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MHD Two-Phase Flow and Heat Transfer between Two Parallel Porous Walls in a Rotating System

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Abstract

The equations describing the two-phase magnetohydrodynamic steady flow and heat transfer through a horizontal channel consisting of two parallel porous walls subject to the action of uniform magnetic field, applied in the direction normal to the plane of flow/axis of rotation are written down, assuming that the magnetic Reynolds number is small. Also, it is assumed that the fluids in the two regions are incompressible, immiscible and electrically conducting, having different viscosities, thermal and electrical conductivities. Further, assumed that the transport properties of the two fluids are constant having constant boundary wall temperatures. Exact solutions for velocity and temperature distributions are obtained and are calculated numerically for different values of parameters. The results are presented graphically. We observe that the effect of increasing suction parameter is to increase the temperature in both phases. Also, as the suction parameter increases, there is a significant change in the primary velocity at the upper region but is insignificant in the lower region.

Keywords: Two–phase flows, magnetohydrodynamics, heat transfer, rotating fluids, porous walls.

Nomenclature

B⁰ magnetic field strength

The contract of the contract of the \mathbf{J} and \mathbf{J} are all \mathbf{J} and \mathbf{J} \mathcal{L} and \mathcal{L} are the set of \mathcal{L} $\left| h_1 \right|$ $\begin{bmatrix} V & V \end{bmatrix}$ $\left| \begin{array}{c} \overline{\Omega} \end{array} \right|$ $\mathcal V$ $h_1 \sqrt{\frac{2}{\pi}}$

K taylor number (Rotation parameter), , *thermal conductivities of the two fluids*

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Greek symbols

1 Introduction

The simplest case of multiphase flows is the two-phase flows, which are of great practical concern in a large number of engineering disciplines, including the chemical, petroleum and various power generating industries. For instance, questions concerning nuclear-reactor safety have led to a demand for an understanding of the detailed phase-distribution mechanisms involved in two-phase flows. Also, one of the more interesting and potentially useful phenomenon associated with the simultaneous, horizontal pipeline flow of two incompressible fluids is the fact that the pressure gradient and power requirement necessary for the flow of the more viscous phase at a given rate may be substantially reduced by the presence of the less viscous phase. The most common practical example in which this effect is evident is the pipeline flow of petroleum and water. Numerous studies dealing with both the experimental and theoretical aspects of the two-phase

classical hydrodynamic problems have been under taken in the past and made available in literature during the several decades by various investigators [1-10]. But, rigorous formulation of the two-phase flow conservation equations, such as mass, momentum and energy has for some time been a challenge, and the inclusion of both electric and magnetic fields into this theory has not been carried out properly. The difficulty in the formulation of conservation equations is associated with the presence of a large number of interfaces and the apparent random nature of the flow field.

But the requirement of modern technology has stimulated interest in magnetohydrodynamic two phase flow studies more closely, which involve the interaction of several phenomena. As the theoretical and organizing experimental results for predicting design parameters of two–phase flows are becoming increasingly important because of their widespread applications in industry in relation to various energy conversion systems. In particular, the development of conceptual design for fusion reactors and liquid metal magnetohydrodynamic rotating power generators, demands an accurate and reliable knowledge of the heat transfer and thermohydraulic mechanisms of two phase flow in presence of an applied magnetic field. Transportation and extraction of the products of oil are other obvious applications using a two-phase system to obtain increased flow rates in an electromagnetic pump from the possibility of reducing the power required to pump oil in a pipe line by suitable addition of water (see Shail [11]). Consequently, the experimental and theoretical aspects of the two-phase flow systems associated with MHD generators have appeared in the literature. A comprehensive review of these problems is studied by various investigators notably, Shercliff [12], Saito et al. [13], Dobran [14], Malashetty and Umavathi [15], Chauhan and Rastogi [16,17].

Besides these studies, Lohrasbi and Sahai [18] have studied the MHD heat transfer aspects in two phase flow with the fluid in one phase being electrically conducting. Then Malashetty and Leela [19] have carried out a theoretical study on Magnetohydrodynamic heat transfer in two- fluid flow for short circuit case. Subsequently, in 1992 they [20] have analyzed the problem of magnetohydrodynamic heat transfer in two-phase flow by assuming that the fluid in both regions to be electrically conducting for the open circuit case. However, the use of liquid metals as heat transfer agents and as working fluid in a rotating MHD power generator and nuclear reactor technology has created a growing interest in the behaviour of liquid metal flows and in particular the nature of interaction with magnetic and electric fields. Moreover, the interaction with coriolis forces between the conducting and the magnetic field radically modifies the MHD two-phase flow.

On the other hand, numerous studies involving heat transport phenomena from both symmetrical and asymmetrical surfaces under the action of rigid rotation where the electromagnetic and coriolis forces often significantly affect the heat transport rates in case of single-phase flow have been reported in the literature during the last several decades. But, in case of two-phase flow, with the exception of few papers discussed above, this substantial effort has not contributed significantly in the literature either toward the understanding of the physical process involved or toward helping the designer in providing him with design information and criteria of sufficient accuracy, reliability and geometry.

Hence, the purpose of this study is to provide a rigorous mechanistic basis for the theoretical description of MHD two-phase flow driven by a constant pressure gradient and associated heat transfer in a horizontal channel consisting of two parallel porous walls under the action of uniform strong magnetic field, applied in the direction normal to the plane of flow subjected to a constant

suction applied normal to both walls. With these assumptions and considering that the magnetic Reynolds number is small, the resulting governing linear differential equations are solved analytically, using the prescribed boundary and interface conditions to obtain the exact solutions for velocity and temperature distributions. Also, their corresponding numerical results for various sets of values of the governing parameters are obtained to represent them graphically and are discussed in detail.

The presentation of a two-phase MHD flow model in a rotating frame of reference, with porous boundaries subject to normal suction is timely in view of the recent interest in liquid metal flow in MHD power rotating generators, in proper design of cooling blanket in a magnetically confined plasma fusion reactors/thermo nuclear reactors, also this study would provide guidance for the investigation of boundary layer behaviors along porous walls with fluid injection or suction.

2 Formulation of the Problem

A steady magnetohydrodynamic two-phase flow driven by a common constant pressure gradient $\left(-\frac{\partial p}{\partial x}\right)$ in a horizontal channel subject to the uniform suction v_0 applied normal to both walls(i.e., in the y- direction) is considered. A constant magnetic field of strength B_0 is applied in the y-direction. The whole system is rotated with an angular velocity Ω about an axis perpendicular to their planes i.e., y-axis. The physical model and coordinate system shown in Fig. 1 consists of two infinite parallel porous walls along x- and z-directions. The regions $0 \le y \le h_1$ and $-h_2 \le v \le 0$ are occupied by two different immiscible electrically conducting, incompressible fluids having different viscosities, thermal and electrical conductivities. The interface between the two immiscible fluids is assumed to be flat, stress free and undisturbed. The boundaries of the channel are rigid and the channel width is assumed to be very large in comparison with the channel height. The two bounding walls are maintained at constant temperature T_w . Further, it is to be assumed that the magnetic Reynolds number is small, so that the externally applied magnetic field is undisturbed by the flow, namely, the induced magnetic field is small when compared with the applied field, hence is negligible. With these assumptions, the governing equations of motion, energy and the corresponding boundary and interface conditions can be formulated as follows for the two-dimensional steady state problem.

Figure 1. Physical model and coordinate system.

3 Governing Equations with Boundary and Interface Conditions and Mathematical Analysis of the Problem

With the above assumptions, the governing equations of motion and energy as in Malashetty and Leela [20] and Raju and Murty [21] for both phases in a rotating frame of reference are

$$
\mu_i \frac{d^2 u_i}{dy^2} - \frac{dp}{dx} - \sigma_i (B_0^2 u_i + B_0 E_0) = 2\rho \Omega w_i - \rho v_0 \frac{du_i}{dy}
$$
 (1)

$$
\mu_i \frac{d^2 w_i}{dy^2} - \sigma_i B_0^2 w_i = -2\rho \Omega u_i - \rho v_0 \frac{dw_i}{dy}
$$
 (2)

$$
K_i \frac{d^2 T_i}{dy^2} + K_i \frac{\rho v_0}{\mu_i} \frac{dT_i}{dy} + \mu_i \left[\left(\frac{du_i}{dy} \right)^2 + \left(\frac{dw_i}{dy} \right)^2 \right] + \sigma_i \left[E_0 + B_0 \mu_i + B_0 w_i \right]^2 = 0
$$
 (3)

where T_i is the temperature, u_i and w_i are the primary and secondary velocity components along x and z directions respectively. The no slip condition requires that the velocity must vanish at the wall. In addition, the fluid velocity, shear stress, temperature and heat flux must be continuous across the interface.

The boundary and interface conditions are

$$
u_1(h_1) = 0
$$
, $w_1(h_1) = 0$, $u_2(-h_2) = 0$, $w_2(-h_2) = 0$, $u_1(0) = u_2(0)$, $w_1(0) = w_2(0)$. (4)

$$
\mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy} \text{ at y=0}, \quad \mu_1 \frac{dw_1}{dy} = \mu_2 \frac{dw_2}{dy} \text{ at y=0.}
$$
 (5)

$$
T_1(h_1) = T_w, T_2(-h_2) = T_w, T_1(0) = T_2(0), K_1 \frac{dT_1}{dy} = K_2 \frac{dT_2}{dy} \text{ at } y=0.
$$
 (6)

In making these equations dimensionless, the following transformations are used:

$$
u_i^* = u_i / \overline{u}_{1, W_i^*} = w_i / \overline{u}_{1, Y_i^*} = y_i / h_{i, \theta_i} = K_1 (T_i - T_w) / (\overline{u}_1^2 \mu_1), \alpha = \mu_1 / \mu_2, \beta = K_1 / K_2
$$

$$
u_i^* = u_i / u_1, w_i^* = w_i / u_1, y_i^* = y_i / h_i, \theta_i = K_1 (T_i - T_w) / (u_1^2 \mu_1), \alpha = \mu_1 / \mu_2, \beta = K_1 / K_2
$$

$$
h = \frac{h_2}{h_1}, \sigma = \frac{\sigma_2}{\sigma_1}, P = \frac{h_1^2}{(\bar{u}_1 \mu_1)} \left[-\frac{dp}{dx} \right], M = B_0 h_1 \sqrt{\frac{\sigma_1}{\mu_1}}, K = h_i \sqrt{\frac{\Omega}{v}}, R_e = \frac{E_0}{B_0 \bar{u}_1}, \lambda = \frac{h_i v_0}{v}.
$$

With the above non dimensional quantities, the governing equations (1) , (2) and (3) become

Phase-I

$$
\frac{d^2u_1}{dy^2} + P - M^2(R_e + u_1) = 2K^2w_1 - \lambda \frac{du_1}{dy}.
$$
\n(7)

$$
\frac{d^2w_1}{dy^2} - M^2w_1 = -2K^2u_1 - \lambda \frac{dw_1}{dy}.
$$
\n(8)

$$
\frac{d^2\theta_1}{dy^2} + \lambda \frac{d\theta_1}{dy} + \left(\frac{du_1}{dy}\right)^2 + \left(\frac{dw_1}{dy}\right)^2 + M^2 (R_e + u_1 + w_1)^2 = 0.
$$
\n(9)

Phase-II

$$
\frac{d^2u_2}{dy^2} + Ph^2\alpha - \alpha\sigma h^2 M^2 (R_e + u_2) = 2K^2 w_2 - \lambda \frac{du_2}{dy}.
$$
 (10)

$$
\frac{d^2 w_2}{dy^2} - \alpha \sigma h^2 M^2 w_2 = -2K^2 u_2 - \lambda \frac{dw_2}{dy}.
$$
\n(11)

$$
\frac{d^2\theta_2}{dy^2} + \lambda \frac{d\theta_2}{dy} + \frac{\beta}{\alpha} \left[\left(\frac{du_2}{dy} \right)^2 + \left(\frac{dw_2}{dy} \right)^2 \right] + \beta \sigma h^2 M^2 (R_e + u_2 + w_2)^2 = 0. \quad (12)
$$

Boundary and interface conditions are

$$
u_1(+1) = 0, w_1(+1) = 0, u_2(-1) = 0, w_2(-1) = 0,
$$
\n(13)

$$
u_1(0) = u_2(0), w_1(0) = w_2(0),
$$
\n(14)

$$
\frac{du_1}{dy} = \frac{1}{\alpha h} \frac{du_2}{dy}, \frac{dw_1}{dy} = \frac{1}{\alpha h} \frac{dw_2}{dy} \quad \text{at } y = 0,
$$
\n(15)

$$
\theta_1(+1) = 0, \ \theta_2(-1) = 0, \ \theta_1(0) = \theta_2(0), \ \frac{d\theta_1}{dy} = \frac{1}{\beta h} \frac{d\theta_2}{dy} \text{ at } y = 0.
$$
 (16)

The asterisks have been dropped for simplicity.

4 Solutions of the Problem

The governing equations of momentum and the energy are to be solved analytically subject to the boundary and interface conditions for the velocity and temperature distributions. Hence the exact solutions of the governing differential equations (7), (8) and (10), (11) using the boundary and interface conditions (13) to (15) for the primary and secondary velocities u_1 , u_2 and w_1 , w_2 respectively in the two regions are obtained as:

$$
u_1 = \frac{1}{2} \Bigg((C_{12} \cosh C_1 y + C_{13} \sinh C_1 y + C_{17} \cosh C_{16} y + C_{18} \sinh C_{16} y) e^{\frac{-\lambda y}{2}} + A_2 + A_{10} \Bigg), (17)
$$

$$
w_1 = A_{50} \Bigg((C_{12} \cosh C_1 y + C_{13} \sinh C_1 y - C_{17} \cosh C_{16} y - C_{18} \sinh C_{16} y) e^{\frac{-\lambda y}{2}} + A_2 - A_{10} \Bigg),
$$

(18)

$$
u_2 = \frac{1}{2} \Biggl(\left(C_{14} \cosh C_2 y + C_{15} \sinh C_2 y + C_{20} \cosh C_{19} y + C_{21} \sinh C_{19} y \right) e^{-\frac{2y}{2}} + A_4 + A_{16} \Biggr),
$$
(19)

$$
w_2 = A_{50} \Biggl(\left(C_{14} \cosh C_2 y + C_{15} \sinh C_2 y - C_{20} \cosh C_{19} y - C_{21} \sinh C_{19} y \right) e^{-\frac{2y}{2}} + A_4 - A_{16} \Biggr).
$$
(20)

Further, to solve the non-dimensional energy equations (9) and (12) for temperature distributions, we use the solutions (17) to (20), as already obtained above for u_1 , u_2 , w_1 and w_2 along with the boundary and interface conditions (16). So the solutions for the temperature distributions θ_1 and θ_2 in two regions are obtained as

$$
\theta_1 = D_{39}e^{D_{25}y} + D_{40}e^{-D_{26}y} + D_{41}e^{D_{27}y} + D_{42}e^{-D_{28}y} + D_{43}e^{D_{29}y} + D_{44}e^{-D_{30}y} \n+ D_{45}e^{D_{31}y} + D_{46}e^{-D_{32}y} + D_{47}e^{D_{34}y} + D_{48}e^{-D_{35}y} + D_{49}e^{D_{36}y} \n+ D_{50}e^{-D_{37}y} + D_{33}e^{-\lambda y} + D_{38}y^2 + F_{51}y + F_{52},
$$
\n(21)

$$
\theta_2 = F_{32}e^{F_{1}y} + F_{33}e^{F_{2}y} + F_{34}e^{2F_{1}y} + F_{35}e^{2F_{2}y} + F_{36}e^{F_{5}y} + F_{37}e^{F_{6}y} \n+ F_{38}e^{2F_{5}y} + F_{39}e^{2F_{6}y} + F_{40}e^{F_{9}y} + F_{41}e^{F_{10}y} + F_{42}e^{F_{11}y} \n+ F_{43}e^{F_{12}y} + F_{44}e^{F_{13}y} + F_{45}e^{F_{14}y} - \frac{F_{23}}{2}y^2 + F_{53}y + F_{54}.
$$
\n(22)

The constants appearing in equations (17) to (22) are not given for the sake of brevity. The velocity and temperature distributions in both the regions are obtained for different sets of values of governing parameters involved in the study and these results are presented graphically. While computing the results, the value of P is fixed at 2.

5 Results and Discussion

The problem of magnetohydrodynamic two-phase flow and heat transfer in a horizontal channel consisting of two parallel porous walls in a rotating system, assuming that the magnetic Reynolds number is small is investigated analytically. The resulting differential equations are solved analytically to obtain exact solutions for the primary and secondary velocity distributions, such as u_1 , u_2 and w_1 , w_2 respectively in the two regions. Consequently, closed form solutions for temperature distributions, namely, θ_1 and θ_2 in both the regions are determined by making use of the, already obtained solutions of velocity distributions. The graphs for the velocity and temperature distributions are shown in Figs. 2-13.

5.1 Velocity profiles

The effect of suction parameter λ is shown in Figs. 2 and 3. We found that, as λ increases u₁ decreases everywhere except near the center of the channel. While, $u₂$ first decreases and then increases near the channel center line, but it increases below the channel center line as λ increases. Also it is observed that, as λ increases, w₂ increases but w₁ increases except at the upper wall. The effect of λ is felt more on u₂ and w₂. The effect of varying the electrical conductivity ratio σ is represented in Figs. 4 and 5. It is observed that, as σ increases, u_1 , u_2 and w_1 , w_2 also increase. From Figs. 6 and 7, it is found that, the effect of increasing M is to increase u_1 and to decrease w_1 , where as to increase both u₂ and w₂. The effect of α is shown in Figs. 8 and 9. The effect of α is felt more on u_2 and w_2 . As α increase u_1 , u_2 and w_1 , w_2 also increase. For small values of α , the fluid in the lower region is more viscous compared to the upper region and this increased shear stress in lower region retorted the flow in this region and this effect is carried over to the upper region through the interface conditions.

5.2 Temperature profiles

From Fig. 10, it is noticed that θ_1 and θ_2 increase for increasing values of λ . The effect of varying the electrical conductivity ratio σ is represented in Fig. 11. As σ increases, $θ$ decreases. It is noticed that, the effect of this ratio is more pronounced on temperature compared to the velocity. From Fig. 12, it is observed that, as M increases (up to 8), θ_1 and θ_2 also increase, but if M increases further (>8), θ_2 decreases and θ_1 increases. From Fig. 13, it is found that, the effect of increasing α is to decrease θ_1 and θ_2 . For small α , it is observed that, the temperature in the two regions increase with decreasing values of α, this is because, more heat is added to the fluid due to viscous heating.

6 Conclusion

Hence, we conclude that, as the suction parameter increases, there is a significant change in the primary velocity at the upper region but is insignificant in the lower region. While as this parameter increases, the primary velocity first increases up to the middle, there after decreases but the secondary velocity increases. Also, as suction parameter increases, there is an increase of temperature in the two regions. Also, it is found that, the effect of viscosity ratio of the two fluids in the two regions is more pronounced on temperature when compared to the velocities. As the electrical conductivity ratio increases, both the primary and secondary velocities in the two regions also increase but temperature decreases. The effect of increasing Hartmann number is to increase the primary velocity and to decrease the secondary velocity in the upper region where as to increase both velocities in the lower region. By understanding the process involved in this study with this simple geometry, it is hoped that, more understanding can be gained in complex geometries of similar phenomenon.

Competing Interests

Authors have declared that no competing interests exist.

References

- [1] Zuber N. On the Variable Density Single Fluid Model for Two-phase Flow. J. Heat Transfer, Transactions of ASME. 1960;82C:255-257.
- [2] Griffith P, Wallis GB. Two-Phase Slug Flow. Journal of Heat Transfer, Transactions of ASME. 1961;83C:307-320.
- [3] Moissis R, Griffith P. Entrance Effects in a Two-Phase Slug Flow. Journal of Heat Transfer, Transactions of ASME. 1962;84C:29-39.
- [4] Bentwich M. Two-Phase Viscous Axial Flow in a pipe. Journal of Basic Engineering, Transactions of ASME. 1964;86D:669-672.
- [5] Charles ME, Lilleleht LU. Co-current Stratified Laminar Flow of two Immiscible Liquids in a Rectangular conduit. The Canadian Journal of Chemical Engineering. 1965;43:110- 116.
- [6] Packham BA, Shail R. Stratified laminar flow of two immiscible fluids. Proceeding of Cambridge Philosophy. 1971;69:443-448.
- [7] Golding JA, Mah CC. The Effect of Mass Transfer on Flow characteristics in Vertical Two-Phase Unstable flow. The Canadian Journal of Chemical Engineering. 1974;52:36-42.
- [8] Oshinowo T, Charles ME. Vertical Two-Phase Flow part I. Flow pattern Correlations. The Canadian Journal of Chemical Engineering. 1974;52:25-35.
- [9] Shipley DG. Two phase flow in large diameter pipes. Chemical Engineering Science. 1984;39:163-165.
- [10] Chamkha AJ. Flow of two immiscible fluids in porous and non -porous channels*,* ASME. Journal of Fluids Engineering. 2000;122:117-124.
- [11] Shail R. On laminar two-phase flow in magnetohydrodynamics. International Journal of Engineering Science. 1973;11:1103-1108.
- [12] Shercliff JA. The theory of Electromagnetic flow measurement. Cambridge University Press.
- [13] Saito M, Nange H, Inone S, Fujjii EY. Redistribution of gaseous phase of liquid metal two phase flow in a strong magnetic field. Journal of Nuclear Science Technology. 1978;l(15):279-335.
- [14] Dobran F. On the consistency conditions of averaging operators in 2 -phase flow models and on the formulation of magnetohydrodynamic 2-phase flow. International Journal of Engineering Science. 1981;19:1353-1368.
- [15] Malashetty MS, Umavathi JC. Two-Phase magnetohydrodynamic flow and heat transfer in an inclined channel. International Journal of Multiphase flow. 1997;22:545-560.
- [16] Chauhan DS, Rastogi P. Hall current and heat transfer effects on MHD flow in a channel partially filled with a porous medium in a rotating system. J. Turkish J. Eng. Env. Sci. 2009;33:167-184.
- [17] Chauhan DS, Rastogi P. Heat transfer effects on rotating MHD Couette flow in a channel partially filled by a Porous medium with Hall current. Journal of Applied Science and Eng. 2012;15(3):281-290.
- [18] Lohrasbi J, Sahai V. Magnetohydrodynamic heat transfer in two–phase flow between parallel plates. Applied Scientific Research. 1988;45:53-66.
- [19] Malashetty MS, Leela V. Magnetohydrodynamic heat transfer in two-fluid flow, Proc. ASME/AIChE 27th Natioanl Heat Transfer Conference and Exposition. 1991;28-31.
- [20] Malashetty MS, Leela V. Magnetohydrodynamic heat transfer in two-phase flow. Int. J. Engg. Sci. 1992;30(3):371-377.
- [21] Raju TL, Murty PSR. Hydromagnetic two-phase flow and heat transfer through two parallel plates in a Rotating system. J. of Indian Acad. Math. 2006;28(2):343-360.

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