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Approximate Solutions for Non-Linear Evolution Stochastic Equations with Variations of Drift Parameters

Azor, P. A. [a*], Annorzie, M. N. [b] and Amadi, I. U. [c]

[a] Department of Mathematics and Statistics, Federal University, Otuoke, Nigeria.
[b] Department of Mathematics, Imo State University, Owerri, Nigeria.
[c] Department of Mathematics and Statistics, Captain Elechi Amadi Polytechnics, Port Harcourt, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This paper considered system of stochastic differential equations with emphasis on variations of drift parameters as it affects financial markets. The Ito's method was applied in solving the problems analytically which resulted to three different investment solutions accordingly. The appropriate conditions were accomplished which controlled various drift parameters in the assessment financial markets. Hence, the impressions on each solution of investors in financial markets were analyzed graphically. Finally, the influences of the relevant parameters of stochastic variables were effectively discussed all in this paper.

**Corresponding author: Email: azorpa@fuotuoke.edu.ng;*

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1 Introduction

In general sense, the various activities in business, trading and investments are risk induced or influenced

Every day activities of the human beings are risk induced; therefore, for the proper management of investment portfolios, the role played by risk must be factored in, this is because, risk is a major factor in determining the transmogrification of returns on the assets and portfolio, which aids the investor with a proper mathematical structure for investment inferences [1] some examples of the risk connected to securities, include Bonds, stocks, property, and so on.

"Nevertheless, because of the risk involved in the management of investment portfolios, insurance companies deemed it pertinent for lives, properties, etc, to be insured. In point of fact, insurance companies share third party in the management and control of their financial results. Risk transfer or risk sharing is the methodology employed by insurance firm on financial outcomes of its coverage duty in a number of ways with risk transfer agreement, risk among numerous insurance firms globally. therefore, in a situation of astronomical loses from financial situation as insurance company will not encounter risk, particularly, reinsurance means the division and distribution of risk. In general, risk is an established factor as long as humans are concerned, since we secure risky or riskless assets properly" [2]. A more robust approach in modelling these factors is the trajectory or path of a diffusion process defined on many basic or fundamental probability space, having the Geometric Brownian motion, used as the standard reference model [3]. Modelling financial concepts cannot be exaggerated as a result of its multiple applications in science and technology. For instance, [4] examined "the maximization of the exponential utility and the minimization." Of the run probability, and results obtained revealed the same kind of investment scheme or approach for interest rate of zero.

Furthermore,[5] did a study on "an optimal reinsurance and investment problem for insurer with jump diffusion risk process". Liang and Bayraktar [6] examined "the risk reserved for an insurer and a reinsurer to follow Brownian motion with drift and applied optimal probability of survival problem under proportional reinsurance and power utility preference". Also, [7,8] examined "the excess loss of reinsurance and investment in a financial market and obtained optimal strategies". Gu et al. [9] employed "a problem of optimal reinsurance investment for an insurer having jump diffusion risk model when the asset price was control by a CEV model". Lin and Li [10] studied "strategies of optimal reinsurance and investment for exponential utility maximization under different capital markets". Osu and Okoroafor [11] considered "investment problem having multiple risky assets". Okoroafor and Osu [1] examined "an optimal portfolio selection model for risky assets established on asymptotic power law behaviour where security prices follow a Weibull distribution". The study of Davis et al. [12] analyzed "the stability of stochastic model of price fluctuation on the floor of the stock market, where exact steps were derived, which aided the determination of the equilibrium price and growth rate of stock shares". Osu et al. [13] examined "the unstable property of stock market forces, making use of proposed differential equation model". Adeosun et al. [14] studied "a stochastic analysis of stock prices and their characteristics and obtained results which showed efficiency in the use of the proposed model for the prediction of stock prices". Similarly, [15] considered "the stochastic formulations of some selected stocks in the Nigerian Stock Exchange (NSE), and the drift and volatility measures or quantities for the stochastic differential equations were obtained and the Euler-Maruyama technique for system of SDEs was applied in the stimulation of the stock prices". Gu et al. [8] produced "the geometric Brownian motion and assessment of the correctness or exactness of the model, using detailed analysis of stimulated data". Furthermore, [16] looked "stochastic problem of unstable stock market prices obtained conditions for determining the equilibrium price, required and adequate conditions for dynamic stability and convergence to equilibrium of the growth rate of the valued function of stocks". Nevertheless, [17] examine "a stochastic problem of unstable prices at the floor of the stock market. From their evaluation, the equilibrium price and the market growth rate of shares were found out". Hence, a good number researchers like [12-24,2], etc., have written broadly on stock market prices.

Previous researches have thus studied similar problems but did not look the variations of drift parameters as it affects financial markets. To a great extent, some researches, like [20,21] and [2], etc. To the best of our understanding, this is the first study that has assessed disparities of drift parameters and its influences in financial markets. Thus, this paper compliments that of Okpoye et al. [23] as it widens the scope of applicability in this dynamic field of mathematical finance.

The organization of this paper is set as follows: Section 2.1 presents the mathematical formulations, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2 Mathematical Formulation

A differential Equation which possesses a stochastic term is termed a stochastic differential equation. Thus, assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space having filteration $\{f_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), ..., W_m(t))^T, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g *as* follows ;

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t),\ 0 \leq t \leq T,$$

$$X(0) = x_0,$$

Where, $T > 0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f : [0, \mathrm{T}] \times ¡^n$ *and* $g : [0, T] \times ¡^n \to \times ¡^{n \times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))dS + \int_0^t g(S, X(S))dZ(S)$$

Where, dX, dZ are terms known as stochastic differentials. The $¡^n$ is a valued stochastic process $X(t)$.

Theorem 2.1: let $T > 0$, be a given final time and assume that the coefficient functions $f : [0, \mathrm{T}] \times ¡^n \to ¡^n$ *and* $g : [0, T] \times ¡^n \to \times ¡^{n \times n}$ are continuous. Moreover, $\exists$ finite constant numbers λ *and* β such that $\forall t \in [0, \mathrm{T}]$ and for all $x, y \in ¡^n$, the drift and diffusion term satisfy

$$\| f(t, x) - f(t, y) \| + \| \mathrm{g}(t, x) - g(t, y) \| \leq \lambda \| x - y \| ,$$

$$\| f(t, x) \| + \| \mathrm{g}(t, x) \| - g(t, \mathrm{x}) \| \leq \beta (1 + \| x \|).$$

Suppose also that x_0 is any $¡^n$ -valued random variable such that $E(\| x_0 \|^2) < \infty.$ then the above SDE has a unique solution X in the interval $[0, T].$ Moreover, it satisfies $E\left(\sup_{0 \leq t \leq T} \| X(t) \|^2 \right) < \infty$. the proof of the theorem 2.1 is seen in Lambert and Lapeyre [24].

Theorem 2.2: (Ito's lemma). Let $f(S, t)$ be a twice continuous differential function on $[0, \infty) \times \mathsf{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0 ,$$

Applying Taylor series expansion of F , gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(dS_t\right)^2 + \text{higer order terms } \left(h.\text{o},\text{t}\right) ,$$

So, ignoring h.o.t and substituting for dS_t ,we obtain;

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(a_t dt + bdz(t)\right)^2$$

$$= \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2 dt,$$

$$= \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2\right) dt + \frac{\partial F}{\partial S_t} b_t dz(t)$$

Let $S(\text{t})$ be stock price, then following GBM, implies (5) and hence, the function $F\left(S,t\right)$,Ito’s lemma gives:

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2}\right) dt + \sigma S \frac{\partial F}{\partial S} dz(t)$$

However, the stochastic analysis of the oscillations of stock drift and its influences in financial markets is herein looked into. The volatility fluctuations and other drift coefficients of stock prices were taken to be constant throughout the days of trading. The initial stock price, which is assumed to follow different trend series was categorized the entire origin of stock dynamics is found in a complete probability space $\left(\Omega, \text{F}, \wp\right)$ with a finite time investment horizon $T > 0$. Hence, we have the following system of stochastic differential equations below;

$$dX_t = -\beta\mu X_t dt + \sigma X_t dW_t^1 \tag{0.1}$$

$$dX_\phi = K \tanh X_\phi dt + \sigma X_\phi dW_t^2 \tag{0.2}$$

$$dX_\varpi = \left(-\beta\alpha + K \tanh\right) X_\varpi dt + \sigma X_\varpi dW_t^3 \tag{0.3}$$

3 Methods of Solutions

The proposed models (1.1) - (1.3) consist of a system of variable coefficient system of stochastic differential equations whose solutions are not trivial. we solve equations independently as follows using Ito’s theorem 2.2:
From (1.1) let $f\left(X_t, t\right) = \ln X_t$

If the partial derivatives are taken, we have;

$$\frac{\partial f}{\partial X_t} = \frac{1}{X_t}, \quad \frac{\partial^2 f}{\partial X_t^2} = \frac{-1}{X_t^2}, \quad \frac{\partial f}{\partial t} = 0 \tag{0.4}$$

Following Ito’s lemma, we have;

$$df\left(X_t,t\right)=\sigma X_t\frac{\partial f}{\partial X_t}dW_t^1+\left(-\beta\mu X_t\frac{\partial f}{\partial X_t}+\frac{1}{2}\sigma^2X_t^2\frac{\partial^2 f}{\partial X_t^2}+\frac{\partial f}{\partial t}\right)dt \tag{0.5}$$

Substituting (1.4) into (1.5), gives;

$$df\left(X_t,t\right)=\sigma X_t\frac{1}{X_t}dW_t^1+\left(-\beta\mu X_t\frac{1}{X_t}+\frac{1}{2}\sigma^2X_t^2\left(-\frac{1}{X_t^2}\right)+0\right)dt \tag{0.6}$$

$$=\sigma\frac{X_t}{X_t}dW_t^1+\left(-\beta\mu X_t\frac{X_t}{X_t}-\frac{1}{2X_t^2}\sigma^2X_t^2\right)dt=\sigma dW_t^1+\left(-\beta\mu-\frac{1}{2}\sigma^2\right)dt=\left(-\beta\mu-\frac{1}{2}\sigma^2\right)dt+\sigma dW_t^1$$

The integration of the expression above, gives;

$$\int_0^t d\ln X_t=\int_0^t df\left(X_tu,u\right)=\int_0^t\left(-\beta\mu-\frac{1}{2}\sigma^2\right)du+\int_0^t\sigma dW_t^1 \tag{0.7}$$

$$\ln X_t-\ln X_0=\left[-\beta\mu u-\frac{1}{2}\sigma_u^2\right]_0^t+\left[\sigma W_u^1\right]_0^t=\ln\left[\frac{X_t}{X_0}\right]=\left[-\beta\mu-\frac{1}{2}\sigma^2\right]t+\sigma W_t^1$$

If the ln of the both sides are taken, we have;

$$X_t=X_0e\left(-\beta\mu-\frac{1}{2}\sigma^2\right)t+\sigma W_t^1 \tag{0.8}$$

Where, W_t^1 is a Brownian Motion

From (1.2), let $f\left(X_\phi,t\right)=\ln X_\phi$

If the partial derivatives are taken, we have;

$$\frac{\partial f}{\partial X_\phi}=\frac{1}{X_\phi},\ \frac{\partial^2 f}{\partial X_\phi^2}=\frac{-1}{X_\phi^2},\ \frac{\partial f}{\partial t}=0 \tag{1.9}$$

Following Ito’s lemma, we have;

$$df\left(X_\phi,t\right)=\sigma X_\phi\frac{\partial f}{\partial X_\phi}dW_t^2+\left(K\tanh X_\phi\frac{\partial f}{\partial X_\phi}+\frac{1}{2}\sigma^2X_\phi^2\frac{\partial^2 f}{\partial X_t^2}+\frac{\partial f}{\partial t}\right)dt \tag{1.10}$$

Substituting (1.9) into (1.10), gives

$$df\left(X_\phi,t\right)=\sigma X_\phi\frac{1}{X_\phi}dW_t^2+\left(K\tanh X_\phi\frac{1}{X_\phi}+\frac{1}{2}\sigma^2X_\phi^2\left(-\frac{1}{X_\phi^2}\right)+0\right)dt \tag{1.11}$$

$$=\sigma\frac{X_\phi}{X_\phi}dW_t^2+\left(K\tanh X_\phi\frac{X_\phi}{X_\phi}-\frac{1}{2X_\phi^2}\sigma^2X_\phi^2\right)dt=\sigma dW_t^2+\left(K\tanh-\frac{1}{2}\sigma^2\right)dt=\left(K\tanh-\frac{1}{2}\sigma^2\right)dt+\sigma dW_t^2$$

The integration of the expression above, gives;

$$\int_0^t d\ln X_{\phi} = \int_0^t df\left(X_{\phi} u,u\right) = \int_0^t \left(K\tanh - \frac{1}{2}\sigma^2\right)du + \int_0^t \sigma dW_t^2 \quad (1.12)$$

$$\ln X_{\phi} - \ln X_0 = \left[K\tanh u - \frac{1}{2}\sigma_u^2\right]_0^t + \left[\sigma W_u^2\right]_0^t = \ln\left[\frac{X_{\phi}}{X_0}\right] = \left[K\tanh - \frac{1}{2}\sigma^2\right]t + \sigma W_t^2$$

If the ln of the both sides are taken, we have;

$$X_{\phi} = X_0 e\left(k\tanh - \frac{1}{2}\sigma^2\right)t + \sigma W_t^2 \quad (1.13)$$

Where, W_t^2 is a Brownian Motion

From (1.3) let $f\left(X_{\varpi},t\right) = \ln X_{\varpi}$

If the partial derivatives are taken, we have;

$$\frac{\partial f}{\partial X_{\varpi}} = \frac{1}{X_{\varpi}}, \quad \frac{\partial^2 f}{\partial X_{\varpi}^2} = \frac{-1}{X_{\varpi}^2}, \quad \frac{\partial f}{\partial t} = 0$$

(1.14)

Following Ito's lemma, we have;

$$df\left(X_{\varpi},t\right) = \sigma X_{\varpi}\frac{\partial f}{\partial X_{\varpi}}dW_t^3 + \left((-\beta\alpha + K\tanh)X_{\varpi}\frac{\partial f}{\partial X_{\varpi}} + \frac{1}{2}\sigma^2 X_{\varpi}^2\frac{\partial^2 f}{\partial X_{\varpi}^2} + \frac{\partial f}{\partial t}\right)dt \quad (1.15)$$

Substituting (1.14) into (1.15), gives;

$$df\left(X_{\varpi},t\right) = \sigma X_{\varpi}\frac{1}{X_{\varpi}}dW_t^3 + \left((-\beta\alpha + K\tanh)X_{\varpi}\frac{1}{X_{\varpi}} + \frac{1}{2}\sigma^2 X_{\varpi}^2\left(-\frac{1}{X_{\varpi}^2}\right) + 0\right)dt \quad (1.16)$$

$$= \sigma\frac{X_{\varpi}}{X_{\varpi}}dW_t^3 + \left((-\beta\alpha + K\tanh)X_{\varpi}\frac{X_{\varpi}}{X_{\varpi}} - \frac{1}{2X_{\phi}^2}\sigma^2 X_{\varpi}^2\right)dt = \sigma dW_t^3 + \left((-\beta\alpha + K\tanh) - \frac{1}{2}\sigma^2\right)dt$$

$$= \left((-\beta\alpha + K\tanh) - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t^3$$

Also, the integration of the expression above, gives;

$$\int_0^t d\ln X_{\varpi} = \int_0^t df\left(X_{\varpi} u,u\right) = \int_0^t \left((-\beta\alpha + K\tanh) - \frac{1}{2}\sigma^2\right)du + \int_0^t \sigma dW_t^3 \quad (1.17)$$

$$\ln X_{\varpi} - \ln X_0 = \left[(-\beta\alpha + K\tanh)u - \frac{1}{2}\sigma_{\varpi}^2\right]_0^t + \left[\sigma W_u^3\right]_0^t = \ln\left[\frac{X_{\phi}}{X_0}\right] = \left[(-\beta\alpha + K\tanh) - \frac{1}{2}\sigma^2\right]t + \sigma W_t^3$$

Similarly, If the ln of the both sides are taken, we have;

$$X_{\varpi} = X_0 e\left(\left(-\beta\alpha + k\tanh - \frac{1}{2}\sigma^2\right)t + \sigma W_t^3\right) \tag{1.18}$$

Where, W_t^3 is a Brownian Motion

The expected value of the solutions (1.8), (1.13) and (1.18), gives;

$$\left. EX_t(t) = X_0 e\left(-\beta\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t^1 = X_t(0) = e - \beta\mu t \right\} \tag{1.19}$$

$$\left. EX_\phi(t) = X_0 e\left(K\tanh - \frac{1}{2}\sigma^2\right)t + \sigma W_t^1 = X_\phi(0) = e - K\tanh t \right\} \tag{1.20}$$

$$\left. EX_\varpi(t) = X_0 e\left((-\beta\alpha + K\tanh) - \frac{1}{2}\sigma^2\right)t + \sigma W_t^1 = X_\varpi(0) = e(-\beta\alpha + K\tanh)t \right\} \tag{1.21}$$

4 Results and Discussion

This Section presents the graphical results for whose solutions are in (1.8), (1.13), (1.18) and (1.19-1.21) respectively. Hence, the following parameter values were used in the simulation study:

$$X_0 = 52.25,\ \beta = 25.6\ ,\sigma = 0.03,\ \mu = 0.88,\ \mathrm{t} = 1,\ \mathrm{W}_t^1 = \mathrm{W}_t^2 = \mathrm{W}_t^3 = 1, \alpha = 0.95, K = 30.7 \text{ and } h = 0.75$$

Fig. 1 and 3, show that the market is experiencing exponential growth or decay. This type of growth or decay is often seen in financial markets when there is a strong feedback loop between prices and expectations. For instance, if investors expect prices to go up, they may start buying more, which can lead to an exponential increase in prices. Conversely, if investors expect prices to go down, they may start selling, which can lead to an exponential decrease in prices. It’s important to note that exponential growth or decay in financial markets.

Fig. 2,4 and 6 describe a market which grows in value but of highly volatile. This means that the value of the market is increasing over time, but there is also a lot of risk associated with investing in this market. Volatility is a measure of how much the markets value is changing over time and high volatility means that there is a lot of uncertainty in the market.

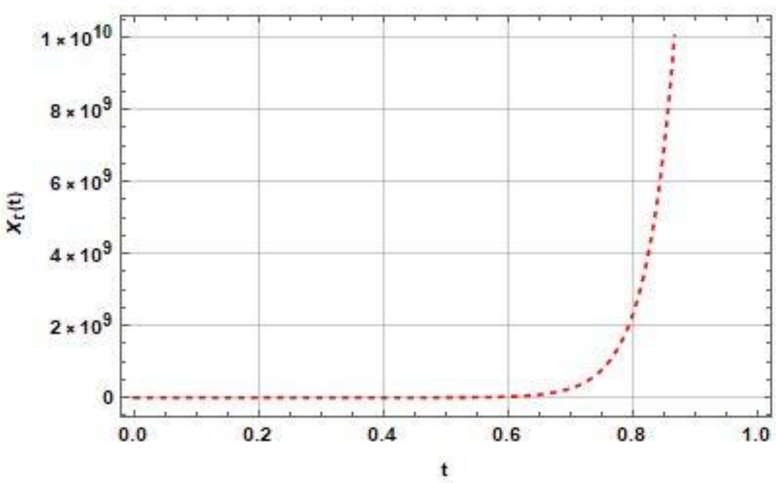

Fig. 1. The effect of negative drift coefficient on financial market against time

Clearly in Fig. 5 connotes that the market is experiencing a constant drift. This means that the market is expected to move in a certain direction at a constant ratio.

However, Fig. 7,8 and 9 describe the average value of the SDE over time. This value is calculated by taking the average of the SDE solution at different points in time. It can be thought of as the expected value of the market at any given time and it can be used to predict future market movement. For instance, if the expected mean is increasing over time, it means that the market is expected to move up, conversely, if the expected mean is decreasing, it means that the market is expected to move down.

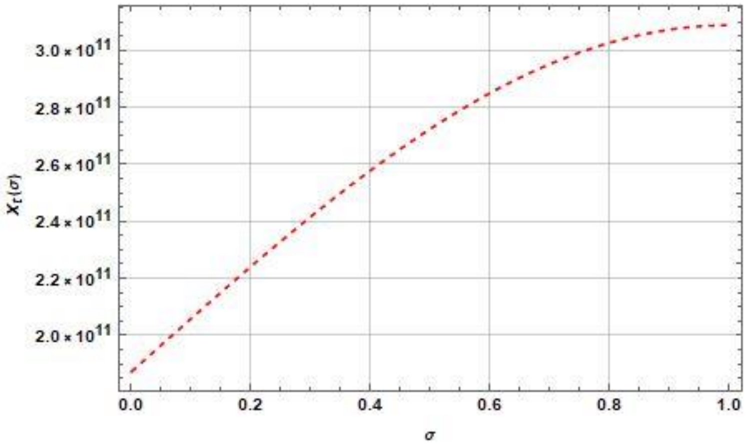

Fig. 2. The effect of negative drift coefficient on financial market against volatility

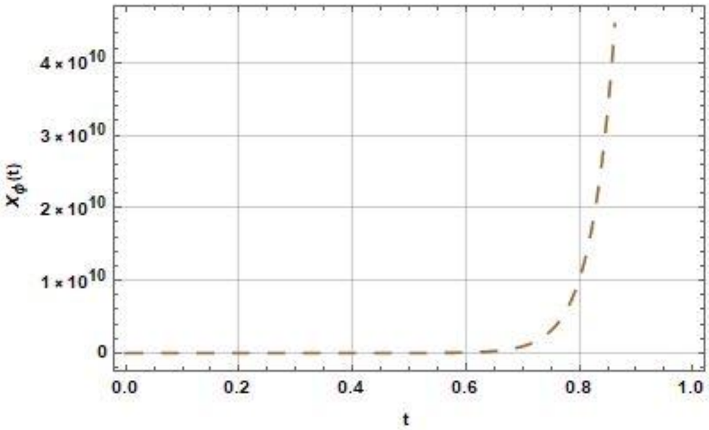

Fig. 3. The effect of periodic drift coefficient on financial market against time

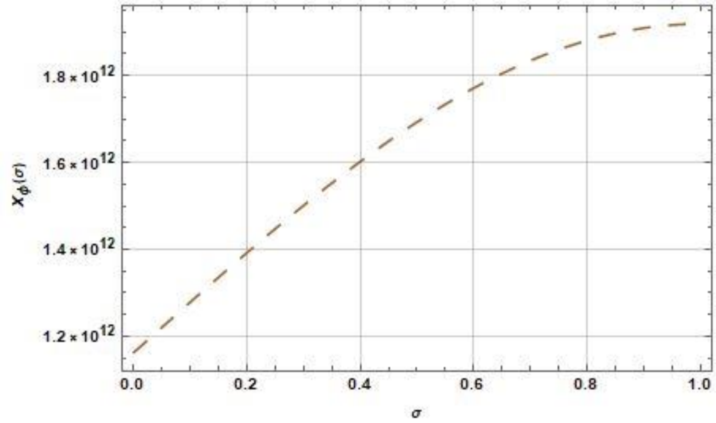

Fig. 4. The effect of periodic drift coefficient on financial market against volatility

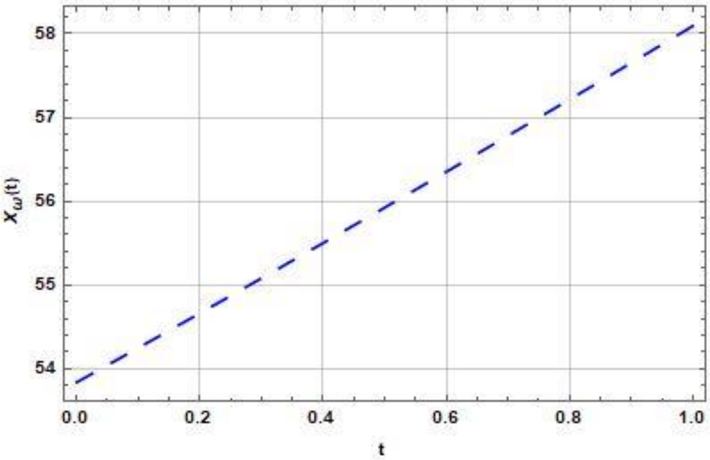

Fig. 5. The effect of constant terms with periodic drift coefficient function on financial market against time

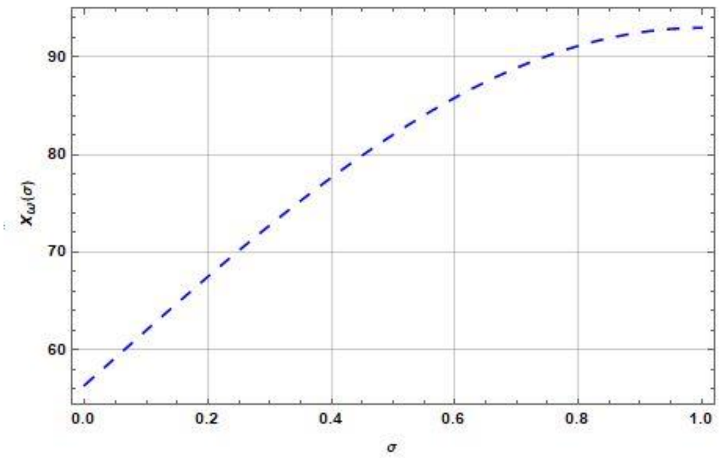

Fig. 6. The effect of constant terms with periodic drift coefficient function on financial market against volatility

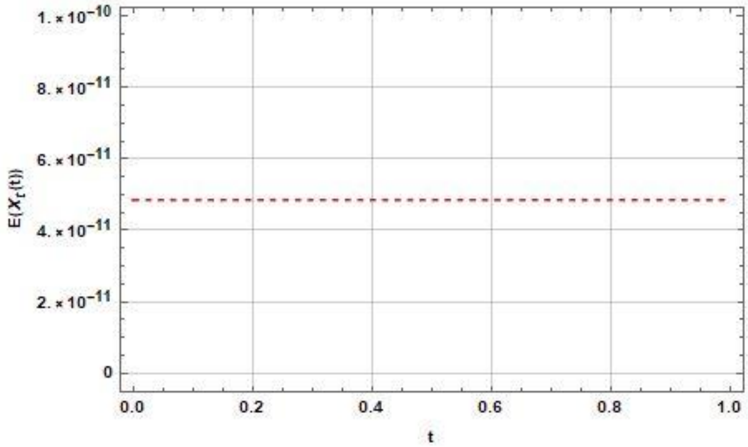

Fig. 7. The effect of expected negative drift coefficient on financial market against time

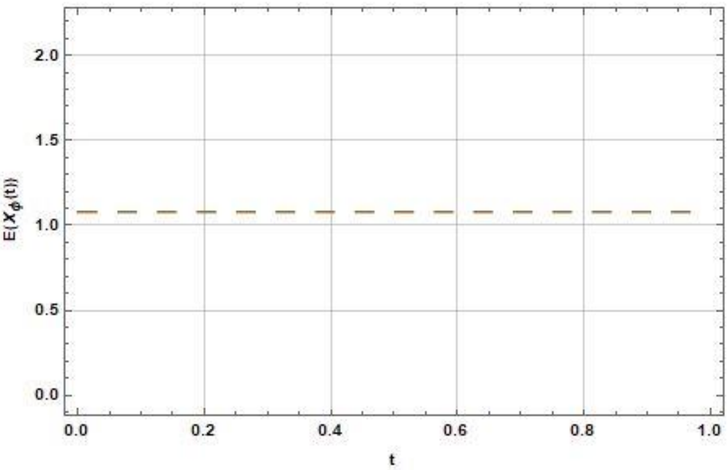

Fig. 8. The effect of expected periodic drift coefficient on financial market against time

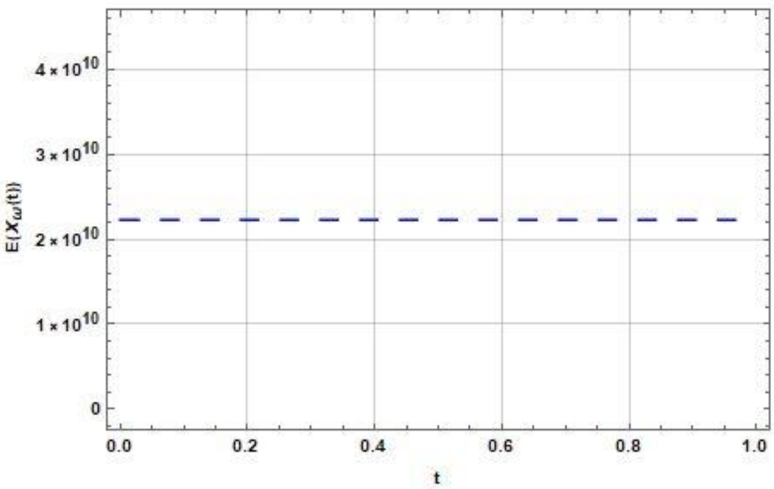

Fig. 9. The effect of expected constant terms with periodic function drift coefficient on financial market against time

5 Conclusion

Stochastic differential equations are useful mathematical tools for estimating stock market variables. This helps investors in making quality decisions that will enhance improvements on the financial dealings. Hence, we looked at the system of stochastic differential equations with disparities of drift parameters in the model. These problems were solved analytically by applying the Ito's lemma approach of solution and three different solutions with their expected mean solutions were obtained accurately. From the analysis of the graphical solutions, we conclude that; there is an effect of exponential growth during the period of trading, it reveals a market that is growing in value but is also highly volatile, a market experiencing a constant drift and finally describes the average value of the SDE over time as it affects financial markets.

Consequently, we recommend that combining ordinary differential and stochastic differential equations in the assessment of drift variations in the next study.

Competing Interests

Authors have declared that no competing interests exist.

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