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Synchronization of symbols as the construction of times and places

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Abstract

We demonstrate an unsuspected freedom in physics, by showing an essential unpredictability in the relation between the behavior of clocks on the workbench and explanations of that behavior written in symbols on the blackboard. In theory, time and space are defined by clocks synchronized as specified by relations among clock readings at the transmission and reception of light signals; however spacetime curvature implies obstacles to this synchronization. Recognizing the need to handle bits and other symbols in both theory and experiment, we offer a novel theory of symbol handling, centered on a kind of ‘logical synchronization’, distinct from the synchronization defined by Einstein in special relativity.

We present three things:

1. We show a need in physics, stemming from general relativity, for physicists to make choices about what clocks to synchronize with what other clocks.
2. To exploit the capacity to make choices of synchronization, we provide a theory in which to express timing relations between transmitted symbols and the clock readings of the agent that receives them, without relying on any global concept of ‘time’. Dispensing with a global time variable is a marked departure from current practice.
3. The recognition of unpredictability calls for more attention to behavior on the workbench of experiment relative to what can be predicted on the blackboard. As a prime example, we report on the ‘horse race’ situation of an agent measuring the order of arrival of two symbols, to show how order determinations depart from any possible assignment of values of a time variable.

Keywords: synchronization, agent, symbol, clock, balance, flip-flop, unpredictability

(Some figures may appear in colour only in the online journal)

1. Introduction

This work began with a vision of an essential unpredictability in physics, beyond quantum uncertainty. A milestone in our pursuit of this vision was our 2005 proof that in quantum physics, whatever evidence may be on hand leaves open choices of explanations of that evidence. That proof depends only on the rule (the Born trace rule) that expresses evidence as probabilities of outcomes and expresses explanations by

density operators paired with measurement operators. Thus, resting only on the basic structure of quantum theory, we find that the writing of a density operator or a wave function to explain evidence involves an element of choice undetermined by any application of logic to measured data. As discussed in [1–3], the proof is made possible by noticing that physicists communicate in symbols, such as the numerals and the letters of the alphabet on the page before you. The proof and the recognition of the dependence of physics on communicating sequences of symbols makes an opening in physics for agents and their symbols. The opening demands departing from the prevalent habit of theorists by respecting experimental behavior on the workbench as conceptually distinct from any



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of its theoretical expressions on the blackboard, leading to unexpected freedom in the construction of times and places.

The next step in pursuing the vision is to pose the right question. Our first try was

How does physics change if we recognize that its equations are written by people who have choices undetermined by physical evidence?

This question presents a stumbling block: the equations, once written, look the same, regardless whether they reflect a person's free choice or not. We find a more promising question as

Does essential unpredictability show up in material behavior, for example in the behavior of clocks, in a way that warrants theoretical attention?

To face the question, we have go back to basics. Scientists try to picture how the world works. They build on what has gone before. Their experiments and theories co-evolve in a context of hitherto unappreciated unpredictability.

The history of science over the past centuries cycles between unifications and fragmentation. Here we discuss how several fragments can now jell into an unexpected unity, based on recognizing that: (1) laws of physics do not write themselves, but are products of an evolving species of organisms, namely people, and (2) discrepancies among clocks as devices on the work bench call for more extensive description than can be expressed by 'uncertainty'. Together these mean that the theories and experimental procedures promoted by physicists contend among themselves for a place in cultural evolution. Support for this point of view will emerge as we proceed.

The great unification in physics came half a millennium ago, with the invention of the pendulum clock and the telescope on the experimental side and of the mathematical formulation of derivatives and integrals—the calculus—of Newton and Leibniz, on the theoretical side. A great unification sometimes requires, however, the *inattention* to certain realities that might embarrass it. Newton saw the diversity of rhythms that can be compared with one another not as the rich source of physics that we shall find it to be, but as an embarrassment to be resolved by leading his readers away from the diverse and independent pendulums they can see into an abstraction (in Newton's words):

Although time, space, place, and motion are very familiar to everyone, it must be noted that these quantities are popularly conceived solely with reference to the objects of the sense perception. And this is the source of certain preconceptions; to eliminate them it is useful to distinguish these quantities into absolute and relative, true and apparent, mathematical and common.

1. Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. Relative, apparent, and common time is any sensible and external measure (exact or nonuniform) of duration by means of motion; such a measure—for example, an hour, a day, a month, a year—is commonly used instead of true time [4].

We take objection to Newton's picture, and are by no means the first to object to it. We aim to respect clocks on the experimental work bench as having contributions to make distinct from what can be asked of mathematical formulations of 'time'; indeed we view the purpose of clocks as 'telling time' as a secondary purpose, by no means a defining purpose.

Einstein broke away from Newton's concepts of time and of space, but kept more of them than one might think. Although Einstein made the special relativistic concept of time depend on 'clocks', these are not clocks on the workbench but *proper clocks*, which are just as mathematical and abstract as Newton's mathematical time. The special-relativistic 'time' defined by the use of (idealized) light signals to synchronize proper clocks was relative to a choice of frame (thus 'relativised') but this 'time' inherits from the mathematical tradition of Newton the suppression of the diversity of rhythms on the workbench.

That would not matter if all the bench rhythms could all be related to some standard rhythm in any simple, 'objective' way, but that is not the case. The best we have are the time broadcasts supplied via the Global Navigation Systems, the internet and cell phones. Time broadcasts involve National Metrology Institutes (NMIs). There is not a single clock for for the world's time; each NMI has several clocks, and these drift apart, so that the NMI nudges the clock rates to keep them from excessive drift, both within an individual NMI and in the relations between an NMI and clock reading transmitted to one NMI from another NMI. This is no 'objective' business but a matter of intense negotiation. As one of the experts puts it:

The fact is that time as we now generate it is dependent upon defined origins, a defined resonance in the cesium atom, interrogating electronics, induced biases, timescale algorithms, and random perturbations from the ideal. Hence, at a significant level, time—as man generates it by the best means available to him—is an artifact. Corollaries to this are that every clock disagrees with every other clock essentially always, and no clock keeps ideal or 'true' time in an abstract sense except as we may choose to define it [5].

So much for the gap between relativity theory and the implementation of time broadcasts. Returning to general relativity, the very simplification of assuming proper clocks leads to a shocking consequence. Einstein's theory of proper clocks, once he extended it into curved spacetime, challenges its own conception of 'time' in a way that, curiously, supports recognizing choice and agency in the organization of even mathematically expressed proper clocks. But why should non-experts care about irregularities that generally affect 'time on Earth' only through its partition into time zones and in the technology of the Global Navigation Systems that are adjusted on the scale of nanoseconds to accommodate spacetime curvature? Even for non-experts, appreciating the collision of today's physical theory of time with itself presents an opportunity think about some interesting situations free of prevalent conceptual errors. Section 2, backed up by the appendix, tells this story.

In the century after Einstein's theories of clocks in special and general relativity came the digital communications

revolution, along with startling improvements in physical clocks, first atomic clocks and more recently optical atomic clocks. But a curious invariance showed up along the stages of improvements in clock precision: getting any two clocks to tick as close together as their evolving technology allows, requires steering their clock rates. Arranging for any pair of clocks to agree as closely as possible continues to require, in effect, that agents adjust the ‘pendulums’ of their clocks in response to unpredictable discrepancies between the clocks.

The business of the computer networks and other digital networks that pervade modern scientific life is to manipulate and communicate symbols. Symbols include numerals that convey readings of clocks that step computers through their cycles of operation. The clock of a computer is not primarily needed to ‘tell time’ but to tell the computer when to step. A main purpose of this paper is to introduce the concept of a clock as a tool of a symbol-handling agent, primarily a tool for managing relations between chosen rhythms in the face of unpredictable effects, and only secondarily as a tool for ‘telling time’.

While viewing clocks as tools of agents has potential advantages, realizing that potential requires a novel conceptual framework in which to think about and measure one rhythm in relation to another, without assuming any globally available ‘time’. In earlier work we introduced a mathematical form for expressing relations among biological rhythms [6]. In section 3 we repeat, with only minor changes, this mathematical form, no longer confined to applications in biology, to make it available for its relevance to fundamental and applied physics. The basic ideas of the mathematical form are (1) the notion of an agent that handles symbols sequentially, one after another, and records the symbols handled on what we call a *clock tape*, and (2) the relation, called a *transmission relation* to express how symbols received by an agent fit into the sequence on the agent’s clock tape, as illustrated in figure 1(a). For present theoretical purposes, a network is a set of agents linked by transmission relations. This theoretical formulation is applicable to networks in a large variety of situations and levels of detail of description, from the world wide web to biochemical networks within a bacterium. A major novelty introduced in this paper is the means to express communication networks without reference to any globally available ‘time coordinate’. Transmission relations serve to express the timing aspect of communications among agents of a network, without requiring any particular assumption of how symbols are propagated; neither a metric nor indeed any spacetime manifold need be assumed. These transmission relations offer a conceptual foundation for constructing ‘times’ and ‘places’.

The freedom to explore the construction of times and places stands in marked contrast to prevalent designs for synchronizing digital sensor networks—designs that approximately implement ‘time’ as defined in special relativity. An example of prevalent designs is the synchronization of the global network of eight radio telescopes that produced the recent picture of a black hole [7]. Another example is the synchronization of dispersed detecting devices in the Compact Muon Solenoid (CMS) [8]. A third example is an undersea network of sensors for which GPS signals are unavailable, in which synchronization is implemented using the Precision Time Protocol

(IEEE 1588) [9]. A more recent example is in [10]. We make no claim to improve the efficiency by which synchronization is managed in these designs. Rather, we offer an alternative approach to synchronization, previously unappreciated, that opens up novel avenues to investigation. The avenues we have thought of so far center on transmission relations involved in biological organisms, where different rhythms come into and drop out of synchronization with other such rhythms, as discussed in [6].

Computer to computer communications offer another lesson from the workbench that warrants theoretical attention: communicating digits from one computer to another requires relations among clocks quite distinct from those defined in special relativity, having to do with phasing of digit arrivals relative to the clock that steps a receiving computer. Section 4 describes the need for agents to adjust the rates of their clocks so that symbols arrive during a suitable phase of the agent’s clock, the condition of *logical synchronization*. Maintaining logical synchronization requires that agents respond to timing gradations beyond the reach of machinery used to recognize distinct symbols, a finding well known to engineers of digital hardware, but deserving more attention in theoretical physics.

Section 5 deals with what might be called the ‘the machinery of logic in motion’. Critical to machinery for symbol manipulation is a tiny balancing device called a *flip-flop*. Occurring by the millions on the silicon chips of digital systems, the flip-flop stores an elementary symbol—a single bit. The flip-flop works like a hinge that, flipped one way, shows a 1, or if flopped the other way, shows a 0. We discuss the flip-flop as a balancing device that not only holds an elementary logical value, but moves in response to changes in that value. In a computer, a flip-flop decides on the temporal order of a clock tick and a symbol arrival: did the symbol arrive before or after the tick? In a close race between the clock tick and the symbol, the flip-flop can be tipped into an unstable equilibrium, a condition that leads to *logical confusion* more complex than anything expressed by ‘measurement uncertainty’. The experimental demonstration of this logical confusion, illustrated by figures 6 and 7, prompts us to see clocks and their management as a topic on its own, separable from what now strikes us as the problematical concept of ‘global time’. Concluding remarks occupy section 6.

2. Agency and the theory of time and length

Much of this paper, especially section 4, is concerned with introducing the concept of *logical synchronization*, but current theoretical physics hinges on a quite different form of synchronization, defined by Einstein in special relativity, that we refer to as *Einstein synchronization*. Here we show how, in the curved spacetime of general relativity, Einstein synchronization encounters obstacles, in a way that makes an opening in theoretical physics for agents that make choices beyond the reach of logic. In this section we sketch the story, relegating its justification to the appendix.

From Einstein, theoretical physics inherits not just one but two theories of time, space, and spacetime. Special relativity

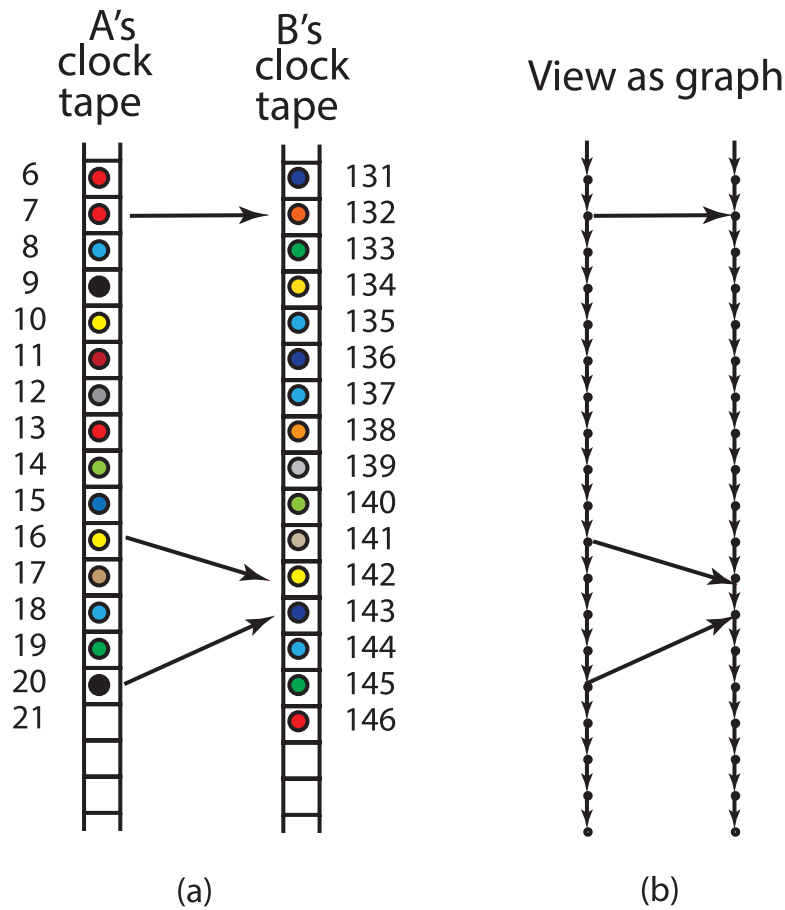


Figure 1. (a) Transmission from A to B from [6, 12]. (b) Graph showing transmission relation from [6, 12]. Reproduced with permission from [12]. © (2019) COPYRIGHT Society of Photo-Optical Instrumentation Engineers (SPIE).

postulates inertial frames as free of acceleration and of gravitational influences. Then ‘time’ and ‘length’ are elegantly defined, relative to a choice of inertial frame, in terms of the Einstein synchronization of proper clocks. Einstein synchronization is defined as a condition on readings of proper clocks when they transmit and receive (theoretical) light signals. One can picture the time coordinate relative to an inertial frame as made available by an infinitely fine, three-dimensional grid of Einstein-synchronized proper clocks, so that every event coincides with a unique tick of a unique clock of the grid.

The special-relativistic definitions of time and length in terms of Einstein-synchronized proper clocks are the theoretical basis of the units of measurement for time and length in the International System (SI). But corrections are needed. To deal with acceleration and gravitation, Einstein made special relativity hold only in vanishingly small spacetime regions of globally curved spacetime. The curvature of spacetime proved not just to be theoretically attractive, for example in astrophysics, but modern navigation systems, such as the global positioning system (GPS), depend on the theory of light signals and clocks articulated in the theory of general relativity.

In the theory of curved spacetime, there can be no inertial frame and no infinitely fine grid of Einstein-synchronized proper clocks by which to define time and length. One still

has the notion, discussed in the appendix, of *observer fields*, any of which is a set of not-necessarily proper clocks so that every event coincides with a unique reading of a unique clock of the observer field. The notion of an observer field allows for theoretical clocks that are *improper* in the sense of generating readings at a rate that varies relative to co-present proper clocks. But even allowing for an observer field of improper clocks, curvature presents an obstacle to having the clocks of an observer field be Einstein synchronized with one another. An observer field can be chosen such that subsets of a few of its clocks can be Einstein-synchronized with one another, but that choice precludes other choices that would Einstein synchronize other small sets of clocks.

Proposition 1. *For a generic curved spacetime, Einstein synchronization can be achieved, even with clock adjustment allowed, only for selected pairs of clocks; that is, the selection of some pairs of clocks to be synchronized excludes Einstein synchronization among other pairs of clocks.*

The requirement to select which clocks to Einstein synchronize with which other clocks raises the question of who or what does the selecting, leading us to the notion of an *agent*. That requirement is also a hint that times are necessarily local times, where by *local* we mean dependent on choices made by agents.

3. Symbol-handling agents

In this section we offer a theory of symbol handling by which to express relations among symbols communicated among clock-using agents, relations that constitute a system of times and places adapted to their communication. The type of synchronization required for agents to communicate is the topic of the following section 4.

As the term is used here, an *agent* has a ‘local clock’ consisting of a cyclic motion, e.g. a swinging pendulum that the agent can adjust, along with the means to count cycles. The count is a ‘local time’. Only in special cases, however, is the clock of one agent Einstein-synchronized to the clock of any other agent. Thus, in general, the agent has available no ‘time’ as defined in special relativity. In step with the ticks of its clock, the agent deals with symbols sequentially. We consider agents that communicate symbols among themselves, as well as to and from an environment, in rhythms set by their (adjustable) clocks. For such agents we offer a mathematical framework for expressing (1) the record of the sequence of symbols that an agent has dealt with, and (2) the timing of symbol exchange among agents. Agents linked in a communications network can work at very different clock rates, and the framework offered needs no assumption of a global time coordinate, nor of spacetime.

As its adjustable clock ticks, an agent executes moves, one after another, each move involving a symbol. The adjustable clock drives a tape, which we call a *clock tape*, reminiscent of the tape of a Turing machine [11]. (We drop the assumption, made in our prior work [2], that agents have the capability of a universal Turing machine.) If one does think of a Turing machine with its infinite tape, then the clock tape is an additional ‘write-only’ tape. The agent, as we now think of it, has a memory, separate from the clock tape, that holds strings of symbols, and the agent’s action can depend on symbols held in its memory. The symbol that an agent records on a square of its clock tape at a move might be read from its memory, written into its memory, received from another agent, transmitted to another agent, or emerge from contact with an unknown realm (which we associate with acts of guesswork [2], but will not discuss further here).

Like the tape of a Turing machine, the clock tape is pictured as marked off in squares, with only one square immediately visible to the agent at any move. As its clock ticks, the agent’s clock tape advances by one square, always in the same direction. (Unlike the tape of a Turing machine, the clock tape is *not* erasable.) By recording one symbol after another on the squares of the clock tape an agent converts its temporal sequence of symbols into a spatial sequence, like a film strip, amenable to mathematical expression.

3.1. Transmission relations

Based on the above picture, we offer a mathematically expressed theory of agents transmitting and receiving symbols. Applications of this theory to some simple, engineered digital networks is transparent, while other applications, for

example to cases of symbol transmission in biology, call for making assumptions tailored to the case.

We express the timing of the transmission of symbols from agent *A* to an agent *B* by a *transmission relation* comprised of ordered pairs, each pair consisting the square on *A*’s clock tape that records the transmitted symbol and the square of *B*’s clock tape that records the reception of the symbol transmitted by *A*, as illustrated in figure 1(a). Each arrow from *A* to *B* indicates an ordered pair. We can label successive squares of *A*’s clock tape by successive integers and do the same for *B*’s clock tape, so that integers serve as names for squares. For example the arrow from square 7 of *A*’s clock tape to square 132 of *B*’s clock tape expresses the ordered pair (7|132), indicating that a symbol on square 7 of *A*’s clock tape was transmitted to *B* and recorded as received on square 132 of *B*’s clock tape. This labeling by integers is ‘local’ in that one integer larger than another on *A*’s tape means one square recorded later than another, but an integer larger on an *A*-tape than an integer on a *B*-tape says nothing about temporal order of the squares on those two distinct tapes. Cross-tape temporal order is expressed *only* by transmission relations.

Transmission relations that link the sequence of squares of the clock tape of one agent to the sequence of squares of the clock tape of another agent are a basic unit of analysis for the timing aspect of symbol handling. It is the clock tape that makes it possible to relate the rhythm of one agent to that of another agent, without assuming any global time variable, thereby opening up what we like to call ‘two-clock physics’. A transmission relation is indifferent to which symbol rests on which square of a clock tape; thus it can be expressed by a graph, as illustrated in figure 1(b).

Now we generalize the transmission relation by allowing the transmission relation to express influences weaker than literal transfer of a symbol:

The transmission relation from *A* to *B* links a symbol on *A*’s clock tape to a (possibly other) symbol on *B*’s clock tape if the symbol from *A* directly influences the writing of a symbol on *B*’s clock tape.

We emphasize again that transmission relations need no assumption of any spacetime manifold. One is free to make whatever assumptions one wants to explain how a symbol moves from one agent to another agent, conditioned of course by traditions, notably the tradition of treating the speed of light as an absolute limit to the speed of propagation.

3.2. Specialized properties of transmission relations

We let \overrightarrow{AB} denote a transmission relation from *A* to *B*. According to the application, one or another property from the following list can be of interest.

1. A transmission relation \overrightarrow{AB} from *A* to *B* is *order-preserving* if links from *A* to *B* never cross. That is, given (a, b) and $(a', b') \in \overrightarrow{AB}$, it is never the case that $a < a'$ while $b' < b$. That means a symbol from *A* to *B* cannot be

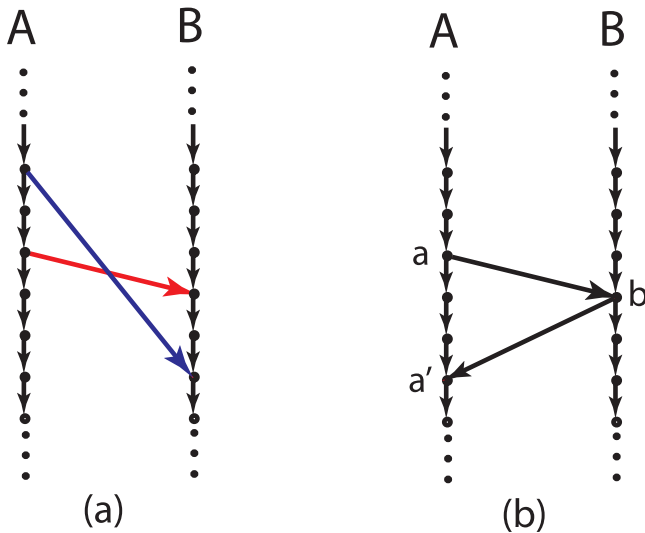


Figure 2. (a) Fragment of a transmission relation showing non-preservation of order from [6]; (b) example of $b \in B$ with an echo count relative to A of 3 from [6]. Reproduced with permission from [6]. ©2019 Institute for Cognitive Science, Seoul National University.

overtaken by another such symbol. The transmission relation illustrated in figure 1(b) is order-preserving; figure 2 shows an instance of a transmission relation that does *not* preserve order: the link shown in red starts later than that shown in blue but arrives earlier.

2. A transmission relation from A to B is *sub-1-to-1* if no A -square is linked to more than one B -square and vice versa.
3. A transmission relation from A to B is *periodically timed* if $(\exists m, n)$ s.t. $(a, b) \in \overrightarrow{AB} \Rightarrow (a + m, b + n) \in \overrightarrow{AB}$. (Notes: (1) The case $n > m$ corresponds to B stepping at a relative frequency n/m with respect to A . (2) We say periodically timed rather than just periodic, because it is not required that the symbols be periodically distributed along the clock tapes, only that the links be periodic.)
4. Given a sub-1-to-1, order-preserving transmission relations from A to B and from B to A , with $a, a' \in A$ and $b \in B$, if $(a, b) \in \overrightarrow{AB}$ and $(b, a') \in \overrightarrow{BA}$, then we say the b has an *echo count* relative to A given by $a' - a$. (This definition of echo count differs from that in [2, 13].)

3.3. Networks of symbol-handling agents

A set of agents and transmission relations among them specify a *network*. Networks engender directed graphs (in the sense of vertices connected by arrows). Corresponding to (the clock tape of) an agent is a graph consisting of a single path with a node for each square on the clock tape and an arrow (directed edge, in graph terminology) from each node to its successor (if it has one). Two agents linked by a transmission relation make a graph consisting of the two paths of the agents, linked by arrows expressed by the transmission relation from one agent to the other. Two agents and the two transmission relations back and forth between them make a directed graph with arrows in both directions. Similarly, networks with more

agents correspond to graphs that have pairs of agents and their transmission relations as subgraphs. Graphs representing networks have non-intersecting paths for clock tapes of agents.

In the use of networks of symbol-handling agents to model various physical and biological networks, a few concepts taken from graph theory, especially Petri nets [14], are helpful.

1. Supposing a network of agents A_ℓ with ℓ in some index set of integers, the *forward reach of square j of agent ℓ* is

$$A_\ell(j)^\bullet := \{(A_k, m) | k \neq \ell \wedge (j, m) \in \overrightarrow{A_\ell A_k}\}. \quad (1)$$

The red dots in figure 3(a) illustrate the forward reach of square j of agent A_1 .

2. Supposing a network of agents A_ℓ with ℓ in some index set of integers, the *backward reach of square j of agent ℓ* is

$${}^\bullet A_\ell(j) := \{(A_k, m) | k \neq \ell \wedge (m, j) \in \overrightarrow{A_k A_\ell}\}. \quad (2)$$

The red dots in figure 3(b) illustrate the backward reach of square j of agent A_1 .

3. $|A_\ell(j)^\bullet|$ denotes the number of clock-tape squares of other agents influenced by symbol j of agent ℓ 's tape. This is a measure of fan-out. In figure 3(a) $|A_1^\bullet(j)| = 3$.
4. $|{}^\bullet A_\ell(j)|$ denotes the number of symbols arriving at square j of the clock tape of A_ℓ . This is a measure of fan-in. In figure 3(b) $|{}^\bullet A_1(j)| = 4$.
5. A network is sub-1-to-1 if

$$(\forall \ell, j) |A_\ell(j)^\bullet| \leq 1 \text{ and } |{}^\bullet A_\ell(j)| \leq 1. \quad (3)$$

6. A network without closed circuits of arrows expresses a partial order which allows one to speak of 'later' and of 'concurrent' [14]. A square $B_k(i)$ is *later* than a square $A_\ell(j)$ if there is a path of arrows from $A_\ell(j)$ to $B_k(i)$. Two squares for which there is no such path from one to the other are *concurrent*.

Networks (based on the clock tapes of agents) that are not partial orders are *acausal* in the sense that a later symbol can influence the writing of an earlier symbol. All applications that we so far envisage for networks rule out acausal networks.

The case $|A_\ell(j)^\bullet| > 1$ (forward reach greater than 1) corresponds to broadcasting by A_ℓ of the symbol on square j . The case $|{}^\bullet A_\ell(j)| > 1$ (backward reach greater than 1) corresponds to the writing of a symbol on square j of the clock tape of A_ℓ being influenced by more than 1 symbol arriving during period j . (Think of listening to a symphony.)

3.4. Picturing a population of agents

In evolutionary biology, one considers populations of organisms that are born and that die. Symbolic communications among agents representing organisms of such a population involve no fixed network, but instead involve the entrance and termination of agents with their clock tapes, leading to a dynamically evolving network. Viewing such a dynamic

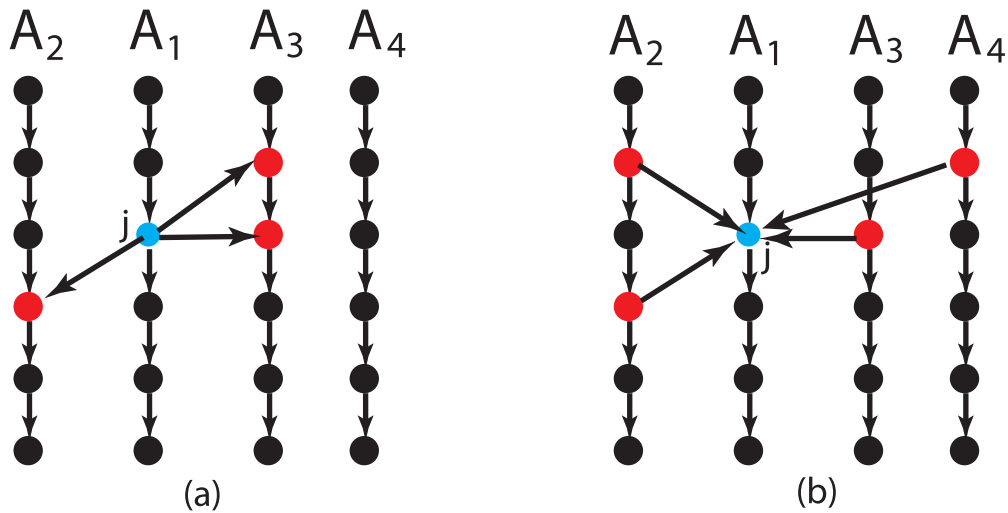


Figure 3. Forward and backward reach from [6]. (a) Forward reach, (b) backward reach. Reproduced with permission from [6]. ©2019 Institute for Cognitive Science, Seoul National University.

network in terms of the clock-tape records, we can portray the entrance and termination of agents as in figure 4.

3.5. Cycles

Sometimes it is desirable to emphasize the cyclical nature of agents, whether as investigators or as subjects of investigation, or both. For example, section 5 recounts an investigation of memory elements, involving a sequence of trials, each of which proceeds through periodically timed phases. To portray the cyclical aspect of such a case, one can ignore any non-periodic transmissions, thereby arriving at a periodic graph (or several disconnected periodic graphs.) A periodic graph can be wound into a cyclic graph, as shown in figure 5. Winding wraps a repeating stretch of the periodic partial order into a graph in which each agent is mapped into a cycle.

In contrast to a partial order, in a cyclic graph, there is no two-place *before* relation, as in ‘*a* before *b*’. Instead one has a three-place *between* relation, e.g ‘*b* between *a* and *c*’. If the period of a graph wound into a cyclic graph corresponds to three or more squares of every agent, the winding is called *loop-free* [15]. A *loop-free* winding of a periodic partial order generates a cyclic partial order—a structure with a myriad of interesting features [15]. (If the winding is not loop-free, there are too few nodes on some cycle to admit the relation of ‘between’.) Haar points out depth of the mathematics concerning windings of partially ordered sets [15].

3.6. Avenues of application of the theory of symbol-handling agents

We turn now from the mathematics of the theory of symbol handling agents to examples of avenues of application of that theory. Transmission relations on clock tapes offer an underpinning to the theory of spacetime, as well as opening up alternatives to spacetime. In the theory of relativity, a mathematical spacetime is a set of events, and one associates an event of a spacetime with a physical ‘event’, e.g. a flash of light. In place of ‘events’ one

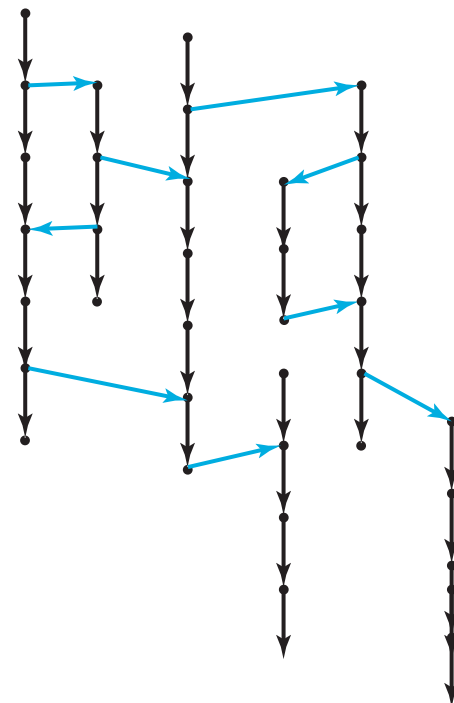


Figure 4. Showing births and deaths in clock tapes of a population of symbol-handling agents from [6, 12]. Reproduced with permission from [12]. © (2019) COPYRIGHT Society of Photo-Optical Instrumentation Engineers (SPIE).

can think of records of symbols on squares of clock tapes linked by transmission relations. This freedom allows the exploration of situations in which times and places can be constructed to suit particular situations. The theory of symbol-handling agents based on transmission relations offers templates for the design of experiments. Here are three examples.

1. Attaching locations to events is a basic activity of physics, typically done by assigning spacetime coordinates, as articulated in 2000 resolutions of the International Astronomical Union:

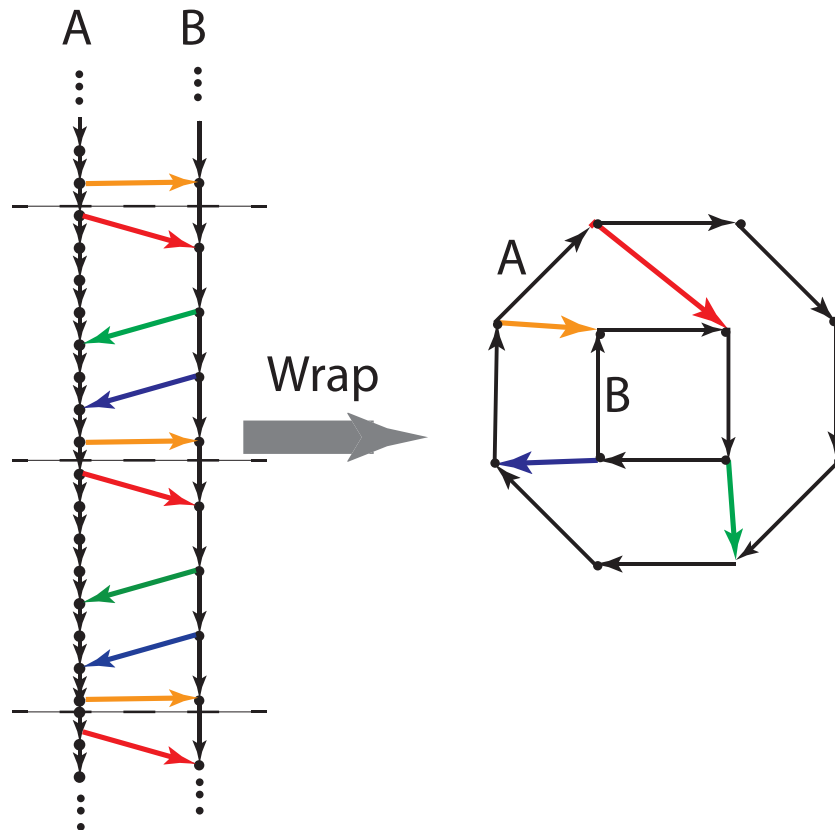


Figure 5. Wrapping a periodically timed network into a cyclic graph from [6, 12]: *A* wraps to octagon; *B* wraps to square. Reproduced with permission from [12]. © (2019) COPYRIGHT Society of Photo-Optical Instrumentation Engineers (SPIE).

The underlying concept in relativistic modeling of astronomical observations is a relativistic four-dimensional reference system. By reference system, we mean a purely mathematical construction (a chart or a coordinate system) giving ‘names’ to spacetime events [16].

As a reference system, the IAU resolutions assume a curved spacetime with a metric tensor field chosen to represent the exterior of the Earth. Thus names of events depend on the assumption of a curved spacetime with a particular choice of metric tensor field. An alternative of theoretical, and in some cases practical, interest is to name events by a clock as the place at which the event occurred and the reading of that clock as the (local) ‘time’ of the event. This alternative makes the names correspond to actual or imaginable measured data, leaving one free to consider how the data might suggest differences from the IAU metric tensor in alternative proposals for Earth’s gravitation.

2. Undersea acoustic networks are of interest for investigating the behavior of cetaceans (e.g. porpoises) that communicate using sound. It appears interesting to use the clock-tape perspective to construct times and places based on sonar communications in rhythms adapted to the communications of the cetaceans.
3. Animal nervous systems function in a variety of rhythms, and, we suspect, involve the manipulation of symbols. As it develops before and after its birth, an animal develops its own system of times and places. The freedom for an investigator to make and to test hypothesis that adapt

times and places for stimulating the animal nervous system to the animals own development of rhythms looks promising.

4. Logical synchronization: how distinctions need gradations

Attention to the necessities of implementation is fostered by the statement, made in the introduction, of a certain independence of the workbench of experiment from any theory. In this section we introduce a kind of synchronization quite different from Einstein’s, arising from behavior found in actual digital systems that implement the transmission relations discussed theoretically in the preceding section.

Unlike the imagined proper clock of relativity theory, a physical clock oscillates through phases—think of a swinging pendulum. Without the phasing there would be no ‘ticks’ to count. Special relativity, however, is based on an abstraction that makes ticks invisible. Early in the paper in which he introduced special relativity, Einstein asserts that judgments in which time plays a role are judgments of the coinciding of events. That assertion comes with an asterisk pointing to an interesting footnote (in our translation from the German):

The inexactitude that lurks in the concept of the coinciding of two events at (approximately) the same place has been skated over by an abstraction that we leave undiscussed [17].

For a theory of symbol handling, this abstraction obscures the a critical issue. An agent, like a digital computer, is stepped by a clock that cycles through periodic phases. If writing a symbol into memory overlapped temporally with reading a symbol from memory, the result would be confusion of logic. In order to avoid this confusion, a symbol transmitted to an agent must arrive only during a particular phase of the receiving agent's clock and not during other phases [2]. This constraint, which we call *logical synchronization*, requires leeway in the arrival time; one cannot ask for a point coincidence. In contrast to Einstein synchronization, the concept of logical synchronization has this leeway built into it. The need for logical synchronization, long known to engineers of digital communications [18], is reminiscent of a game of catch, in which a player cycles through phases of throwing and catching a ball, or more simply, a spoken dialog in which each person alternates between speaking and listening.

4.1. Logical synchronization versus Einstein synchronization

As discussed in [2, 13], logical synchronization has both freedoms and constraints relative to Einstein synchronization. Freedoms include:

1. Unlike Einstein synchronization, clock readings at transmissions and receptions are allowed a certain leeway.
2. Unlike Einstein synchronization, the logically synchronized clocks can differ in frequency. That is because the conditions for logical synchronization are not required for all periods of the clock curves, but only for those periods linked by the transmission of a symbol [13].
3. Because of the freedom to vary clock rate relative to a proper clock, two agents in relative motion in a flat spacetime can maintain logical synchronization, even though Doppler shift precludes Einstein synchronization.

Constraints include:

1. Transmissions and receptions are restricted to appropriate clock phases.
2. Consider several agents thought of as in a spacetime, communicating symbols carried as light pulses. The requirement of logical synchronization strongly constrains the possible transmission relations. This constraint is discussed in [13] as the 'stripes in spacetime' imposed by logical synchronization; it corresponds to 'you cannot synchronize with everybody at once, so you have to make choices'.

4.2. Extra-logical clock adjustment to maintain logical synchronization

In many situations, to maintain the arrival of symbols within the leeway allowed by logical synchronization, agents must more-or-less continually adjust the tick-rates of their clocks. The adjustments of clock rates necessary to the maintenance of logical synchronization are steered by a feedback loop that estimates phase deviations from the aiming point. To sense deviations within the leeway, an agent must reach

beyond logical operations on symbols, for the simple reason that the logic of symbol handling has to be oblivious to those deviations.

Proposition 2. *The timing of symbol arrival within the allowed phase cannot be registered by the process that recognizes distinct symbols.*

Proof: the recognition of a symbol depends on indifference to the timing of arrival within the allowed leeway.

It follows that distinction-bearing symbols cannot be the whole story, for they cannot function without agents attending to gradations. Thus auxiliary mechanisms are necessary to supply an agent with information to guide the steering of its clock rate. Steering of clock rates so as to maintain logical synchronization is often automated to function according to an algorithm that responds to graded deviations of the phases of arriving symbols registered over some running number of cycles. The computational complexity of the algorithm is in many cases minimal. But because, even in principle, deviations are unpredictable [13], no algorithm, no matter how complex, can anticipate deviations so perfectly as to eliminate them. Choosing an algorithm to steer clocks requires that an agent reach beyond logic to make a guess.

In the next section we go into behavior on the work bench that illuminates the gradations necessary to dealing with distinct symbols.

5. When the coin lands on edge

In section 3 we mostly focused on theoretical transmission relations on the clock tapes of agents, relations that can be written to sit still on the blackboard. In section 4 we enriched the theory of symbol handling by considering the need for logical synchronization, essential to implementing designs on the workbench based on theoretical transmission relations. But logical synchronization does not just happen; agents must maintain it by steering clock rates. The steering of clock rates is dynamic, involving not only distinctions but also indistinct arrivals of symbols within a phase. Logical synchronization depends on agents attending to graded transitions between distinctions. Here we discuss the gradations that have to be dealt with in order to implement logical distinctions on the workbench.

We start with the question: what happens when the arrival of a symbol fails to meet the conditions of logical synchronization? We show how an agent's act of receiving a symbol outside a receptive phase is like flipping a coin that lands on edge, resulting in logical confusion, sometimes referred to as a 'glitch'. We go beyond our earlier discussions [1, 2], by relating the glitch to evidence of logical confusion pictured on clock tapes.

Logic on the workbench is built from physical *NAND gates* used to construct a digital computer. On the blackboard, a NAND gate is thought of as implementing the NEGATION of the Boolean function 'AND', but a NAND gate on the workbench moves. It has two input wires and an output wire;

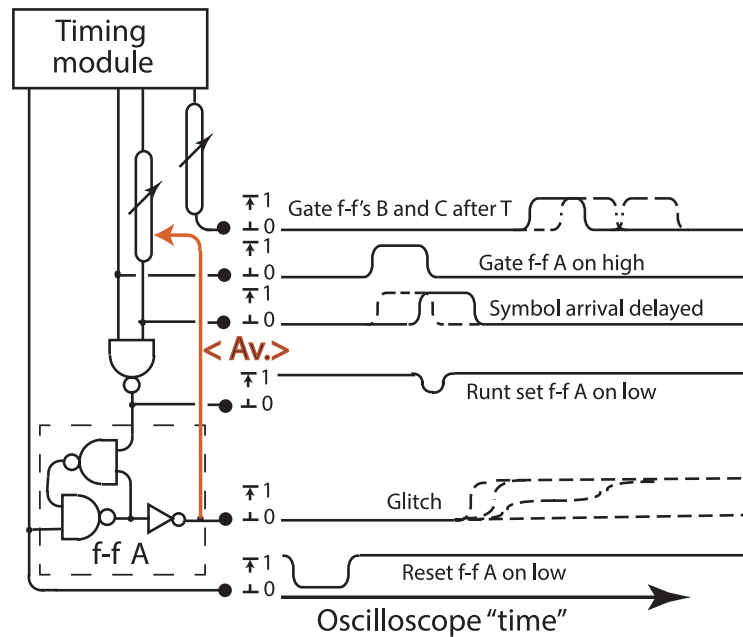


Figure 6. Oscilloscope traces of flip-flop A under test.

on all three wires, voltages implementing Boolean values 0 or 1 undergo changes. When voltages are held constant for a little while on its input wires the NAND gate generates, after a delay, a voltage on its output wire—a high voltage for a 1 unless both input wires have high voltages, in which case the output is a low voltage for 0. The phrase ‘after a delay’ is one hint that logic on the workbench differs from blackboard logic. A digital system, composed of NAND gates must be temporally organized, which requires that some of the inputs of its NAND gates are driven by clocks. Only then can the digital system deal coherently with changes in inputs and outputs.

A pair of cross-coupled NAND gates called a *flip-flop* implements a square of a clock tape on which can be written a single bit as a 0 or a 1. The two NAND gates of a flip-flop form an unstable balancing device, the electronic analog of a hinge that records a 1 if flipped one way or a 0 if flopped the other way. A gated flip-flop is a flip-flop with one input preceded by a third NAND gate that acts as a valve. If open, it allows the ‘hinge’ to flip or flop, and if closed, prevents the ‘hinge’ from flipping or flopping. Figure 6 shows how two NAND gates form a flip-flop. It also shows the NAND gate that precedes the flip-flop and acts as a valve. The cross coupled NAND gates of a flip-flop feedback on themselves, thereby providing another hint of a difference between blackboard logic and bench logic.

As discussed in section 4, the clocks of digital systems step flip-flops through phases of a cycle, so there is a phase for changing the symbols on the inputs and a distinct other phase during which symbols appear on the outputs. Translated into terms that relate to an experiment on flip-flops, our question becomes: what happens if a symbol arrives just as an agent’s clock closes the valve on the phase in which an agent’s flip-flop can accept a symbol?

A trial of the experiment begins by resetting the flip-flop to 0. When the flip-flop has been reset to 0, a 1 arriving during a phase in which the valve is open flips the hinge over, so that the flip-flop generates an output of 1. So, to repeat our question: what happens when the symbol 1 arrives not while the valve is open, but just as the valve is closing? One might guess that what happens is random, i.e. either the flip-flop generates an output of 1 or it generates an output of 0, but that guess is, at best, misleading.

We and others (e.g. [19]) have experimented to find out what happens. Our focus on symbols led us to counting evidence of glitches expressible by relations among clock tapes. We arrange a clock, shown as the ‘Timing module’ in figure 6, to drive a sequence of trials of a flip-flop A that, after a variable delay T , is viewed by a matched pair of flip-flops B and C. Viewing the flip-flops, including their clocking, as agents, we can display the experimental results on clock tapes for A, B, and C. Successive squares on the clock tapes of A, B, and C are generated in lock-step, one each per cycle of the clock that drives the trials. Figure 7 shows the form of evidence, in which the evidence of glitch is seen when a B-square and a C-square linked to a given A-square disagree: one shows a 0 while the other shows a 1. In this form of evidence, any given square of a clock tape holds a distinct symbol and not any other symbol; however, to show how the experiment works, one has to show what goes on within phases of the clock cycle.

The left margin of figure 6 shows the electrical circuit diagram of flip-flop A under test, with the probe points indicated by •. The circuit shows the three NAND gates mentioned above, along with two variable delays. On the right of the figure are oscilloscope traces of voltage versus time for the probe points, as told by the sweep of the electron beam of the oscilloscope. The traces illustrate the effect of a symbol arriving just as the gate for its acceptance is closing. The

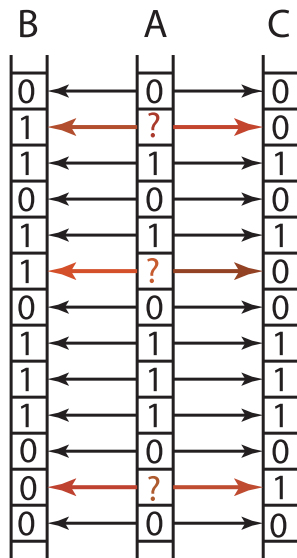


Figure 7. Disagreements between Agents B and C viewing A.

immediate effect is the generation of a runt pulse—enough of a pulse to perturb flip-flop A, but not enough to necessarily set it. The result is various. In some trials, flip-flop A is set promptly; in other trials it is not set at all, and, in the more interesting trials, flip-flop A is put into an unstable equilibrium, where it teeters, indefinitely, before generating a definite high or low output.

Proposition 3. *In some cases in which a symbol arriving at a clocked flip-flop A comes late or early relative to the receptive phase determined by the clock, the flip-flop teeters indefinitely, before, eventually ‘falling over’ to generate a distinct low or a high voltage. When A transmits its output to the inputs of two flip-flops B and C, these can disagree about A’s state; the likelihood of disagreement diminishes with the waiting time T prior to B and C being gated to read A’s output.*

Our experiment required a little invention to make symbol arrivals straddle a boundary between phases (just the opposite of steering clock rates so as to maintain logical synchronization). We needed to shape the runt pulse so as to put flip-flop A into the teetering condition often enough to study it. The runt pulse is shaped by the difference in two delays, and the delays drift unpredictably with temperature variations and other unknown influences. When we set the delays statically, the drift made A register a sequence of all 1’s or a sequence of all 0’s, with no glitches. To counter the effect of drifts we fed back a short-term running average of A’s outcomes to control one (voltage variable) delay line. While trying to adjust the delays statically from the behavior of the oscilloscope traces failed, the use of feedback worked fine.

In summary, to avoid logical inconsistencies ($B \neq C$), agents receiving symbols must ‘avoid looking’ during transitions between symbols. This ‘avoiding looking’ during a transition from between logical values—i.e. logical synchronization—is no passive circumstance, but generally requires agents to actively steer the tick rates of their clocks. Furthermore, and this we emphasize, maintaining logical synchronization requires agents to attend to the intrinsically

unpredictable gradations. The attempt to impose distinctions on these gradations would put a flip-flop into an unstable state.

Proposition 4. *The order of events asserted by a flip-flop in a race condition can be decided by extraneous influences, no matter how small: the teetering of a flip-flop allows, for a little while, the future to affect the record of the past.*

Proposition 5. *The registering of disagreements under fan-out proved a superior measurement technique for recording borderline cases.*

5.1. Extended lessons from the glitch

Now we introduce the following

Assumption (Principle of the balance) *The measurement of the temporal order of symbol arrivals requires balancing one arrival against a single other arrival, e.g. by use of a flip-flop, making measured order of arrival a binary relation.*

From this principle and proposition 4 it follows that

Proposition 6. *The decision of a close race is necessarily beyond logic.*

The principle of the balance also implies

Proposition 7. *Measuring the order of arrival in an n-way race requires measuring pairwise, that is, measuring the order of arrival of $n(n - 1)/2$ two-way races.*

Describing a race in terms of theoretical ‘arrival times’ encounters a conflict with the workbench. On the blackboard, a time is expressed as a real number t and a [20, section 1.1 1.1]. Then for any two such blackboard ‘times’ t_1 and t_2 there are the three mathematical possibilities. $t_1 < t_2$, $t_1 = t_2$, or $t_1 > t_2$. On the workbench, in the absence of logical synchronization, the glitch tells us that ‘=’ is unstable.

Proposition 8. *Indeterminacy in two-way races implies possible non-transitivity in races among three or more; e.g. in a three-way race of a, b, and c, there can be cases of finding $a < b$, $b < c$, with $c < a$.*

Proposition 9. *The use of real numbers on the blackboard to express timing under race conditions conflicts with experimental evidence.*

6. Concluding remarks

‘Mathematics is based on the idea of a distinction’ [21], and the conveying of distinctions by the use of symbols starts with life itself, e.g. in the bases of DNA. The symbols expressing formulas of mathematical logic can sit still on the blackboard, but logic on the workbench is the logic of devices that move. Motion creates a problem on the workbench foreign to logic on the blackboard, namely the deciding of temporal order: which symbol came before the other? We have tried to let the device on the workbench that decides temporal order—the flip-flop, with its teetering—tell its story, a story that binds

the communication of distinctions to logical synchronization with its dependence on the unpredictable teetering, richer than can be captured by ‘measurement uncertainty.’ Three more remarks:

1. With the recognition of symbol-handling as part of physics and part of life, the role of clocks reaches beyond ‘telling time’ to the opening and closing of gates necessary to the coherent communicating of distinctions.
2. Without logical synchronization, agreement about distinctions is impossible.
3. With logical synchronization, the arrival of a symbol, as recorded on a square of a clock tape, is *objective* in the sense that one expects that two agents to which the square fans out will agree on the symbol. Objectivity in this sense endures after we give up any aspiration to final ‘truth,’ as we must in light of the incompleteness theorem discussed in [3].

The work reported here opens a door to dealing with the timing of symbolic communication in a way that supplies a previously unavailable underpinning to concepts and implementations of ‘times and places’. There is a lot more to explore. We have discussed the maintenance of logical synchronization, once that synchronization is in place. Left to the future is the challenging topic of two agents that seek to acquire logical synchronization so that they can communicate. From the engineering world, we learn that there can be no deterministic upper bound on how many cycles that acquisition may require [18].

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Appendix. Synchronization and the theory of general relativity

For the proper clocks of the theory of special relativity, Einstein defined a form of synchronization, *Einstein synchronization*, that permeates theoretical physics. For example in the International System of Units (SI), Einstein synchronization guides the definitions of the measuring units for time and for length.

A.1. The meter in relation to clock readings as defined in special relativity

The SI meter is ‘the length of the path traveled by light in vacuum during a time interval of $1/299\,792\,458$ of a second’ [23]. This ‘time interval’ rests on the concept of an inertial frame of special relativity. Every clock fixed to an inertial frame is Einstein synchronized to every other such clock. Einstein synchronization relates readings of one clock to

readings of another clock. Let t_A be a reading of clock A at the emission of a light pulse that reaches B at t_B , and t'_A the reading of A at the receipt of an echo reflected from B at t_B . Looking at t_A and t'_A as functions of t_B , clock B is Einstein synchronized to clock A , relative to the inertial frame, provided that, for all t_B ,

$$t'_A - t_B = t_B - t_A. \quad (\text{A.1})$$

Proposition 10. *The time interval in the definition of the meter denotes the difference between clock readings of Einstein-synchronized proper clocks at the two ends of a light path.*

For c the speed of light, the SI length of a path from A to B is $c(t_B - t_A)$, and thus invokes readings of separated, Einstein-synchronized proper clocks. Note that even in special relativity, Doppler shift precludes Einstein synchronization of proper clocks moving relative to each other.

B.1. Einstein synchronization drastically restricted by spacetime curvature

Spacetime curvature changes the story. It is known that in a generic curved spacetime of the theory of general relativity, no grid of exactly Einstein-synchronized proper clocks is possible. Because curved spacetimes are locally flat, deviations from Einstein synchronization are often small; however, the astounding stability of today’s optical atomic clocks makes small deviations from synchronization measurable and of physical interest, as in the detection of gravitational effects. For a second example, coordinated universal time (UTC) is distributed by clocks that, even in theory, require their tick rates to be adjusted to compensate for gravitation.

C.1. Clocks as expressed in general relativity

In the theory of special relativity a clock fixed to an inertial frame is expressed by a straight, timelike line. Turning from the flat spacetime of special relativity to the curved spacetimes of general relativity, one expresses a clock in terms of a timelike curve in a manifold [22]:

Here and in the following, our terminology is as follows. A general-relativistic spacetime is a 4-dimensional manifold M with a smooth metric tensor field g of Lorentzian signature and a time orientation; the latter means that a globally consistent distinction between future and past has been made. A clock is a smooth embedding $\gamma: t \rightarrow \gamma(t)$ from a real interval into M such that the tangent vector $\dot{\gamma}(t)$ is everywhere timelike with respect to g and future-pointing. This terminology is justified because we can interpret the value of the parameter t as the reading of a clock. Note that our definition of a clock does not demand that ‘its ticking be uniform’ in any sense. Only smoothness and monotonicity is required [22].

We will speak of reparameterization of the embedding that specifies a clock as ‘an adjustment of the tick rate of the clock’.

Instead of an inertial frame, for a curved spacetime one has an ‘observer field’:

By an observer field on a general-relativistic spacetime we mean a smooth vector field V which is everywhere timelike and future-pointing. An observer field V is called a standard observer field if $g(V, V) = 1$. According to our earlier terminology, integral curves of observer fields are clocks, and integral curves of standard observer fields are standard clocks with the usual choice of time unit. For the sake of brevity, we will refer to the integral curves of an observer field V as to clocks in V . Note that V fixes the parametrization for each of its integral curves uniquely up to an additive constant, i.e. for each clock in V there is still the freedom of choosing the zero point on the clocks dial [22].

For a generic curved spacetime, we can say something about the issue of trying to Einstein-synchronize clocks in a *radar neighborhood*, which is a neighborhood too large to be considered flat, but ‘small’ enough to avoid extreme gravitational effects [22]. More precisely, given clocks A and B within a radar neighborhood, for an event $b \in B$ there is precisely one light ray from A to b , and one light ray from b to A .

Although no inertial frame of Einstein-synchronized proper clocks is possible in a curved spacetime, there exist adjustments of the tick rates of selected pairs of clocks that can make them Einstein synchronized.

Proposition 11. *For any two non-intersecting clocks following given timelike trajectories within a radar neighborhood of a generic curved spacetime, there exist tick rates, in general varying, for which the two ‘improper’ clocks can be Einstein-synchronized.*

For a flat spacetime, the needed adjustment of (possibly moving) clocks is illustrated in figure 4 of [13], and the same procedure works in a radar neighborhood of a curved spacetime. However, when more than two clocks are considered in a curved spacetime, it is in general impossible to Einstein synchronize each clock to all the others.

A ‘radar distance’ can be defined for improper clocks in a curved spacetime, analogous to distance as defined in special relativity, but in a curved spacetime, radar distance is neither transitive nor symmetric [22].

Proposition 12. *Assuming a generic curved spacetime, as the maximum radar-distance across a network of more than two clocks increases, the minimum possible deviations from Einstein synchronization also increase, even when adjustable clocks are allowed.*

From proposition 12 we arrive at proposition 1 of section 2.

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