



## Nonlinearity Estimation and Compensation for Robust Paths and Forces Control of Robot

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## Abstract

For the control of nonlinear robot dynamics a centralized design method so called the method of the exact linearization and decoupling by state feedback is often used. This method has the disadvantage in that an extra out-loop robust control design is needed to handle varied operating conditions such as payload variations. In this research a new control model design based on fictitious linear dynamic of decentralized structure called nonlinearity estimation and compensation is described in which coupling effects and other nonlinearities are estimated by an extended state observer and counteracted using the concept of disturbance rejection control. The controller is structurally robust against the varied operating conditions. Simulation for nonlinear positions and forces results on a PUMA 560 robot manipulator [1] shows the advantages of this approach.

*Keywords: Robot Control; decentralized Control; Nonlinearity; State Observer; Disturbance Rejection.*

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## 1 Introduction

From the viewpoint of the structure of control systems robot controllers can be divided into two different types: one with decentralized structure and the other centralized. By decentralized controllers each joint of a robot is considered as independent from the others and implemented with a simple linear controller, the nonlinear coupling effects between the joints are neglected. This concept is referred to as *independent joint control* and it was one of the first methods used in robot control. As robots have to do more demanding jobs, for example higher working velocities and the accuracy of positioning which have been often required, the various concepts for the improvement of the control-design method have then been introduced in terms of the nonlinear coupling effects. By one group of these improved controllers the decentralized structure is kept and the nonlinear coupling effects are compensated – either with the method of *computed torque* by feedforwarding the coupling torques obtained from the desired trajectories, or with the method of *joint torque control* by the feedback of coupling torques which are measured at each joint. Unlike the above-mentioned methods another group of these improved controllers uses a centralized structure, in which a robot is considered as a whole as a single multivariable system, where nonlinear effects are decoupled and compensated using the method of *exact linearization* and decoupling by state feedback – a method from the nonlinear control theory. The centralized control approach – the method of exact linearization – is on the one hand methodically more precise than one of the decentralized concepts – the method of computed torque, since the actual but not the desired position and velocity information are used; on the other hand it is more involving due to the on-line state feedback than the latter, in which the required feedforwarding torques can be computed off-line. In both cases, however, a complete knowledge of the dynamic model of robots is assumed. Incompletely-known model parameters, varied operating conditions like payload variations, the varied frictional forces and so on lead to the need for an additional robust control design, see [1]. This problem disappears, if another decentralized control concept called the method of joint torque control, is used, since parameter inaccuracies, payload variations and varied frictional effects together with the nonlinear coupling forces are recorded by the measurement of joint torques. However, this method suffers from the drawback that it needs additional measuring devices as commercial robots have. In this paper a new approach of decentralized control design, the so-called *nonlinearity estimation and compensation* is introduced, where these joint torques can be estimated with decentralized state observers and nonlinearities can thus be compensated by the feedback of these estimated values. The idea of nonlinearity estimation and compensation was introduced in [2]. Based on a fictitious model of the time behaviour of the nonlinearities, a state observer of an extended linear dynamic system is designed, which results in an estimate of the nonlinear effects. These nonlinearities are then counteracted by the estimated signals by applying disturbance rejection control techniques. Using this method not only to design robust decentralized position controllers for both rigid-body robots and robots with elastic joints but also to design system diagnosis in a Turbo rotor has been systematically investigated [3] and [4]. Although for simplicity only a single-joint robot was considered in the previous simulation examples, the good quality of estimates of the nonlinear gravitational and frictional forces has been successfully illustrated. Especially, this decentralized control concept has demonstrated its advantages in the case of real robot problems with inaccurate model parameters and payload variations in comparison to the method of exact linearization.

This paper will at first give a whole description how the method of nonlinearity estimation and compensation is used to design a robust decentralized controller for rigid-body robots. Then simulation results of a multi-joint robot – the elbow type robot PUMA 560 [1] – are given, which will confirm the high performance of this control concept, even if the nonlinear coupling effects among various joints are taken into account. Comparisons between this decentralized control concept and the centralized one – the method of exact linearization – are also made, which will demonstrate the advantages of the proposed approach.

In addition to the given method, in the recent years, some other methods for the nonlinear estimation have been used for the fault Tolerant control in nonlinear system [5] and for the improvement of performances in the chaotic system via Chebyshev approximation approach [6].

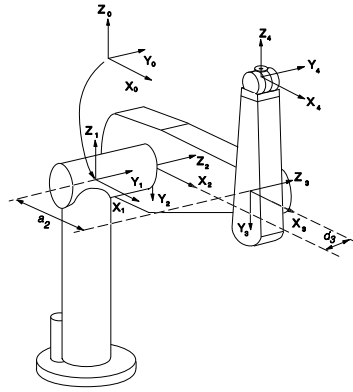


Figure 1: PUMA 560

The Figure 1 shows a universal rigid - body robot with the horizontal axis  $X_0$ , the vertical axis  $Y_0$  and the axis  $Z_0$  which are describing the body of the robot with 6 coordinates. The dynamics of this robot can be modeled according to [1] in terms of the general coordinate system  $\mathbf{q}$  as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{u}. \quad (1.1)$$

Here  $\mathbf{q}$  is the vector of joint coordinates,  $\mathbf{M}(\mathbf{q})$  the non-diagonal, positive definite mass matrix. Since all nonlinear effects will be considered as a whole by the following control design methods, the Coriolis and the centripetal as well as the gravitational forces are summed in one vector  $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$ . The vector  $\mathbf{u}$  represents the drive torques.

If frictional effects are considered – as it will be done below –, then they are also included within the vector  $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$ .

## 2 Control Design Methods

### 2.1 Exact Linearization

According to [1] a multivariable controller can be designed with the method of exact linearization based on (1.1). The drive torques  $\mathbf{u}$  are chosen herewith as

$$\mathbf{u} = \mathbf{M}(\mathbf{q})\mathbf{v} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}). \quad (2.1)$$

Considering the regularity of the mass matrix  $\mathbf{M}(\mathbf{q})$  a decoupled linear system

$$\ddot{\mathbf{q}} = \mathbf{v} \quad (2.2)$$

can be obtained using (1.1). Here  $\mathbf{v}$  is a new input vector.

For (2.3) simple linear controllers can be designed. If, for example, a desired trajectory  $\mathbf{q}_d(t)$  with the desired velocity  $\dot{\mathbf{q}}_d(t)$  and acceleration  $\ddot{\mathbf{q}}_d(t)$  is to be realized, it can thus be reached by the feedback

$$\mathbf{v} = \ddot{\mathbf{q}}_d + \mathbf{k}_{\dot{q}}(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{k}_q(\mathbf{q}_d - \mathbf{q}), \quad (2.3)$$

i.e.

$$\mathbf{u} = \mathbf{M}(\mathbf{q})(\ddot{\mathbf{q}}_d + \mathbf{k}_{\dot{q}}(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{k}_q(\mathbf{q}_d - \mathbf{q})) + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}), \quad (2.4)$$

here  $\mathbf{k}_{\dot{q}}$  and  $\mathbf{k}_q$  are diagonal matrices with positive diagonal elements which determine the dynamics of the controllers.

## 2.2 Nonlinearity Estimation and Compensation

Starting-point for the design of decentralized controllers with the method of nonlinearity estimation and compensation is still the coupled system (1). At first it is written down in a decoupled form relative to the single axes. For this the mass matrix  $\mathbf{M}(\mathbf{q})$  is divided into a constant diagonal matrix  $\mathbf{M}_0$  of the mean values of the moments of inertia and a remaining position-and force dependent part  $\Delta\mathbf{M}(\mathbf{q})$ :

$$\mathbf{M}(\mathbf{q}) = \mathbf{M}_0 + \Delta\mathbf{M}(\mathbf{q}). \quad (2.5)$$

These mean values can be chosen, for example, with regard to a typical working-space of the robot or along the desired trajectory. Summarizing further for each axis  $i$  all nonlinear terms in  $n_i$ ,

$$n_i = \sum_{j=1}^N \Delta M_{ij}(\mathbf{q})\ddot{q}_j + h_i(\mathbf{q}, \dot{\mathbf{q}}), \quad i = 1, \dots, N, \quad (2.6)$$

where  $N$  is the number of joints, equation (1) can thus be separately considered for a single axis:

$$\ddot{q}_i = \frac{1}{M_{0i}}(u_i - n_i), \quad i = 1, \dots, N. \quad (2.7)$$

Leaving out the index  $i$  for the sake of brevity, a state-space description of this one-axis model can be obtained with the state vector  $\mathbf{x} = [q \quad \dot{q}]^T$ :

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{N}n + \mathbf{B}u, \quad (2.8)$$

$$y = \mathbf{C}\mathbf{x} \quad (2.9)$$

with the matrices

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{B} = -\mathbf{N} = \begin{bmatrix} 0 \\ \frac{1}{M_0} \end{bmatrix}, \quad (2.10)$$

$$\mathbf{C} = [1 \quad 0]. \quad (2.11)$$

Here the measurement of the joint coordinate  $q$  is assumed.

The objective of the position control of a robot is often that the joint coordinate  $q$  has to track along a desired trajectory  $r(t) = q_d(t)$  determined by path planning. The control error is thus defined as  $q(t) - q_d(t)$  or in state-space form:

$$z = \mathbf{F}\mathbf{x} + Rr, \quad (2.12)$$

where

$$\mathbf{F} = [-1 \quad 0], \quad R = 1. \quad (2.13)$$

The design of each decentralized controller is based on the state-space model (2.8-2.13). On the one hand the coupling effects and other nonlinearities contained in  $n$  are compensated with the method of nonlinearity estimation and compensation. On the other hand the tracking control is reached by a feedforward, which is determined similar to the design of a disturbance rejection control [7]. Altogether an asymptotically stable control with

$$z(t) \rightarrow 0 \quad \text{for} \quad t \rightarrow \infty \quad (2.14)$$

is the whole design objective.

The time signal  $n$  of the nonlinearities and couplings is approximated appropriately by a time function, which is itself solution of a fictitious linear dynamic system:

$$n(t) \approx H_1 w_1(t), \quad (2.15)$$

$$\dot{w}_1(t) = W_1 w_1(t). \quad (2.16)$$

It was shown in [2] that this approximation can be at best carried out with step functions, so that an integrator model is chosen in (16-17):

$$H_1 = 1, \quad W_1 = 0. \quad (2.17)$$

The desired trajectory  $r(t) = q_d(t)$  is known. For the feedforward control the variables  $\dot{q}_d(t)$  and  $\ddot{q}_d(t)$  are additionally needed. Although they are theoretically available too, but usually they don't exactly correspond to the derivatives of the actually requested trajectory due to possible disturbances. Therefore, they are estimated by an observer which is constructed like (2.15-2.16) as well, see [4]:

$$r(t) \approx \mathbf{H}_2 \mathbf{w}_2(t), \quad (2.18)$$

$$\dot{\mathbf{w}}_2(t) = \mathbf{W}_2 \mathbf{w}_2(t). \quad (2.19)$$

The approximation is based on step and ramp functions in [8] and [9] is one way to solve them. The alternative method is the step function and harmonic  $\omega$  functions, here

$$\mathbf{H}_2 = [1 \ 0 \ 0], \quad \mathbf{W}_2 = \begin{bmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega \\ 0 & 0 & 0 \end{bmatrix} \quad (2.20)$$

and the approximation for the nonlinear forces is based on step function and  $sgn$  functions, here

$$\mathbf{H}_2 = [1 \ 0 \ 0], \quad \mathbf{W}_2 = \begin{bmatrix} 0 & sgn & 0 \\ 0 & 0 & sgn \\ 0 & 0 & 0 \end{bmatrix} \quad (2.21)$$

is selected.

The design of observers for the estimation of the signal  $n$ ,  $\dot{q}_d$  and  $\ddot{q}_d$ , as well as  $\dot{q}$  if needed, is based on a linear system, which is obtained by inserting (2.15-2.21) in (2.8-2.14):

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{w}_1 \\ \dot{\mathbf{w}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{N}H_1 & \mathbf{0} \\ \mathbf{0} & W_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ w_1 \\ \mathbf{w}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \\ 0 \end{bmatrix} u, \quad (2.22)$$

$$\begin{bmatrix} y \\ r \end{bmatrix} = \begin{bmatrix} \mathbf{C} & 0 & \mathbf{0} \\ \mathbf{0} & 0 & \mathbf{H}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ w_1 \\ \mathbf{w}_2 \end{bmatrix}, \quad (2.23)$$

$$z = [\mathbf{F} \quad \mathbf{0} \quad R\mathbf{H}_2] \begin{bmatrix} \mathbf{x} \\ w_1 \\ \mathbf{w}_2 \end{bmatrix}. \quad (2.24)$$

This extended system with the given system matrices is completely observable. Therefore, an observer can be designed according to one of the usual methods, a quadratic optimal identity observer is determined here. It is shown at the same time that this observer can be separated into two parts, one for  $q, \dot{q}, n$  and the other for  $q_d, \dot{q}_d, \ddot{q}_d$ :

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{w}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{L}_x\mathbf{C} & \mathbf{N}H_1 \\ -L_{w1}\mathbf{C} & W_1 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{w}_1 \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{L}_x \\ L_{w1} \end{bmatrix} y, \quad (2.25)$$

$$\dot{\hat{\mathbf{w}}}_2 = (\mathbf{W}_2 - \mathbf{L}_{w2}\mathbf{H}_2)\hat{\mathbf{w}}_2 + \mathbf{L}_{w2}\mathbf{r}. \quad (2.26)$$

The desired estimated values are then obtained from e.g.  $\hat{n} = H_1\hat{w}_1$ .

With the estimated variables  $\hat{\mathbf{x}}$ ,  $\hat{w}_1$  and  $\hat{\mathbf{w}}_2$  a feedback

$$u = -\mathbf{K}_x\hat{\mathbf{x}} - K_{w1}\hat{w}_1 - \mathbf{K}_{w2}\hat{\mathbf{w}}_2 \quad (2.27)$$

is constructed. The gain matrix  $\mathbf{K}_x$  of the state feedback can be set with standard methods such as pole assignment, the complete controllability of the matrices  $(\mathbf{A}, \mathbf{B})$  is fulfilled here. The feedback gain  $K_{w1}$  for the compensation of the nonlinearities and coupling effects and the gain matrix  $\mathbf{K}_{w2}$  for the feed forwarding of the desired trajectory are determined from the following equations:

$$(\mathbf{A} - \mathbf{B}\mathbf{K}_x)\mathbf{X}_1 - \mathbf{X}_1W_1 - \mathbf{B}K_{w1} = -\mathbf{N}H_1, \quad (2.28)$$

$$\mathbf{F}\mathbf{X}_1 = 0 \quad (2.29)$$

and

$$(\mathbf{A} - \mathbf{B}\mathbf{K}_x)\mathbf{X}_2 - \mathbf{X}_2W_2 - \mathbf{B}\mathbf{K}_{w2} = \mathbf{0}, \quad (2.30)$$

$$\mathbf{F}\mathbf{X}_2 = -R\mathbf{H}_2, \quad (2.31)$$

which are resulted from (2.14), see also [2]. The solutions for  $K_{w1}$  and for  $\mathbf{K}_{w2}$  are:

$$K_{w1} = -1, \quad (2.32)$$

$$\mathbf{K}_{w2} = [-K_{x1} \quad -K_{x2} \quad -M_0]. \quad (2.33)$$

$\mathbf{X}_1$  and  $\mathbf{X}_2$  are matrices which characterize the stationary behaviour of  $\mathbf{x}(t)$  depending on  $w_1(t)$  and  $\mathbf{w}_2(t)$ .

With this the controller is finally obtained:

$$u = M_0(\hat{q}_d) + K_{x2}(\hat{q}_d - \hat{q}) + K_{x1}(\hat{q}_d - \hat{q}) = \hat{n}. \quad (2.34)$$

## 2.3 Comparison

Comparing the control law (2.34) with the controller (2.4) from the method of exact linearization in Section 3.1, a formal agreement between both of them is found out. Whereas (2.4) needs an exact knowledge of the model parameters of  $\mathbf{M}(\mathbf{q})$  and  $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$ , (2.33-2.34) shows the advantage that unprecise models can be taken, here only the values of different  $M_0$ 's have to be chosen and all the unmodeled effects can be grasped in  $n$ 's by the used state observers.

The robustness of the controller (2.34) can easily be verified. If the parameters in the system description (1.1) are inaccurate, for example due to varied payload or unknown friction torques appear, the real system behavior must then be described with a modified model

$$\mathbf{M}'(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}'(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{u} \quad (2.35)$$

with a changed mass matrix  $\mathbf{M}'(\mathbf{q})$  and a changed vector of nonlinear effects  $\mathbf{h}'(\mathbf{q}, \dot{\mathbf{q}})$ .

Whereas the controller (2.4) is still based on the nominal model (1.1) so that mismatches between  $\mathbf{M}'$  and  $\mathbf{M}$  as well as between  $\mathbf{h}'$  and  $\mathbf{h}$  exist, the controller (2.34) has obviously regard to the changed system behavior. As in (2.5), let

$$\mathbf{M}'(\mathbf{q}) = \mathbf{M}_0 + \Delta\mathbf{M}'(\mathbf{q}), \quad (2.36)$$

then one has only to replace the term  $n_i$  in the description (8) by

$$n'_i = \sum_{j=1}^N \Delta M'_{ij}(\mathbf{q})\ddot{q}_j + h'_i(\mathbf{q}, \dot{\mathbf{q}}), \quad (2.37)$$

$$i = 1, \dots, N$$

similar to (2.6). Because the design of the controller (2.34) is based on the fact that  $n_i$  – and thus  $n'_i$  – is interpreted as an unknown function and to be estimated by  $\hat{w}_1$ , the controller fulfills its task also in the case of changed system behavior. The controller (2.34) is structurally robust against parameter inaccuracies and initial effects.

## 3 Simulation on Puma 560

The proposed control methods are demonstrated with simulations on a dynamic model of PUMA 560 robot manipulator. For the simulation the inertial parameters reported in [10] are used. Besides of this, frictional effects are modeled like that in [11] and the model parameters given in this thesis are used. For the modeling of payload variations it is assumed that a load is attached to the last link of the robot and the inertial parameters in the dynamic model are changed by this, see [10].

The end-effector is to track a circular path in three-dimensional space during the simulation, see Figure 1. This path is proposed in [12] to verify the performance characteristics of industrial robots. A similar path was also used in [13] with the performance comparison of different manipulator servo schemes. This is a circle within the diagonal plane of a quadratic cube, whose sides are arranged parallel to the axes of reference coordinate system. The middle point  $(X_0, Y_0, Z_0)$  of the circle is so defined that all three main axes of the robot are located roughly in the middle of their working areas at this point. The radius  $r_0$  of the circle is proportional to the largest reachable distance between the Tool-Center-Point and the axis of the first joint of robot. Altogether this trajectory is described by

$$X(t) = r_0 \sin(\omega(t)) \sin(\phi_0) + x_0, \quad (3.1)$$

$$Y(t) = r_0 \cos(\omega(t)) + y_0, \quad (3.2)$$

$$Z(t) = r_0 \sin(\omega(t)) \cos(\phi_0) + z_0, \quad (3.3)$$

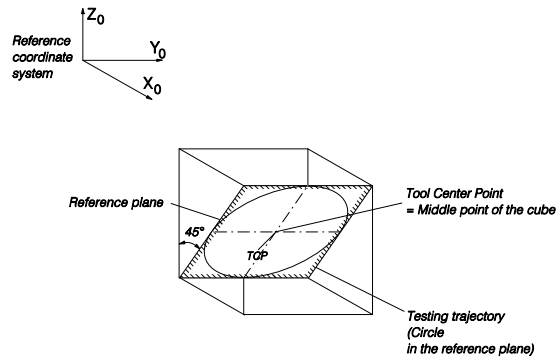


Figure 2: A three-dimensional circular path

in which  $\phi_0 = 45^\circ$  is the tilt angle, the angular displacement  $\omega(t)$  is here planned according to a general form of parametrization [13] is given by

$$\begin{aligned}
 0 \leq t \leq 1 : \quad & \omega(0) = 0, \quad \omega(1) = \frac{\pi}{2}, \\
 & \dot{\omega}(0) = 0, \quad \dot{\omega}(1) = \frac{\pi}{2}, \\
 & \ddot{\omega}(0) = 0, \quad \ddot{\omega}(1) = 0; \\
 1 \leq t \leq 3 : \quad & \omega(1) = \frac{\pi}{2}, \quad \omega(3) = \frac{3\pi}{2}, \\
 & \dot{\omega}(1) = \frac{\pi}{2}, \quad \dot{\omega}(3) = \frac{\pi}{2}, \\
 & \ddot{\omega}(1) = 0, \quad \ddot{\omega}(3) = 0; \\
 3 \leq t \leq 4 : \quad & \omega(3) = \frac{3\pi}{2}, \quad \omega(4) = 2\pi, \\
 & \dot{\omega}(3) = \frac{\pi}{2}, \quad \dot{\omega}(4) = 0, \\
 & \ddot{\omega}(3) = 0, \quad \ddot{\omega}(4) = 0.
 \end{aligned} \tag{3.4}$$

Here one circle is drawn in four seconds.

The orientation of the end-effector is commanded to stand constant throughout the path.

To evaluate the trajectory-tracking performance of the two different control design methods presented in Section 3, the following performance index [13] is used:

*Absolute position error* – the Cartesian distance between the desired and the actual end-effector position:

$$Q_e = \sqrt{(X_d - X)^2 + (Y_d - Y)^2 + (Z_d - Z)^2}. \tag{3.5}$$

If the actual position tracks the desired one exactly, then  $Q_e = 0$ .

At first the control behavior is showed when the parameters of the system model are exactly known, see Figure 3. The method of exact linearization gives no position error as expected, whereas this appears by the control design with the method of nonlinearity estimation and compensation especially at the beginning of the simulation, because the observers require a certain time until they can give the right estimates. In the figure 3, the horizontal axis shows the time progress of operation in sec [s] and the vertical axis presents the position error in [mm]. The Figure 3 shows the force nonlinearity with a friction by given model and their compensation of force.



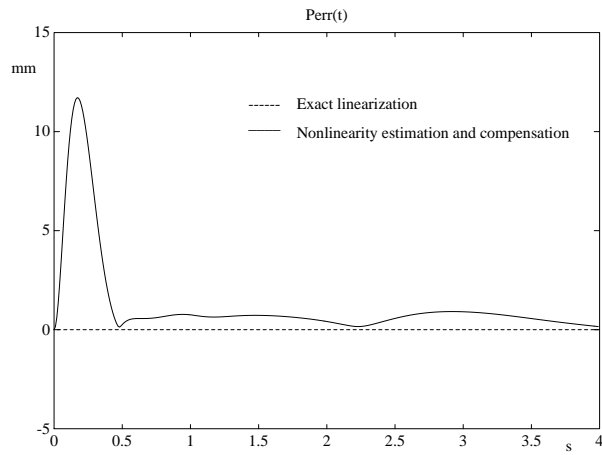


Figure 3: Position error by exact model knowledge

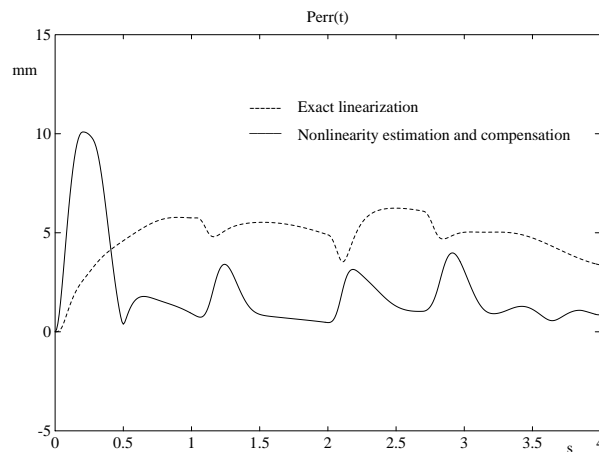


Figure 4: Position error by real problem

In the Figure 4 like the Figure 3, the horizontal axis shows the time progress of operation in sec [s] and the vertical axis presents the position error in [mm]. The advantage of the control method of nonlinearity estimation and compensation is then recognized, when real robot problems are considered. The Figure 3 and the Figure 4 give the comparison of this method with the method of exact linearization under the conditions that a load with  $2kg$  are taken into account respectively. In the Figure 5, the horizontal axis shows the time progress of operation in sec [s] and the vertical axis presents the nonlinear force in [N] The Figure 5 illustrates the force nonlinearity with a friction in the X axis by given model( blue color) and their compensation of force(green color) and give the comparison of this method with the method of exact linearization under the conditions that a load with  $2kg$  are

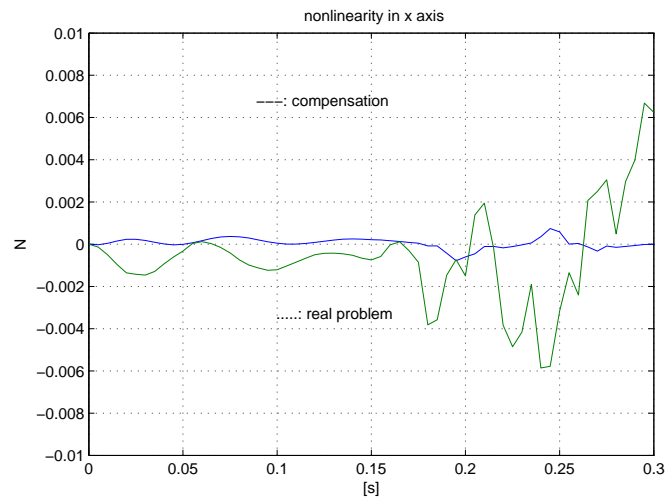


Figure 5: Force nonlinearity by x axis

considered. After 3 [s] the compensation is started. In the Figure 6 like Figure 5, the horizontal axis

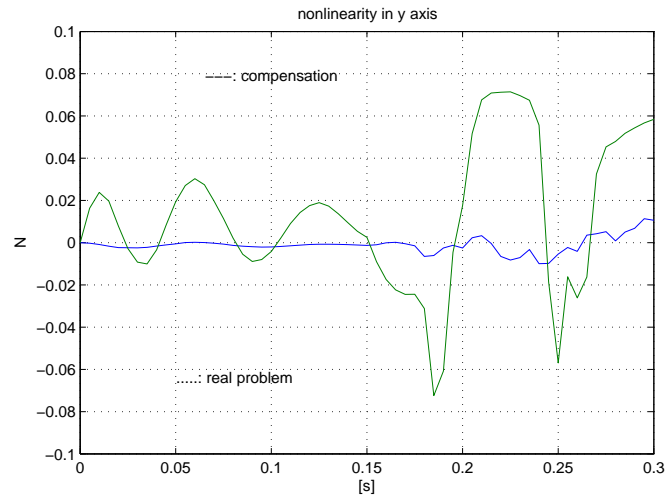


Figure 6: Force nonlinearity by y axis

shows the time progress of operation in sec [s] and the vertical axis presents the nonlinear force in [N]. The Figure 6 shows the force nonlinearity with a friction in the y axis by the same model and their compensation of force and give the comparison of this method with the method of exact linearization under the the same conditions like the Figure 5. The position error and force nonlinearity of the proposed method are much smaller as the existed one except at the very beginning.

## 4 Conclusion

The method of nonlinearity estimation and compensation has been proved to be a suitable approach for the design of robust position- and force nonlinearities controllers for robots. In addition to its robustness its decentralized structure offers additional advantages such that the concept of independent joint control can further be used, even in an improved manner. The requirements on the modeling of the robot dynamics are very low. And the control design can be easily done because it is based only on linear system theory. Simulation results for the compensation of the position errors- and nonlinear forces have shown the efficiency of the proposed control design method.

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## Competing Interests

The author declares that no competing interests exist.

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## Appendix

The sum of the elementary matrices used in the equations can be described as following

$$\begin{aligned}
 M_{(j_k, j_k)}^{(g)}(i_e) &= \sum_{i_e=1}^N \left[ \sum_{j_k=1}^{(i_e-1)\frac{n}{2}+1} \left( \sum_{i, j=j_k} M_e(ii, jj) \right) \right]_{(i_e)} \\
 &\quad + M_{(dime, dime)}^0 \\
 K_{(j_k, j_k)}^{(g)}(i_e) &= \sum_{i_e=1}^N \left[ \sum_{j_k=1}^{(i_e-1)\frac{n}{2}+1} \left( \sum_{i, j=j_k} K_e(ii, jj) \right) \right]_{(i_e)} \\
 &\quad + K_{(dime, dime)}^0 \\
 G_{(j_k, j_k)}^{(g)}(i_e) &= \sum_{i_e=1}^N \left[ \sum_{j_k=1}^{(i_e-1)\frac{n}{2}+1} \left( \sum_{i, j=j_k} G_e(ii, jj) \right) \right]_{(i_e)} \\
 &\quad + G_{(dime, dime)}^0 \\
 D_{(j_k, j_k)}^{(g)}(i_e) &= \sum_{i_e=1}^N \left[ \sum_{j_k=1}^{(i_e-1)\frac{n}{2}+1} \left( \sum_{i, j=j_k} D_e(ii, jj) \right) \right]_{(i_e)} \\
 &\quad + D_{(dime, dime)}^0
 \end{aligned}$$

The system matrices used in equation are follows

$$A = \begin{bmatrix} 0 & \vdots & I_{(nn)} \\ \dots & \dots & \dots \\ -M_g^{-1}K_g & \vdots & -M_g^{-1}(D_{dg} + G_g) \end{bmatrix}_{(64 \times 64)}$$

The vectors of the gravity and the vector of nonlinearity input are

$$u(t) = \begin{bmatrix} 0 \\ \dots \\ M_g^{-1} f_{eg} \end{bmatrix}, N_R = \begin{bmatrix} 0 \\ \dots \\ -M_g^{-1} Ls_{(i_e)} \end{bmatrix};$$

They are of order (64 x 1). The vector  $N_u$  is as follows

$$N_u = \begin{bmatrix} 0 \\ \dots \\ -M_g^{-1} Ls_{(i_e)} \end{bmatrix}_{(i_e=2,4,6)}$$

where the matrices  $M_g, K_g, D_g$  and  $G_g$  are the sums of the elementary matrices of  $M_e, K_e, D_e$  and  $G_e$ . The vector of the excitation consists of gravity and the sgn function presented by

$$f_{(g, i_e=1, \dots, N)} = 0,$$

$$sgn(x) = 1 \quad x > 1, \quad 0, \quad x = 0, \quad -1 \quad x < 1$$

The order of the  $f_g$  is of (32 x 1) and  $f_u$  is of (32 x 1).

$$\begin{aligned}
 f_{(u;1)} &= -e_m \Omega^2 m_{(ex)} \sin(\Omega t + \beta) \\
 f_{(u;2)} &= e_m \Omega^2 m_{(ex)} \cos(\Omega t + \beta)
 \end{aligned}$$

The angle of the phase is  $\beta = 0$ , the length of the subsystem of rotor  $l_e = 0.5m$ , the diameter of the subsystem of link makes  $ed = 0.25m$ . The mass of elementary subsystem:  $m = \pi l_e \rho \frac{d_e^2}{4}$ , The density is  $\rho = 7860 \frac{kg}{m^3}$ , excentricity  $e_m = 0.0001$ , the mass of the excentricity:  $m_{(ex)} = 3$  kg respectively. The modulus  $E$  is of  $2.1 * 10^5 N/mm^2$ . The stiffness of bearing:  $K_{bearing} = 15 * 10^5 N/mm^2$ . The measurement matrix of order  $(4 \times 32)$ ,  $C_{(i=1, \dots, 4, j=1, \dots, 32)} = 0$ , except  $C_{(1,1)} = C_{(2,2)} = C_{(29,29)} = C_{(30,30)} = 1$ . The number of the nonlinearities  $n_f$  is 1 and the number of the measurements  $m_e$  makes 4. The elementary matrices  $K_e, M_e$  depend on the geometry. The damping matrix is  $D_e = \alpha K_e + \gamma M_e$  with  $\alpha = 0.000001, \gamma = 0$ . The elementary mass matrix, stiffness matrix and damping matrix are of order  $(8 \times 8)$

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