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On Curvatures of Lorentzian Concircular Structure Manifolds

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Abstract

In this paper, we study the conditions satisfying $\widetilde{C}(\xi, X)P = 0$, $P(\xi, X)S = 0$, $P(\xi, X)\widetilde{C} = 0$, and $P(\xi, X)P = 0$ on a Lorentzian concircular structure manifold. According to these cases, (LCS)-manifolds are categorized and an example is used to demonstrate that the method presented in this paper is effective.

Keywords: (LCS)-Manifold; projective curvature Tensor and Concircular curvature.
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1 Introduction

Recently, in [1], A.A. Shaikh introduced the notion of Lorentzian concircular structure manifolds(briefly, (LCS)-manifolds) with an example, which generalizes the notion of LP-Sasakian manifolds were introduced by Matsumoto.

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Generalized φ-recurrent $(LCS)_n$-manifold was studied in [2, 3]

Works on the P-Sasakian manifolds satisfying certain conditions was studied in [4]. After then, In [5], C. özgür and M.M. Tripathi researched P-Sasakian manifolds satisfying certain conditions on the concircular curvature tensor.

In [6], Authors classified Lorentzian concircular structure manifolds satisfying the conditions $\widetilde{C}(\xi, X)\widetilde{C} = \widetilde{C}(\xi, X)R = \widetilde{C}(\xi, X)S = 0$ and $\widetilde{C}(\xi, X)C = 0$.

Again, recently, S.K. Yadav, P.K Dwivedi and D. Suthar have studied Lorentzian concircular structure manifolds which satisfy the certain conditions. This is related to the subject in the paper [7].

Motivated by the studies of the above authors, the aim of the this paper is to study on an $(LCS)_n$-manifold satisfying certain conditions the curvature tensors which have not been attempted in related papers. The present paper contain $(LCS)_n$-manifolds have the scalar curvature, constant sectional curvature, η-Einstein manifold and projective flat.

2 Preliminaries

In this section, we will give some notations used throughout this paper. We recall some necessary facts and formulas from the theory of $(LCS)_n$.

An n-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric tensor g, that is, M admits a smooth symmetric tensor field g of the (0,2) such that for each $p \in M$,

$$g_p : T_M(p) \times \ T_M(p) \longrightarrow \mathbb{R}$$

is a non-degenerate inner product of signature $(-, +, +, ..., +)$. A non-zero vector $X_p \in T_M(p)$ is said to be time-like(resp., non-spacelike, null, spacelike) if it satisfies $g_p(X_p, X_p) < 0$(resp., $\leq 0,\ = 0,\ > 0$). The category to which a given vector falls is called its casual character [8].

Definition 2.1. In a Lorentzian manifold (M, g), a vector field ρ defined by

$$g(X, \rho) = A(X), \tag{2.1}$$

for any $X \in \Gamma(TM)$, is said to be a concircular vector field if

$$(\nabla_X A)Y = \alpha\{g(X, Y) + \omega(X)A(Y)\}, \tag{2.2}$$

where α is a non-zero scalar function, A is a 1-form, ω is a closed 1-form and $\Gamma(TM)$ denote the differential vector fields set on M.

Let M be a Lorentzian manifold admitting a unit time-like concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi, \xi) = -1. \tag{2.3}$$

Since ξ is the unit Concircular vector field, there exists a non-zero 1-form such that

$$g(X, \xi) = \eta(X) \tag{2.4}$$

and its the covariant derivative

$$(\nabla_X \eta)Y = \alpha\{g(X, Y) + \eta(X)\eta(Y)\}, \quad \alpha \neq 0, \tag{2.5}$$

for all $X, Y \in \Gamma(TM)$, where ∇ denote the operator of covariant differentiation with respect to Lorentzian metric tensor g and α is a non-zero scalar function satisfying

$$\nabla_X \alpha = X(\alpha) = \rho\eta(X), \tag{2.6}$$

where ρ being a scalar function. If we set

$$\nabla_X \xi = \alpha.\phi X, \tag{2.7}$$

then from (2.5) and (2.7), we can derive

$$\phi X = X + \eta(X)\xi, \tag{2.8}$$

from which it follows that ϕ is a symmetric (1,1)-type tensor. Thus the Lorentzian manifold M together with the unit time-like concircular vector field ξ, its associated 1-form η and (1,1)-tensor field ϕ is said to be Lorentzian concircular structure manifold(briefly $(LCS)_n$-manifold). In particular, choosing $\alpha = 1$, then (LCS)-manifold becomes LP-Sasakian manifold of Matsumoto [9].

In an $(LCS)_n$-manifold, the following relations hold [8]:

$$\phi^2 = I + \eta \otimes \xi, \;\; \eta(\xi) = -1, \;\; \phi\xi = 0, \eta o \phi = 0, \tag{2.9}$$
$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{2.10}$$
$$\eta(R(X, Y)Z) = (\alpha^2 - \rho)(g(Y, Z)\eta(X) - g(X, Z)\eta(Y)), \tag{2.11}$$
$$R(X, Y)\xi = (\alpha^2 - \rho)(\eta(Y)X - \eta(X)Y), \tag{2.12}$$
$$R(\xi, X)Y = (\alpha^2 - \rho)(g(Y, X)\xi - \eta(Y)X), \tag{2.13}$$
$$(\nabla_X \phi)Y = \alpha\{g(X, Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}, \tag{2.14}$$
$$S(X, \xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{2.15}$$
$$S(\phi X, \phi Y) = S(X, Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{2.16}$$

for any $X, Y \in \Gamma(TM)$, where R and S denote the Riemannian curvature and Ricci tensors of M, respectively. For the papers in this subject, we refer to references.

Lemma 2.1. *In an $(LCS)_n$-manifold, the following relations hold.*

$$\rho = -\xi(\alpha) \;\; and \;\; X(\rho) = -\xi(\rho)\eta(X), \tag{2.17}$$

for any $X \in \Gamma(TM)$.

Definition 2.2. A Lorentzian concircular structure manifold M is said to be η-Einstein if the Ricci operator Q of M satisfies

$$Q = aI + b\eta \otimes \xi,$$

where a and b are smooth functions on M. If particular $b = 0$, then M becomes an Einstein manifold [10, 11].

Now, let (M, g) be an n-dimensional Riemannian manifold. The Concircular curvature tensor $\widetilde{C}$, the Weyl conformal curvature tensor C and Projective curvature tensor P are, respectively, given by

$$\widetilde{C}(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y], \tag{2.18}$$

$$\begin{aligned} C(X, Y)Z &= R(X, Y)Z - \frac{1}{n-2}[S(Y, Z)X - S(X, Z)Y \\ &+ g(Y, Z)QX - g(X, Z)QY] \\ &+ \frac{r}{(n-1)(n-2)}[g(Y, Z)X - g(X, Z)Y], \end{aligned} \tag{2.19}$$

and

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[g(Y,Z)QX - g(X,Z)QY], \tag{2.20}$$

for any $X, Y, Z \in \Gamma(TM)$, where r is the scalar curvature of M.

3 Some New Results on $(LCS)_n$-Manifolds

In this section, the necessary and sufficient conditions are given in an $(LCS)_n$-manifolds satisfying the derivations $\widetilde{C}(\xi, X)P = 0$, $P(\xi, X)S = 0$, $P(\xi, X)\widetilde{C} = 0$, and $P(\xi, X)P = 0$. Results are evaluated.

Theorem 3.1. *Let M be an n-dimensional $(LCS)_n$. Then the Projective curvature tensor P and the Concircular curvature tensor $\widetilde{C}$ of M satisfy*

$$\widetilde{C}(\xi, X)P = 0 \tag{3.1}$$

if and only if M either the scalar curvature of M is $r = n(n-1)(\alpha^2 - \rho)$ or M is locally isometric to the Euclidean space $\mathbb{E}^n(0)$.

Proof. The condition $\widetilde{C}(\xi, X)P = 0$ implies that

$$\begin{aligned} \widetilde{C}(\xi, X)P(Y, Z,)W &- P(\widetilde{C}(\xi, X)Y, Z)W - P(Y, \widetilde{C}(\xi, X)Z)W \\ &- P(Y, Z)\widetilde{C}(\xi, X)W = 0, \end{aligned}$$

for any $X, Y, Z, W \in \Gamma(TM)$. So by using (2.11), (2.13), (2.14), (2.18) and (2.20), we arrive at

$$\begin{aligned} &[\alpha^2 - \rho - \frac{r}{n(n-1)}][g(P(Y,Z)W, X) + \frac{1}{n-1}\{g(Z,W)g(QY,X) \\ -\ & g(Y,W)g(QZ,X)\} + (\alpha^2 - \rho)\{-g(X,Y)g(Z,W) \\ +\ & g(X,W)\eta(Y)\eta(Z) + g(X,Z)\eta(W)\eta(Y) + g(X,Z)g(Y,W) \\ -\ & g(X,W)\eta(Z)\eta(Y) + g(X,W)\eta(Y)\eta(Z) - g(X,Z)\eta(Y)\eta(W) \\ -\ & g(X,W)\eta(Y)\eta(Z) + g(Y,W)\eta(Z)\eta(X) - g(Z,W)\eta(Y)\eta(X) \\ +\ & g(X,Y)\eta(W)\eta(Z) + g(Z,W)g(X,Y) - g(X,W)\eta(Z)\eta(Y) \\ +\ & g(X,W)\eta(Y)\eta(Z) - g(Y,W)\eta(X)\eta(Z) \\ +\ & g(X,Z)\eta(W)\eta(Y) - g(X,Y)\eta(W)\eta(Z)\} + g(W,Z)\eta(X)\eta(Y) \\ -\ & g(X,Z)g(Y,W) - g(X,Z)\eta(Y)\eta(W)] = 0. \end{aligned}$$

Thus M is of either the scalar curvature $r = n(n-1)(\alpha^2 - \rho)$ or the equation

$$g(P(Y,Z)W, X) + \frac{1}{n-1}\{g(W,Z)g(QY,X) - g(Y,W)g(QZ,X)\} = 0,$$

which implies that M is a flat space. Thus M is locally isometric to the Euclidean space $\mathbb{E}^n(0)$. □

Theorem 3.2. *Let M be an n-dimensional $(LCS)_n$-manifold. Then the projective curvature tensor P and the Ricci tensor S of M satisfy*

$$P(\xi, X)S = 0$$

if and only if M is an Einstein manifold with scalar curvature $r = n(n-1)(\alpha^2 - \rho)$.

Proof. The condition $P(\xi, X)S = 0$ yields to

$$\begin{aligned}(P(\xi, X)S)(Y, Z) &= P(\xi, X)S(Y, Z) - S(P(\xi, X)Y, Z) \\ &- S(Y, P(\xi, X)Z) = 0,\end{aligned}$$

for any $X, Y, Z \in \Gamma(TM)$. From the condition $P(\xi, X)S = 0$ for $Z = \xi$, we have

$$S(P(\xi, X)Y, \xi) + S(Y, P(\xi, X)\xi) = 0, \tag{3.2}$$

for any $X, Y \in \Gamma(TM)$. By using (2.11), (2.12) and (2.20), we obtain

$$P(\xi, X)Y = \frac{1}{n-1}\eta(Y)QX - (\alpha^2 - \rho)\eta(Y)X \tag{3.3}$$

and

$$\begin{aligned}P(X, Y)\xi &= (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y] - \frac{1}{n-1}[\eta(Y)QX \\ &- \eta(X)QY].\end{aligned} \tag{3.4}$$

Substituting (3.3) and (3.4) into (3.2) and using (2.15), we obtain

$$S(QX, Y) - (n-1)(\alpha^2 - \rho)S(X, Y) = 0,$$

which implies that

$$QX = (n-1)(\alpha^2 - \rho)X.$$

Thus we conclude that

$$S(X, Y) = (n-1)(\alpha^2 - \rho)g(X, Y),$$

which proves our assertion. □

Theorem 3.3. *Let M be an $(LCS)_n$-manifold. Then the Projective curvature tensor P and the Concircular curvature tensor $\widetilde{C}$ of M satisfy*

$$P(\xi, X)\widetilde{C} = 0$$

if and only if either the scalar curvature of M is $r = n(n-1)(\alpha^2 - \rho)$ or M is an Einstein manifold.

Proof. The condition $P(\xi, X)\widetilde{C} = 0$ yields to

$$\begin{aligned}P(\xi, X)\widetilde{C}(Y, Z)U &- \widetilde{C}(P(\xi, X)Y, Z)U - \widetilde{C}(Y, P(\xi, X)Z)U \\ &- \widetilde{C}(Y, Z)P(\xi, X)U = 0,\end{aligned} \tag{3.5}$$

for any $X, Y, Z, U \in \Gamma(TM)$. By using (2.11), (2.12), (2.18) and (2.20), we obtain

$$\begin{aligned}P(\xi, X)\widetilde{C}(Y, Z)U &= \frac{1}{n-1}\eta(\widetilde{C}(Y, Z)U)QX \\ &- (\alpha^2 - \rho)\eta(\widetilde{C}(Y, Z)U)X,\end{aligned} \tag{3.6}$$

$$\begin{aligned}\eta(\widetilde{C}(Y, Z)U) &= [\alpha^2 - \rho - \frac{r}{n(n-1)}] \\ &\times [g(Z, U)\eta(Y) - g(Y, U)\eta(Z)],\end{aligned} \tag{3.7}$$

$$\begin{aligned}\widetilde{C}(P(\xi,X)Y,Z)U &= \frac{1}{n-1}\eta(Y)\widetilde{C}(QX,Z)U \\ &- (\alpha^2-\rho)\eta(Y)\widetilde{C}(X,Z)U, \end{aligned} \tag{3.8}$$

and

$$\begin{aligned}\widetilde{C}(Y,Z)P(\xi,X)U &= \frac{1}{n-1}\eta(U)\widetilde{C}(Y,Z)QX \\ &- (\alpha^2-\rho)\eta(U)\widetilde{C}(Y,Z)X. \end{aligned} \tag{3.9}$$

Substituting (3.6), (3.7), (3.8) and (3.9) into (3.5), we conclude that

$$[\alpha^2-\rho-\frac{r}{n(n-1)}]\eta(U)[S(X,Y)+(n-1)(\alpha^2-\rho)g(X,Y)]=0.$$

This implies that $r=n(n-1)(\alpha^2-\rho)$ or

$$S(X,Y)+(n-1)(\alpha^2-\rho)g(X,Y)=0,$$

which proves our assertion. □

Theorem 3.4. *Let M be an $(LCS)_n$-manifold. If the Projective curvature tensor P satisfies*

$$P(\xi,X)P=0,$$

then M is an η-Einstein manifold.

Proof. The condition $P(\xi,X)P=0$ implies that

$$\begin{aligned}P(\xi,X)P(Y,Z)U &- P(P(\xi,X)Y,Z)U-P(Y,P(\xi,X)Z)U \\ &- P(Y,Z)P(\xi,X)U=0, \end{aligned} \tag{3.10}$$

for any $X,Y,Z,U\in\Gamma(TM)$. By using (2.20), we obtain

$$P(\xi,X)Y=(\alpha^2-\rho)g(X,Y)\xi-\frac{1}{n-1}S(X,Y)\xi,, \tag{3.11}$$

$$\begin{aligned}P(P(\xi,X)Y,Z)U &= [(\alpha^2-\rho)g(X,Y)-\frac{1}{n-1}S(X,Y)] \\ &\times [(\alpha^2-\rho)g(Z,U)\xi-\frac{1}{n-1}S(Z,U)\xi] \end{aligned} \tag{3.12}$$

and

$$P(Y,Z)P(\xi,X)U=0. \tag{3.13}$$

Thus (3.11), (3.12) and (3.13) are used in (3.10), we reach

$$\begin{aligned}0 &= (n-1)(\alpha^2-\rho)[g(X,Z)S(Y,U)+g(X,Y)S(Z,U) \\ &+ S(X,Z)g(Y,U)]+(n-1)^2(\alpha^2-\rho)^2[g(X,Z)S(Y,U) \\ &+ g(Y,U)g(X,Z)+S(X,Y)S(Z,U)-S(Y,U)S(X,Z) \end{aligned} \tag{3.14}$$

From (3.14), we conclude

$$S(X,Y)=(n-1)(\alpha^2-\rho)[-g(X,Y)+(2+(n-1)(\alpha^2-\rho))\eta(X)\eta(Y)],$$

which proves our assertion. □

In this section, an example is used to demonstrate that the method presented in this paper is effective. While we are creating this example, the works of A. A. Shaikh and his co-authors were inspired. We are grateful to the authors.

Example 3.5. *We consider a 4-dimensional manifold*

$$M = \{(x, y, z, u) \in \mathbb{R}^4\},$$

where (x, y, z, u) are the standard coordinates in $\mathbb{R}^4$. In this space, let $\{E_1, E_2, E_3, E_4\}$ linearly independent global frame on M given by

$$E_1 = e^u(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}),\ \ E_2 = e^u\frac{\partial}{\partial y},\ \ E_3 = e^u\frac{\partial}{\partial z},\ \ E_4 = \frac{\partial}{\partial u}.$$

Let g be the Lorentzian metric defined by $g(E_1, E_1) = g(E_2, E_2) = g(E_3, E_3) = -g(E_4, E_4) = 1$ and $g(E_i, E_j) = 0$ for $i \neq j$. Let η be the 1-form defined by $\eta(U) = g(X, E_4)$ for any $X \in \Gamma(TM)$. Let ϕ be the (1,1) tensor defined by $\phi E_1 = E_1$, $\phi E_2 = E_2$, $\phi E_3 = E_3$ and $\phi E_4 = 0$. Then we have $\phi^2 X = X + \eta(X)\xi$, $\eta(X) = -1$ and $g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y)$. Thus (ϕ, ξ, η, g) defines an Lorentzian paracontact metric structure on M.

Let ∇ be the Levi-Civita connection on M, then we have

$$\begin{aligned} [E_1, E_2] &= -e^u E_2,\ \ [E_1, E_3] = -e^u E_3,\ \ [E_1, E_4] = -E_1, \\ [E_2, E_4] &= -E_2,\ \ [E_3, E_4] = -E_3,\ \ [E_2, E_3] = 0. \end{aligned}$$

Using Koszul formula for the Lorentzian metric g, one can easily get

$$\begin{aligned} \nabla_{E_1}E_4 &= -E_1,\ \ \nabla_{E_4}E_1 = 0,\ \ \nabla_{E_2}E_4 = -E_2,\ \ \nabla_{E_4}E_2 = 0, \\ \nabla_{E_3}E_4 &= -E_3,\ \ \nabla_{E_4}E_3 = 0,\ \ \nabla_{E_1}E_1 = -E_4, \\ \nabla_{E_2}E_1 &= e^u E_2,\ \ \nabla_{E_1}E_2 = 0,\ \ \nabla_{E_3}E_1 = e^u E_3,\ \ \nabla_{E_1}E_3 = 0, \\ \nabla_{E_1}E_4 &= -E_1,\ \ \nabla_{E_4}E_1 = 0,\ \ \nabla_{E_2}E_3 = \nabla_{E_3}E_2 = 0, \\ \nabla_{E_2}E_2 &= -e^u E_1 - E_4,\ \ \nabla_{E_3}E_3 = -e^u E_1 - E_4,\ \ \nabla_{E_4}E_4 = 0. \end{aligned}$$

So (ϕ, ξ, η, g) is an $(LCS)_4$ structure on M, that is, M is a 4-dimensional $(LCS)_4$ manifold with $\alpha = -1$ and $\rho = 0$. Using the above relations, we can easily calculation the the non-vanishing components of the curvature tensor R as follows:

$$\begin{aligned} R(E_1, E_2)E_1 &= (e^{2u} - 1)E_2,\ \ R(E_1, E_2)E_2 = -(e^{2u} - 1)E_1, \\ R(E_2, E_3)E_3 &= -(e^{2u} - 1)E_2,\ \ R(E_1, E_4)E_1 = -E_4, \\ R(E_1, E_4)E_4 &= -E_1,\ \ R(E_2, E_3)E_2 = (e^{2u} - 1)E_3, \\ R(E_3, E_4)E_3 &= -E_4,\ \ R(E_3, E_4)E_4 = -E_3, \\ R(E_1, E_3)E_3 &= -(e^{2u} - 1)E_1,\ \ R(E_2, E_4)E_4 = -E_2, \\ R(E_1, E_3)E_1 &= (e^{2u} - 1)E_3,\ \ R(E_2, E_4)E_2 = -E_2, \\ R(E_2, E_3)E_3 &= -(e^{2u} - 1)E_2,\ \ R(E_3, E_4)E_4 = -E_3, \\ R(E_2, E_4)E_4 &= -E_2. \end{aligned}$$

By direct calculations, Ricci tensor can be written as

$$S(X, Y) = (2e^{2u} - 1)g(X, Y) + 2(e^{2u} + 1)\eta(X)\eta(Y),$$

for all $X, Y \in \Gamma(TM)$. This tells us that $(LCS)_4$ manifold is an η-Einstein.

Thus the scalar curvature $r = \sum_{i=1}^{4} g(E_i, E_i)S(E_i, E_i)$ of M is given by

$$r = 6(e^{2u} - 1).$$

4 Conclusion

In this paper we have categorized Lorentzian concircular structure manifolds satisfying the conditions $\widetilde{C}(\xi,X)P = P(\xi,X)S = P(\xi,X)\widetilde{C} = P(\xi,X)P = 0$. This derivative operators are very important. It provides information about the structure of the manifold and manifold can be reduce known space.

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Competing Interests

The author declares that no competing interests exist.

References

[1] Shaikh AA, Lorentzian almost para contact manifolds with structure of concircular type. Kyungpook Math. J. 2003;43:305-314.

[2] Shaikh AA, Hui SK, On generalized φ-recurrent $(LCS)_n$-manifolds. Amercan Institute of Physisc Conf. Proc. 2010; vol.1309:419-429, ICMS Int. Conf. on Maht. Sci.: doi:10.1063/1.3525143.

[3] Shaikh AA, Basu T, Eyasmin S, On the existence ϕ-recurrent $(LCS)_n$-manifolds. Extracta Mathematicae. 2008;231:305-314.

[4] Adati T, Miyasawa T, On P-Sasakain manifolds satisfying certain condition. Tensor(N.S). 1979;33:173-178.

[5] Özgür C, Tripathi MM, On P-Sasakain manifolds satisfying certain conditions on the concircular curvature tensor. Turk. J. Math. 2007;31:171-179.

[6] Yadav SK, Dwived PK, Suthar D, On $(LCS)_{2n+1}$-manifolds satisfying certain conditions on the concircular curvature tensor. Thai J. Math. 2011;9(3):597-603.

[7] Sreenivasa GT, Venkatesha and Bagewadi CS, Some Results on $(LCS)_{2n+1}$-Manifolds. Bulletin of Mathematical Analysis and Applications. 2009;1(3):64-70.

[8] Shaikh AA, Binh TQ, On weakly symmetric ϕ-recurrent $(LCS)_n$-manifolds. J. Adv. Math. Studies 2. 2009;2:103-118.

[9] Matsumoto K, On Lorentzian paracontact manifolds. Bull. Yamagata Univ. Natur Sci. 1989;12(2):151-156.

[10] Shaikh AA, Baishya KK, On concircular structures spacetimes. J. of Math. and Stat. 2005;1:129-132. New-York, USA.

[11] Shaikh AA, Baishya KK, On concircular structre spactimes, II. American J. of Appl. sciences. 2006;3(4):1790-1794. New-York, USA.

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