Advances in Research
7(1): 1-13, 2016, Article no.AIR.25478
ISSN: 2348-0394, NLM ID: 101666096

SCIENCEDOMAIN *international*
www.sciencedomain.org

Alternative Layout and Automation of Optimality Results for Machine Replacement Problems Based On Stationary Data and Age Transition Perspectives

Ukwu Chukwunenye[1*]

[1]*Department of Mathematics, University of Jos, P.M.B. 2084, Jos, Plateau State, Nigeria.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/AIR/2016/25478
Editor(s):
(1) Martin Kroger, Professor, Computational Polymer Physics, Swiss Federal Institute of Technology (ETH Zürich), Switzerland.
Reviewers:
(1) Stephen Akandwanaho, University of KwaZulu-Natal, South Africa.
(2) Raad Y. Qassim, Federal University of rio de Janeiro, Brazil.
Complete Peer review History: http://sciencedomain.org/review-history/14189

Original Research Article

Received 4th March 2016
Accepted 8th April 2016
Published 15th April 2016

ABSTRACT

Aim: This investigation aimed at automating the computations of optimal replacement policies and rewards for a class of equipment replacement problems based on time perspectives and stationary pertinent data.
Methodology: The aim was achieved by the exploitation of the structure of the states given as functions of decision periods, in age-transition dynamic programming recursions.
Results: Alternative Excel solution implementation templates were designed and automated for the determination of the optimal replacement policies in machine replacement problems, with pertinent data given only as functions of machines' ages.
Conclusion: The automation of these templates obviates the need for manual inputs of the states and stage numbering, as well as the inherent tedious and prohibitive manual computations associated with dynamic programming formulations and may be optimally exploited for sensitivity analyses on such models.

Keywords: Age transition dynamic programming recursions; automation of optimality results; decision period; machine replacement problems; pertinent data; sensitivity analyses.

**Corresponding author: E-mail: cukwu@hotmail.com;*

1. INTRODUCTION

The Equipment Replacement Problem is a subject of considerable and diverse research interests.

Consider the problem of researching an optimal Equipment Replacement policy over an n -period planning horizon. At the start of each year a decision is made whether to keep the equipment in service an extra year or to replace it with a new one at some salvage value. As remarked by Taha [1], "the determination of the feasible values for the age of the machine at each stage is somewhat tricky". The latter went on to obtain the optimal replacement ages using network diagrammatic approach, with machine ages on the vertical axis and decision years on the horizontal axis. In an alternative time perspective approach, Winston [2] initiated the determination process for the optimal replacement time with network diagrams consisting of upper half-circles on the horizontal axis, initiating from each feasible time of the planning horizon and terminating at feasible times, with the length of successive transition times at most, the maximum operational age of the equipment. Sequel to this, Winston [2] formulated dynamic recursions as functions of the decision times, the corresponding feasible transition times, the problem data and the cash-flow profile. Unfortunately network diagrams are unwieldy, cumbersome and prone to errors, especially for large problem instances; consequently the integrity of the desired optimal policies may be compromised. Verma [3] and Gupta & Hira [4] used the average annual cost criteria to determine alternative optimal policies and the corresponding optimal rewards in a non-dynamic programming fashion. Gress et al. [5] modeled the equipment replacement problem using a Markov decision process and a reward function that can be more helpful in processing industries. Unfortunately, the key issues of large-scale implementation and sensitivity analyses were not discussed by the afore-mentioned authors.

A new impetus was provided for sensitivity analyses and implementation paradigm shift by Ukwu [6], with respect to optimal solutions to machine replacement problems. Ukwu [6] pioneered the development of computational formulas for the feasible states corresponding to each decision year in a certain class of equipment Replacement problems, thereby eliminating the drudgery and errors associated with the drawing of network diagrams for such determination. Ukwu [6] went further to design prototypical solution templates for optimal solutions to such problems, complete with an exposition on the interface and solution process. Ukwu [7] extended the formulations and results in [6] to a class of machine replacement problems, with pertinent data given as functions of machine ages and the decision periods of the planning horizon. By restructuring the data in three – dimensional formats Ukwu [7] appropriated key features of the template in Ukwu [6] for the extended template. Finally Ukwu [7] solved four illustrative examples of the same flavour that demonstrated the efficiency, power and utility of the solution template prototype. In Ukwu [7] it was pointed out that the template could be deployed to solve each equipment replacement problem in less than 10 percent of the time required for the manual generation of the alternate optima. However a major draw-back of the templates in Ukwu [6,7] is that for any problem instance, the inputs of the states and stage numbering are manually generated. Moreover, the templates require row updating of the formulas for the optimal criterion function values for problems of larger horizon lengths. Evidently this functionality needs to be improved upon for more speedy solution implementations, especially for practical problems of long planning horizons. This article sets out to remedy the above situation. The major contributions of the articles are as follows: The work provides alternative layout and solution templates to those in Ukwu [6], with full automation of all computations for $t_1 = 1$. The case $t_1 \geq 2$ requires only trivial repositioning of the last automated state $i-1+t_1$, t_1-1 places to the right, with the cell values in-between deleted in each of stages $m+1+t_1, m+t_1, \ldots, 2, 1$ of the process, where $i \in \{1, 2, \cdots, n\}$ is the decision year, m is the mandatory equipment replacement age, n is the length of the planning horizon and t_1 is the starting age of the equipment. The article also gives an exposition on the solution template incorporating the outputs for given problem instances, as reflected in Tables 1, 2, 3, 4 and 5. The outputs are consistent with the general exposition.

2. MATERIALS AND METHODS

In this section, the problem data, working definitions, elements of the DP model and the dynamic programming (DP) recursions are laid out as follows:

Equipment Starting age $= t_1$

Equipment Replacement age $= m$

S_i = The set of feasible equipment ages (states) in decision period i (say year i), $i \in \{1, 2, \dots, n\}$

$r(t)$ = annual revenue from a $t-$ year old equipment

$c(t)$ = annual operating cost of a $t-$ year old equipment

$s(t)$ = salvage value of a $t-$ year old equipment; $t = 0, 1, \dots, m$

I = fixed cost of acquiring a new equipment in any year

The elements of the DP are the following:

1. Stage i, represented by year $i, i \in \{1, 2, \dots, n\}$
2. The alternatives at stage (year) i. These call for keeping or replacing the equipment at the beginning of year i
3. The state at stage (year) i, represented by the age of the equipment at the beginning of year i.

Let $f_i(t)$ be the maximum net income for years $i, i+1, \dots, n-1, n$ given that the equipment is t years old at the beginning of year i.

Note: The definition of $f_i(t)$ starting from year i to year n implies that backward recursion will be used. Forward recursion would start from year 1 to year i.

The template will implement the following theorem formulated in [1] and exploited in [6], using backward recursive procedure.

2.1 Theorem 1: Dynamic Programming Recursions for Optimal Policy and Rewards [1]

$$f_i(t) = \max \begin{cases} r(t) - c(t) + f_{i+1}(t+1); \text{ IF KEEP} \\ r(0) + s(t) - I - c(0) + f_{i+1}(1); \text{ REPLACE} \end{cases}$$

$$f_{n+1}(x) = s(x), \quad i = 0, 1, \dots, n-1, \ x = \text{age of machine at the start of period } n+1$$

3. RESULTS AND DISCUSSION

3.1 Theorem on Analytic Determination of the Set of Feasible Ages at Each Stage. Ukwu [6]

Let S_i denote the set of feasible equipment ages at the start of the decision year i. Let t_1 denote the age of the machine at the start of the decision year i, that is, $S_1 = \{t_1\}$. Then for $i \in \{1, 2, \dots, n\}$,

$$S_i = \begin{cases} \left\{\min\limits_{2 \le j \le i}\{j-1, m\}\right\} \cup \left\{1 + (i - 2 + t_1)\operatorname{sgn}\left(\max\{m + 2 - t_1 - i, 0\}\right)\right\}, \text{if } t_1 < m \\ \\ \left\{\min\limits_{2 \le j \le i}\{j-1, m\}\right\}, \text{ if } t_1 \ge m \end{cases}$$

An Excel template will now be designed and deployed to solve the practical problems below with a prescribed starting age less than the replacement age. In the sequel an exposition on the template will be given using the above problem as an illustrative example.

3.2 Application Problems on Theorem 3.1 and the Implementation of the Solution Templates

A company needs to determine the optimal replacement policy for a current t_1- year old equipment over the next eight years. The following table gives the data of the problem. The company requires that a 6 – year old equipment be replaced. The cost of a new machine is $100,000.

Table 1. Pertinent data for optimal policy and reward determination

Age: *t* yrs.	**Revenue: *r*(*t*) ($)**	**Operating cost: *c*(*t*) ($)**	**Salvage value: *s*(*t*) ($)**
0	20,000	200	-
1	19,000	600	80,000
2	18,500	1,200	60,000
3	17,200	1,500	50,000
4	15,500	1,700	30,000
5	14,00	1,800	10,000
6	12,200	2,200	5,000

Solve the above problems for $t_1 \in \{1, 2, 3, 4\}$, using dynamic programming recursions.

The article arbitrarily singles out two among the alternate optimal results for $t_1 = 3$. All template outputs are self-explanatory.

Table 2. Template solution of the equipment replacement problem with starting age of 1 year

Equipment replacement problem solution template					***n***	**Starting age**	
Replacement age =			**6**	**yrs**	**8**	**1**	
	Given data			**Stage**	8		
	***I* =**	**100000**	***V(0) = r(0)-c(0) - I =***		**-80200**		
Age *t* (yrs.)	0	1	2	3	4	5	6
revenue: *r(t)* ($)	20000	19000	18500	17200	15500	14000	12200
Mnt. cost, *c(t)* ($)	200	600	1200	1500	1700	1800	2200
Salvage value, s(t)		80000	60000	50000	30000	10000	5000
K		78400	67300	45700	23800	17200	Must Replace
R		79800	59800	49800	29800	9800	4800
Opt. value: f(t)		79800	67300	49800	29800	17200	4800

Opt. decision	*R*	*K*	*R*	*R*	*K*	*R*
State	1	2	3	4	5	6
			Stage	7		
K	85700	67100	45500	31000	17000	Must Replace
R	79600	59600	49600	29600	9600	4600
Opt. value: f(t)	85700	67100	49600	31000	17000	4600
Opt. Decision	*K*	*K*	*R*	*K*	*K*	*R*
State	1	2	3	4	5	6
			Stage	6		
K	85500	66900	46700	30800	16800	Must Replace
R	85500	65500	55500	35500	15500	10500
Opt. value: *f(t)*	85500	66900	55500	35500	16800	10500
Opt. Decision	K/R	K	R	R	K	R
State	1	2	3	4	5	6
			Stage	5		
K	85300	72800	51200	30600	22700	
R	85300	65300	55300	35300	15300	
Opt. value: *f(t)*	85300	72800	55300	35300	22700	
Opt. Decision	*K/R*	*K*	*R*	*R*	K	
State	1	2	3	4	5	
			Stage	4		
K	91200	72600	51000	36500		
R	85100	65100	55100	35100		
Opt. value: *f(t)*	91200	72600	55100	36500		
Opt. Decision	*K*	*K*	*R*	*K*		
State	1	2	3	4		
			Stage	3		
K	91000	72400	52200			
R	91000	71000	61000			
Opt. value: *f(t)*	91000	72400	61000			
Opt. Decision	*K/R*	*K*	*R*			
State	1	2	3			
			Stage	2		
K	90800	78300				
R	90800	70800				
Opt. value: *f(t)*	90800	78300				
Opt. Decision	*K/R*	*K*				
State	1	2				
			Stage	1		
K	96700					
R	90600					
Opt. value: *f(t)*	96700					
Opt. Decision	*K*					
State	1					

Table 3. Template solution of the equipment replacement problem with starting age of 2 years

Equipment replacement problem solution template					***N***	**Starting age**	
Replacement age =			**6**	**yrs**	**8**	**2**	
	Given data			**Stage**	8		
	***I* =**	**100000**	***V(0) = r(0)-c(0) - I* =**		**-80200**		
Age *t* (yrs.)	0	1	2	3	4	5	6
Revenue: *r(t)* ($)	20000	19000	18500	17200	15500	14000	12200
Mnt. cost, *c(t)* ($)	200	600	1200	1500	1700	1800	2200
Salvage value, s(t)		80000	60000	50000	30000	10000	5000
K		78400	67300	45700	23800	17200	Must replace
R		79800	59800	49800	29800	9800	4800
Opt. value: f(t)		79800	67300	49800	29800	17200	4800
Opt. decision		*R*	*K*	*R*	*R*	*K*	*R*
State		1	2	3	4	5	6
				Stage	7		
K		85700	67100	45500	31000	17000	Must replace
R		79600	59600	49600	29600	9600	4600
Opt. value: f(t)		85700	67100	49600	31000	17000	4600
Opt. decision		*K*	*K*	*R*	*K*	*K*	*R*
State		1	2	3	4	5	6
				Stage	6		
K		85500	66900	46700	30800	16800	
R		85500	65500	55500	35500	15500	
Opt. value: *f(t)*		85500	66900	55500	35500	16800	
Opt. Decision		K/R	K	R	R	K	
State		1	2	3	4	5	
				Stage	5		
K		85300	72800	51200	30600		Must replace
R		85300	65300	55300	35300		10300
Opt. value: *f(t)*		85300	72800	55300	35300		10300
Opt. Decision		*K/R*	*K*	*R*	*R*		R
State		1	2	3	4		6
				Stage	4		
K		91200	72600	51000		22500	
R		85100	65100	55100		15100	
Opt. value: *f(t)*		91200	72600	55100		22500	
Opt. Decision		*K*	*K*	*R*		K	
State		1	2	3		5	
				Stage	3		
K		91000	72400		36300		

R	91000	71000		41000
Opt. value: *f(t)*	91000	72400		41000
Opt. decision	*K/R*	*K*		R
State	1	2		4
			Stage	2
K	90800		56700	
R	90800		60800	
Opt. value: *f(t)*	90800		60800	
Opt. Decision	*K/R*		R	
State	1		3	
			Stage	1
K		78100		
R		70600		
Opt. value: *f(t)*		78100		
Opt. Decision		K		
State		2		

Table 4. Template solution of the equipment replacement problem with starting age of 3 years

Equipment replacement problem solution template					*n*	**Starting age**	
Replacement age =			**6**	**yrs**	**8**	**3**	
	Given data			**Stage**	8		
	***I* =**	**100000**	***V(0) = r(0)-c(0) - I =***		**-80200**		
Age *t* (yrs.)	0	1	2	3	4	5	6
Revenue: *r(t)* ($)	20000	19000	18500	17200	15500	14000	12200
Mnt. cost, *c(t)* ($)	200	600	1200	1500	1700	1800	2200
Salvage value, s(t)		80000	60000	50000	30000	10000	5000
K		78400	67300	45700	23800	17200	Must replace
R		79800	59800	49800	29800	9800	4800
Opt. value: f(t)		79800	67300	49800	29800	17200	4800
Opt. decision		*R*	*K*	*R*	*R*	*K*	*R*
State		1	2	3	4	5	6
				Stage	7		
K		85700	67100	45500	31000	17000	Must replace
R		79600	59600	49600	29600	9600	4600
Opt. value: f(t)		85700	67100	49600	31000	17000	4600
Opt. decision		*K*	*K*	*R*	*K*	*K*	*R*
State		1	2	3	4	5	6
				Stage	6		
K		85500	66900	46700	30800	16800	
R		85500	65500	55500	35500	15500	
Opt. value: *f(t)*		85500	66900	55500	35500	16800	
Opt. decision		K/R	K	R	R	K	
State		1	2	3	4	5	

			Stage	5		
K	85300	72800	51200	30600		
R	85300	65300	55300	35300		
Opt. value: *f(t)*	85300	72800	55300	35300		
Opt. Decision	*K/R*	*K*	*R*	*R*		
State	1	2	3	4		
			Stage	4		
K	91200	72600	51000			Must replace
R	85100	65100	55100			10100
Opt. value: *f(t)*	91200	72600	55100			10100
Opt. decision	*K*	*K*	*R*			R
State	1	2	3			6
			Stage	3		
K	91000	72400			22300	
R	91000	71000			21000	
Opt. value: *f(t)*	91000	72400			22300	
Opt. decision	*K/R*	*K*			K	
State	1	2			5	
			Stage	2		
K	90800			36100		
R	90800			40800		
Opt. value: *f(t)*	90800			40800		
Opt. decision	*K/R*			R		
State	1			4		
			Stage	1		
K			56500			
R			60600			
Opt. value: *f(t)*			60600			
Opt. decision			R			
State			3			

The following optimality results are obtained in one fell swoop:

Some Alternate optimal paths: **3R1K2K3R1K2K3R1S; 3R1K2K3R1R1K2K3S**
Others are readily available from the template solutions.

The optimal net income from the beginning of year 1 to the end of year 8 is **$60,600.00.**

Table 5. Template solution of the equipment replacement problem with starting age of 4 years

Equipment replacement problem solution template					*n*	**Starting age**	
Replacement age =			**6**	**yrs**	**8**	**4**	
	Given data			**Stage**	8		
	***I* =**	**100000**	***V(0) = r(0)- c(0) - I* =**		**-80200**		
Age *t* (yrs.)	0	1	2	3	4	5	6
Revenue: *r(t)* ($)	20000	19000	18500	17200	15500	14000	12200
Mnt. cost, *c(t)* ($)	200	600	1200	1500	1700	1800	2200

Salvage value, s(t)	80000	60000	50000	30000	10000	5000
K	78400	67300	45700	23800	17200	Must replace
R	79800	59800	49800	29800	9800	4800
Opt. value: f(t)	79800	67300	49800	29800	17200	4800
Opt. decision	*R*	*K*	*R*	*R*	*K*	*R*
State	1	2	3	4	5	6
			Stage	7		
K	85700	67100	45500	31000	17000	Must replace
R	79600	59600	49600	29600	9600	4600
Opt. value: f(t)	85700	67100	49600	31000	17000	4600
Opt. decision	*K*	*K*	*R*	*K*	*K*	*R*
State	1	2	3	4	5	6
			Stage	6		
K	85500	66900	46700	30800	16800	
R	85500	65500	55500	35500	15500	
Opt. value: *f(t)*	85500	66900	55500	35500	16800	
Opt. Decision	K/R	K	R	R	K	
State	1	2	3	4	5	
			Stage	5		
K	85300	72800	51200	30600		
R	85300	65300	55300	35300		
Opt. value: *f(t)*	85300	72800	55300	35300		
Opt. decision	*K/R*	*K*	*R*	*R*		
State	1	2	3	4		
			Stage	4		
K	91200	72600	51000			
R	85100	65100	55100			
Opt. value: *f(t)*	91200	72600	55100			
Opt. decision	*K*	*K*	*R*			
State	1	2	3			
			Stage	3		
K	91000	72400				Must replace
R	91000	71000				16000
Opt. value: *f(t)*	91000	72400				16000
Opt. decision	*K/R*	*K*				R
State	1	2				6
			Stage	2		
K	90800				28200	
R	90800				20800	
Opt. value: *f(t)*	90800				28200	
Opt. decision	*K/R*				K	
State	1				5	
			Stage	1		
K				42000		
R				40600		
Opt. value: *f(t)*				42000		
Opt. Decision				K		
State				4		

3.3 An Exposition on the Solution Templates for Tables 2, 3, 4 and 5 and Generic Problems

Step 1: Documentation, stage numbering automation, Data and fixed value storage

Use Excel references A1:H2 for documentation. Save the problem data in the indicated cells using the Copy and Paste functionality. Under the decision R, save the fixed value $V(0) = r(0) - c(0) - I$ under the fixed cell reference $F$4, using the code: = $B$6-$B$7-$C$4, <ENTER>

Store m, n and t_1, in the fixed (absolute) cell references $D$2, $F$2, and $G$2 respectively.

To automate the stage numbering, perform the following actions:

Store last stage number n under the relative cell reference $F3, by typing: =$F$2 there, followed by <Enter>.

Secure the stage number $n-1$ under the relative cell reference $F15, by typing: =$F$2 - 1 there, followed by <Enter>.

Secure the stage number $n-2$ under the relative cell reference $F22, by typing: =$F15 - 1 there, followed by <Enter>.

Step 2: Automation of the states in all *n* stages

Blank out column B.

Type the following code in C13:

```
= IF ( B13 >= $D$2,"", IF (AND ($F3-1+$G$2 > $D$2, B13 < $F3-1),1+B13,
  IF (AND ($F3-1+$G$2  > $D$2, B13 >= $F3-1),"", IF (AND ($F3-1+$G$2 <= $D$2, B13 <
$F3-1), 1+B13,
  IF (AND ($F3-1+$G$2 <= $D$2, B13 = $F3-1), $F3-1+$G$2,""))))) <Enter>.
```

Perform the clerical duty form C13 to N13, to secure the stage n states and the accompanying blank spaces.

Now copy C13:N13 and paste it successively to the cell references

$\mathrm{C}[13+7(n-i)]:\mathrm{N}[13+7(n-i)],$ for $i \in \{n-1,\ n-2, \cdots, 2, 1\}$, to secure the states in the remaining $n-1$ stages.

Step 3: Stage *n* computations (Here *n* = 8)

For $t = 1$, under REPLACE, type the following code in the cell reference C10:

=If(C13 = “”,””, $F$4+ $C$8+C$8) <ENTER> to secure $f_8^R(1)$.

Click back on cell C10, position the cursor at the right edge of the cell until a crosshair appears. Then drag the crosshair across to the last the cell N8 to secure

$f_8^R(2),\ f_8^R(3), \cdots, f_8^R(6)$ and blank spaces.

Henceforth, the act of clicking back on a specified cell, positioning the cursor at the right edge of the cell until a crosshair appears and the crosshair-dragging routine will be referred to as clerical routine/duty.

For $t = 1$, under KEEP, type the following code in the cell reference C9:

=If ($C13 =$D$2,”Must Replace”, if (C13= “”,””,C$6-C$7+D$8)) <ENTER> to secure $f_8^K(1)$.

Perform the clerical duty to secure $f_8^K(2), f_8^K(3), \cdots, f_8^K(6)$ .To secure $f_8(t)$, for $t \in \{1, 2, \cdots, 6\}$, type the following code in the cell reference C11:

=If (C13 = “ ”,” ”,if (C9 = “Must Replace”, C10, max(C9,C10))) <ENTER> to secure $f_8(1)$.

Then perform the clerical routine across to N13 to secure $f_8(2), f_8(3), \cdots, f_6(6)$ and blank spaces

3.3.1 Remarks on segment code redundancy

In Excel, the max and min functions return values for only numeric expressions, ignoring string constants; for example if the number 5 is saved in B2 and the string constant "**Must**" is saved in C2, Then in D2, the code: = max(B2, C2) <Enter> returns 5. In E2, the code: = max (B2, C2) <Enter> also returns 5. Therefore the code segment involving "if (C9 = "Must Replace", C10" may be dispensed with throughout the template.

To obtain the optimal decision for each of the stage 8 states $t \in S_8 = \{1, 2, \cdots, 6\}$, type the following code in the cell reference C12:

> =If (C13 = " "," ",if(C13 = $D$2, "R", if(C9 = C10, "K/R", if(C9 > C10, "K", "R")))) <ENTER> to secure $D_8(1)$.

Then perform the clerical routine to secure $D_8(2), D_8(3), \cdots, D_8(6)$ and blank spaces in sequence

Step 4: Stage (*n* - 1) computations (Here *n* - 1 = 7)

For $t = 1$, under REPLACE, type the following code in the cell reference C17:

> =If (C20 = "","", $F$4+ C$8+$C11) <ENTER> to secure $f_7^R(1)$.

Perform the clerical duty to secure $f_7^R(2), f_7^R(3), \cdots, f_7^R(6)$ and succeeding blank spaces

For $t = 1$, under KEEP, type the following code in the cell reference C16:

> =If (C20 =$D$2,"Must Replace", if (C20 = "","", C$6-C$7+D11)) <ENTER> to secure $f_7^K(1)$. Perform the clerical duty to secure $f_7^K(2), f_7^K(3), \cdots, f_7^K(6)$ and succeeding blank spaces

To secure $f_7(t)$, for $t \in \{1, 2, \cdots, 6\}$, type the following code in the cell reference C18:

> =If (C20 = " "," ", if (C16 = "Must Replace", C17, max(C16,C17)))<ENTER> to secure $f_7(1)$. Then perform the clerical routine to secure $f_8(2), f_8(3), \cdots, f_8(6)$ and succeeding blank spaces.

To obtain the optimal decision for each of the stage 6 states $t \in S_3 = \{1, 2, \cdots, 6\}$, type the following code in the cell reference C19:

> =If (C20 = " "," ",if(C20 =$D$2, "R", if(C16 = C17, "K/R", if (C16 > C17, "K", "R"))))<ENTER> to secure $D_7(1)$.

Then perform the clerical routine to secure $D_7(2), D_7(3), \cdots, D_7(6)$ and the blanks.

Step 5: Stage (*n* - 2) computations (Here *n* - 2 = 6)

Copy the contiguous region $A15:N20 of stage *n*-1 into the contiguous region $A22:N27 of stage *n*– 2 to secure stage (*n* - 2) computational values

Step 6: Stage *i* Implementations, $i \in \{n-3, \cdots, 2, 1\}$, in One Fell Swoop

This is a crucial step involving a single Copy and $n-3$ Paste Operations, using the contiguous region region $A22:N27 of stage $(n-2)$.

Simply use the Copy and Paste functionality to copy and paste the contiguous region $A22:N27 successively into stages $(n-3)$ to 1 regions.

Note: Consecutive stages should be separated by a blank row. In other words, for $i \in \{n-3, n-4, \cdots, 1\}$ use the Copy and Paste functionality to copy and paste the contiguous region \$A22:N27 successively into stages $(n-3)$ to 1 regions:

$$\mathrm{A\$}\left[8+7(n-i)\right]:\mathrm{A\$}\left[13+7(n-i)\right].$$

Step 7: Repositioning of the state $\boldsymbol{i-1+t_1}$, in $\boldsymbol{S_i}$, for $\boldsymbol{1 \le i \le m+1-t_1}$

For states S_j, with $2 \le i \le m+1-t_1$, the states $1, \cdots, i-1$ are already well arranged in increments of 1.

Delete the adjacent remaining state $i-1+t_1$, located in column $2+i$ and type this value in Excel column

$(2+i-1+t_1)$, where Excel column 1 is A and Excel column 26 is Z, in lexicographical order. Then delete other values between $(i-1)$, exclusive and $(i-1+t_1)$, exclusive.

Leave the states S_1, for $m+1-t_1 < i \le n$ the way they are (in serial order).

Note that the stage numbering is automatically implemented, computations in all stages are automatically executed and the problem correctly solved in one fell swoop. Tremendous huh!

<u>3.3.2 Remarks on the use of the templates for large problem sizes</u>

It is clear that the crosshair horizontal-dragging routine must be extended beyond column N, as appropriate, if $m \ge 13$. This can be optimally done before the Copy and Paste operations from stage $n-1$. Hence the template can be adequately appropriated for sensitivity analyses on this class of Equipment Replacement problems in just a matter of minutes, as contrasted with manual investigations that would at best consume hours or days with increasing values of m and/or n and the number of investigations, not to talk of the dire consequences of committing just one error in any stage computations.

4. CONCLUSION

The article designed and automated prototypical solution templates for optimal policy prescription for some equipment replacement problems of the stationary class, complete with an exposition on the interface and solution process. The optimal results were assured and secured by trivial repositioning of the last state in each of the stages 1 to $m+1-t_1$, for a total of $m+1-t_1$ states. Finally the article deployed the template to obtain alternate optimal policy prescriptions with respect to a relevant problem, with a horizon length of 12 years, and four different starting ages. The long horizon length may preclude attempts at manual solutions to these four problem instances. Definitely, all things being equal, it would take no less than forty eight hours to solve the problem manually; the dire consequences of committing just one error in any stage of the process could hardly be contemplated. This would contrast quite sharply with the automated solutions that took no more than eight minutes, subject to correct data input, demonstrating the efficiency, power and utility of the solution template prototype. In general, the template could be deployed to solve each equipment replacement problem in less than 5 percent of the time required for the manual generation of the alternate optima.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Taha HA. Operations research: An introduction. Seventh Edition. Prentice-Hall of India, New Delhi. 2006;418-421.
2. Winston LW. Operations research: Applications and algorithm, Duxbury Press, Boston. 1994;798- 802.
3. Verma AP. Operations research. Kataria SK, Sons. New Delhi. 2010;882-885.
4. Gupta PK, Hira DS. Problems in operations research (Principles and Solutions). S. Chand & Company Ltd. New Delhi. 2012;844-854.
5. Gress ESH, Arango OM, Armenta JRC, Reyes AOO. A Reward Functional to Solve the Replacement Problem. Intelligent Control and Automation. 2012;3:413-418.
6. Ukwu C. Novel state results on equipment replacement problems and excel solution implementation templates. Transactions of

the Nigerian Association of Mathematical Physics. 2015;1:237-254.

7. Ukwu C. Excel solution implementation templates for machine replacement problems with age and decision period variable data. Transactions of the Nigerian Association of Mathematical Physics. 2015;1:255-264.

Peer-review history:
The peer review history for this paper can be accessed here:
http://sciencedomain.org/review-history/14189