworth filter whereas maximum in case of MDT Elliptic filter.

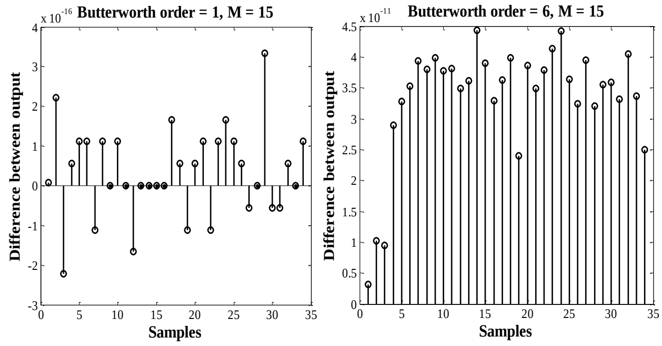
6.5 PEAK ERROR DUE TO MDT-IIR DOWN SAMPLER FILTER

The input signal is a sequence of random numbers. Difference between the output samples from original filter and MDT filter is evaluated using MATLAB as shown in Fig.15. Simulations are carried out with lowest filter order N = 1 and highest filter order N = 6 with Decimation Factor M = 15.

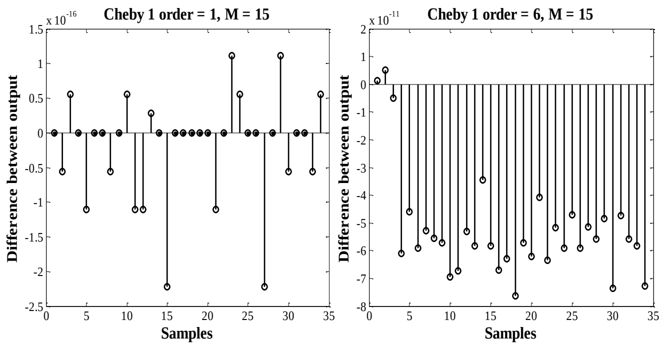
From the simulation results, it is observed that the peak error is in the range of 10^{-10} to 10^{-17}. Output of the simulations can be varied with different values of Decimation Factor; M and filter order N. Peak errors for different filters are given in Table.11.

Table.11. Peak Error at Filter Order N = 1 and Filter Order N = 6 at M = 15

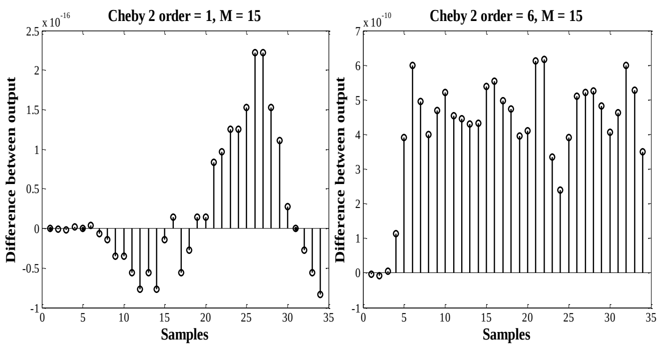
Filter	Peak Error at N = 1	Peak Error at N = 6
Butterworth Filter	10^{-16}	10^{-12}
Chebyshev-1 Filter	10^{-17}	10^{-10}
Chebyshev-2 Filter	10^{-17}	10^{-12}
Elliptic Filter	10^{-16}	10^{-12}



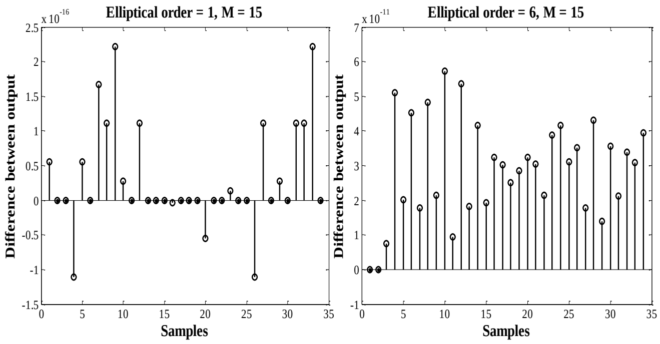
(a)



(b)



(c)



(d)

Fig.15. Difference between Output of Original Filter and Transformed Filter at N = 1 and N = 6 for M = 15

7. CONCLUSIONS

An efficient architecture and cost reduction technique to implement the Decimation IIR filter is introduced and the scheme is simulated and compared for various filters such as; Butterworth, Chebyshev-1, Chebyshev-2 and Elliptic filters. The comparative study shows that using MDT technique in order to implement the Decimated IIR filter, filtering and Sampling Rate Converter can be implemented in a single stage instead of separate stages. This in turn provides lower complexity and less multipliers per output sample when it is compared with the conventional filter. The proposed work shows that there is reduction in computational cost of MDT-IIR filter when compared with FIR, conventional IIR and polyphase IIR filter and it is about 83.56%, 73.56% and 50.83% & 78.71%, 73.85% and 51.89%, respectively with different filter specifications. The computational cost increases with increase in M. The results show that Frequency Responses of MDT filters are same as existing conventional filters for various Decimation Factor and filter order therefore it provides the good agreement with conventional filter. This work elaborates that the MDT filter maintains the stability of the filter and it does not make the system unstable. The simulation results show that Transformed Elliptic filter is more stable and Transformed Chebyshev-2 filter is marginally stable. By the proposed study, it can also be observed that in MDT-IIR filter, the additional zeros have the concurrent position with the additional poles if bits are not truncated or overflow so Frequency Response (Magnitude as well as Phase Response) will not alter. The simulation results show that MDT-Butterworth filter provides more satisfactory result in terms of minimum required bit length whereas MDT- Elliptic filter provides less satisfactory result. The difference between conventional output samples and Transformed output samples have also been calculated in order to find the correctness of MDT. Simulation results show that peak error is in the range of 10^{-10} to 10^{-17} for N = 1 to 6 at Decimation Factor M = 15.

It can also be concluded that the decomposition of higher order system into 1st order and 2nd order systems using parallel implementation architecture with MDT increases the processing rate of filter, reduces the cost of implementation as well as peak error without disturbing the stability issues.

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