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A Study on Fixed Point Results for OWC Self-Maps on C-Metric Space

K. Prudhvi [a*]

[a] Department of Mathematics, University College of Science, Saifabad, Osmania University, Hyderabad, Telangana State, India.

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

We study in this paper fixed point results in cone metric space (c - metric space) for OWC "Occasionally Weakly Compatible mappings". Recently, M. Amari and D. El Moutawakil [1] have obtained fixed point results in metric space with (E.A) property. Our results are generalize; improve and extends the results the results of [1].

Keywords: Occasionally Weakly Compatible (OWC); C- metric space; fixed point.

1 Introduction and Preliminaries

The concept of cone metric space was introduced in 2007 by Haung and Zhang [2] and they have generalized the concept of a metric space, and they replacing the real numbers by on ordered Banach space and obtained some fixed point theorems for mappings satisfying different contractive conditions in cone metric space. Later on many authors were, inspired in these results and generalized, extended and improved these results in many ways (for e.g. see [3,4,5], and [6,7,8]). Recently M. Amari and D. El Moutawakil [1] have proved some fixed point theorems results under strict contractive conditions in metric space and they were using the property (E.A). We have extended these results into cone metric space. In this paper we obtained some results for OWC in cone metric spaces, which are generalized and improved the results of [1].

**Corresponding author: Email: prudhvikasani@rocketmail.com;*

Some of the useful definitions are needed in our main results which are due to in [2].

Definition 1.1 Let $X \neq \emptyset$ set of E. And let: $X \times X \rightarrow B$ be a mapping, if ρ satisfies the following axioms

(1). $\rho(x, y) \geq 0$ for every x, y∈X (Non-negative);
(2). $\rho(x, y) = 0 \Leftrightarrow x = y$, for every x, y∈X;
(3). $\rho(x, y) = \rho(y, x)$ (symmetry) , for every x, y∈X;
(4).$\rho(x, y) \leq \rho(x, z) + \rho(z, y)$ (Triangle inequality) for every x, y, z∈X.

Then, ρ is called a cone metric(c-metric) on X and (X, ρ) is called a c- metric space.

Example 1.1 [2] Let $B = \mathbb{R}^2$, $S = \{(x, y) \in B$ such that : $x, y \geq 0\} \subseteq \mathbb{R}^2$, $X = \mathbb{R}$ and

$\rho : X \times X \rightarrow B$ such that $\rho(x, y) = (|x - y|, \alpha|x - y|)$, where $\alpha \geq 0$ is a constant. Then (X,) is a c- metric space.

Definition 1.2. Let B be a Banach Space and S be a cone. We define a partial ordering ≤ w. r .t to S by $x \leq y \Leftrightarrow y - x \in S$. We write $x<y$ to $x \leq y$, but $x \neq y$, while $x \ll y$ stands for $y-x\in$ interior of S, where interior of S denotes the interior of the set S. Then this cone is called an order cone.

Definition 1,3. Let $S \subset B$ be an order cone and B be a Banach space. Then the order cone S is said to be called a Normal Cone if there exists a constant M > 0 such that for all x, y∈B, $0 \leq x \leq y \Rightarrow ||x|| \leq M\,||y||$.

Where, M is the least +ve number satisfying this inequality is said to be a normal constant of S.

Definition 1.4 Let (X, ρ) be a cone metric space .The sequence $\{x_n\}$ is said to be

(i) a convergent sequence if for any c>>0, there is a $\mathbb{N}$ such that for all n> $\mathbb{N}$,
$\rho(x_n, x) \ll c$, for x ∈X . We denote this $x_n \longrightarrow x$, as $n \rightarrow .\infty$.
(ii) a Cauchy sequence if for every c in B with c>>0, there is a $\mathbb{N}$ such that for all n, m> $\mathbb{N}$,
$\rho(x_n, x_m) \ll c$.

Note: A c- metric space (X,) is said to be complete, if every Cauchy sequence in X is convergent in X.

Definition 1.5 [3]. Let A, B : $X \rightarrow X$ be mappings of a set X. If Ax = Bx = z, for some x in X, then we call x is a coincidence point of A and B, z is said to be a point of coincidence of A and B.

Proposition 1.6 [9]. Let A and B be Occasionally Weakly Compatible (OWC) self-mappings of a set X ⇔ there is a point x in X which is coincidence point of A and B at which A and B are commute.

Lemma 1.1 [9]. A, and B are said to be OWC self-map of the set X. If A and B have a unique point of coincidence Ax = Bx= z, then, z is said to be the unique common fixed point of A and B.

2 Main Results

In this section, we proved fixed point theorems, which are generalized and improved the results of [1].

Theorem 2.1. Suppose (X, ρ) is a c- metric space and S is a normal cone. And let A and B are two OWC self-mappings of X , if

(i) $AX \subset BX$
(ii) $\rho(Ax, By) < \max\{\rho(Bx, By), [\rho(Bx, Ax)+ \rho(By, Ay) / 2], [\rho(By, Ax)\ \rho(Bx, Ay) / 2]\}$,

for all $x \neq y \in X$.

Then , A and B have a unique common fixed point in X.

Proof: There **e**xists a point x ϵX such that Ax = Bx. And let a point y ϵ X for which Ay = By. Then from (ii) we get that

$$\rho(Ax, Ay) < \max\{ \rho(Ax, Ay), [\rho (Ax, Ax) + \rho(Ay, Ay) / 2], [\rho(Ay, Ax) + \rho(Ax, Ay) / 2] \},$$
$$< \max \{\rho(Ax, Ay), 0 , \rho(Ax, Ay)\},$$
$$= \rho(Ax, Ay) < \rho(Ax, Ay),$$

which is a contradiction.

Therefore, Ax = Ay , and Ax is unique. From the above Lemma 1.1, we get that A and B have a unique common fixed point in X. Hence. Proved and this completes the proof of the theorem

Theorem 2.2 . Suppose (X, ρ) be a c- metric space , let S be a normal cone. And let A , B , M and N are self-mappings of X such that

(i) the pairs {A, M} and {B, N}are OWC.
(ii) ρ (Ax, By)< max{ ρ(Mx,My), [ρ (Mx, My) + ρ (Ny, Ay)/ 2] , [ρ(Mx, By) + ρ(Ny, Ax) / 2]} for each x, y ϵX for which Ax ≠ By, then A, B, M and N have a unique common fixed point in X.

Proof: By (i) {A, M} and {B, N} are OWC, then there exists x, y ϵ X such that Ax = Mx and By = Ny. We claim that Ax = By. For otherwise by (ii)

$$\rho(Ax, By) < \max\{ \rho (Mx, Ny), [\rho (Mx, Ax) + \rho (Ny, By) / 2] , [\rho (Mx, By) + \rho(Ny, Ax) / 2]\}$$

since, Ax = Bx = w and By = Ny = z are points of coincidence of {A, M} and {B, N} respectively. Then

$$\rho (Ax, By) < \max\{ \rho (Ax, By), [\rho(Ax, Ax) + \rho (By,By) / 2] , [\rho (Ax, By) + \rho (By, Ax) / 2]\}$$
$$< \max\{ \rho (Ax, By), 0, \rho (Ax, By) \} < \rho (Ax, By),$$

which is a contradiction.

Therefore, Ax = By, that is, Ax = Mx = By = Ny.

Moreover, if there exits another point z such that Az = Bz, then by (ii), we get that

Az = Mz = By = Ny or Ax = Az.

Therefore, Ax = Mx = w is the unique point of coincidence of A and M.

By the above Lemma 1.1. , w is the unique common fixed point of A and M. Similarly, there exists a unique point z ϵ X such that Bz = Nz = z.

Finally we shall prove that uniqueness. Suppose that if possible assume that w ≠ z. hen by (ii) we get that

$$\rho(w, z) = \rho(Aw, Az)$$
$$< \max\{\rho(Mw, Nz), \rho[(Mw, Aw)+ \rho(Nz, Bz)/ 2], [\rho(Mw, Bz)+\rho(Nz,Aw) / 2]\}$$
$$= \max\{ \rho (w, z), [\rho (w,Aw) + \rho (z, z) / 2] , \rho [(w,z) + \rho(z, w) / 2]\}$$
$$= \max\{ \rho (w, z), \rho (w, z) \} = \rho (w, z) < \rho (w, z), \text{ which is a contradiction.}$$

Therefore, w = z and w is the unique common fixed point of A, B, M, and N.

Hence proved. And this completes the proof of the theorem.

3 Conclusion

We have, generalize, improve and extends the results of [1]. And our results are more general than the results of [1].

Competing Interests

Author has declared that no competing interests exist.

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