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Comparative Study of Elementary School Mathematics Textbooks in China, Japan, South Korea, Singapore, America, Germany: A Case Study on "Fraction Division"

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

The main purpose of this paper is to select 15 primary school mathematics textbooks from China, the United States, Singapore, Japan, South Korea and Germany, take "fraction division" as an example, and clarify the characteristics and similarities and differences of its operational meaning model and revelation methods of arithmetic reasoning in Fraction Division through literature method, content analysis method and comparative research method. The results show that there are great differences in these two aspects between different versions of teaching materials. Therefore, combining the national conditions of various countries, seeking common ground while reserving differences, provides a teaching path of fraction division based on national conditions and absorbing the advantages of different countries, and provides theoretical support for the better implementation of curriculum standards and textbooks.

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Keywords: Elementary school mathematics; fraction division; textbook comparison; division models; revelation methods of arithmetic reasoning in fraction division.

1 Introduction

According to the report of the International Association for the Evaluation of Educational Achievement, China, Japan, South Korea, Singapore, Germany and America have achieved relatively good results in international mathematical achievement comparison tests such as TIMISS and PISA. The reason is that elementary school students' understanding of mathematics basically comes from classroom teaching and textbooks, and the differences in textbooks may partly explain the differences in students' test performance. The textbooks of six countries can concretely present the arrangement and development of curriculum content, carry curriculum culture and transmit curriculum ideas.

Effective teaching of fraction arithmetic is essential. Among them, fraction division, as an important part of "number and algebra", is a key link for students to expand the number system and deepen mathematical understanding, and plays an irreplaceable role in students' subsequent understanding of decimals, ratios, proportions, percentages and other concepts. Dividing by fractions is also a topic related to multiplication and subtraction of fractions. Because the solution to fraction division depends heavily on the inverse multiplication algorithm. Mastering the essence of fraction division is of great significance for cultivating students' number sense. Therefore, the study of fraction division in textbooks is indispensable. Through in-depth study of its teaching methods and strategies, we can help students better grasp this complex and important mathematical topic and lay a solid foundation for their subsequent mathematical learning and application.

In the latest round of mathematics curriculum reforms, understanding arithmetic reasoning is an important goal, which is more abstract and complex for elementary school students at this stage, but understanding arithmetic reasoning can help students truly understand arithmetic reasoning. Many students have many difficulties in understanding arithmetic reasoning, often only remember the arithmetic reasoning, through rote memorization, step-by-step procedural operations to master mathematical knowledge, improve mathematical performance. Therefore, the conflict between the importance of fraction division and the difficulty of understanding fraction arithmetic reasoning makes fraction division a hot topic in mathematics education [1].

To improve students' conceptual understanding of fraction division, we can proceed from two aspects: the meaning of fraction division and the revelation process of arithmetic reasoning. Ma Liping analyzed the meaning model of fraction division in detail in " The mastery and teaching of primary school mathematics ". Jia Suijun systematically sorted out the revelation model of fraction division arithmetic reasoning in " A comparative study of primary school mathematics textbooks from the perspective of Conceptual understanding -- A case study of "fractional Division"". The classification of these two scholars' models is highly consistent with the content of 15 textbooks from 6 countries studied by the author, providing strong theoretical support for the teaching of fraction division.

Based on the considerations outlined above, this paper, based on Jia Suijun's reasoning model [2] and Ma Liping's model describing the meaning of fraction division operation [3], grounded in the two perspectives of the models of the significance of fraction division, the revelation methods of arithmetic reasoning in fraction division compares and analyzes models of fraction division meaning, the revelation methods of arithmetic reasoning in fraction division in the textbooks of six countries. Therefore, it provides some relevant suggestions for elementary school teachers on how to learn fraction division.

2 Research Design

2.1 Research objects

Asian countries such as China, Japan, South Korea and Singapore continue to show excellence in PISA mathematics, with students in these countries performing well in both mathematical literacy and problem-solving skills. In contrast, the performance of Western countries such as Germany and the United States has continued to decline in this field, which makes people wonder why.

In order to further explore this phenomenon, we can select the textbooks of different countries with different levels of mathematics level for comparative study, which is expected to provide us with a more comprehensive perspective, help us to better understand the advantages and disadvantages of mathematics education in different countries, so as to provide useful reference and inspiration for improving the quality of global mathematics education.

According to the wide influence and usage, the author selected 15 primary school mathematics textbooks published by different publishers around the world as research objects.

They are widely available in six countries. We took into account the leading role of the government in the selection to ensure that the samples could reflect different cultural and educational backgrounds. This standard gives us a comprehensive understanding of the content and teaching methods of these textbooks and provides a solid foundation for further study.

At the same time, the author gives priority to textbooks with strong universality. This means that the textbooks selected are not only widely used, but also widely applicable to a variety of educational Settings and student groups. Their content and structure are designed to cater to the needs of different learners, making them suitable for comparison and analysis.

For example, California Mathematics: Concepts, Skils, and Problem Solving is the worm Machmilian (one of the most famous international publications, with its high quality education, academic, research and literature books product is famous for its) and Mc Graw-Hi(the famous Ivy League company in the United States, the world's top 500 enterprises), two major educational publishing institutions jointly published primary school mathematics textbooks, the textbook publishing house is the most influential basic education textbook publishing house in the United States, and California mathematics education plays a pivotal role in the development of mathematics education in the United States.

Table 1. Research objects

Nation	Version abbreviation	Textbook version	Publishing House
China	RJ	People's Education Press Grade 6 Volume 1 [4]	People's Education Press
The United States	CM	California Mathematics Grade6 [5]	The McGraw-Hill companies Glencoe
	M5	My Math Grade5 [6]	The McGraw-Hill companies Education
	HM	Go Math Grade6 [7]	Houghton Mifflin Harcourt
Singapore	TM	Targeting Mathematics 6A [8]	Star Publishing Pte Ltd
	MC	My Pals Are Here! Math 6A 3rd Edition [9]	Marshall Cavendish Education
Japan	Math 634	Math 6 (Math 634) [10]	School Book Press
	Math 631	New Math 6 (Math 631) [11]	Tokyo Books Press
	Math 638	Math 6 (Math 638) [12]	Qilin Guan Press
	Math 633	New Math 6 (Math 633) [13]	Dai Nippon Book Press
	Math 636	elementary school Math 6 (Math 636) [14]	Education Press
South Korea	TC	Math 6-1[15]	Genius Education Press
Germany	WE	Mathematik 6 (Hauptschule Bayern) [16]	Westermann
	NA	Lernstufen Mathematick 6 (Neue Ausgabe) [17]	Cornelsen
	MB	Lernstufen Mathematick 6 (Mittelschule Bayern) [18]	Cornelsen

3 Methodology

The main research methods of this study are literature method, content analysis method and comparative research method.

Literature method: Collect 15 primary mathematics textbooks from 6 countries, sort out their compilation characteristics, sample questions and related research and other relevant literature, obtain a preliminary understanding of primary mathematics textbooks and the compilation characteristics of sample questions, summarize the categories reflected in the compilation characteristics of primary mathematics textbooks, and then establish the text analysis framework of this study.

Content analysis: This method is a systematic and objective analysis of text content research means. When discussing the characteristics and differences of mathematics textbooks in different countries, text analysis can help us deeply analyze the text, charts, examples and other specific content in textbooks. This research is based on the established framework of text analysis of sample questions in primary school mathematics textbooks of six countries, and analyzes the text analysis and statistics of sample questions in the selected version of textbooks, and then probes into their compilation characteristics.

Comparative research method: Based on each category of sample text analysis framework and text analysis of all sample questions, compare the similarities and differences of the sample questions in each category of various versions of textbooks.

3.1 Representation models of fraction division meaning

Ma Liping's research indicates that the meaning of fraction division operations can be revealed through three models: the inclusion model, the part-whole model, and the product-and-factor model [3]. Among them, the part-whole model includes a special type where the divisor is an integer. The meaning of fraction division in such special cases is similar to the meaning of integer division [2]. Many researchers separate this special case from the part-whole model, often referring to it as the equal sharing model. In this study, the 15 elementary school mathematics textbooks from various publishers around the world generally follow these four types to reveal the meaning of fraction division.

(1) ***Inclusion Model***: Given a fixed total quantity of objects, known quantities for each share, the task is to determine the number of shares contained in the total quantity. For example, the context of the The United States's CM textbook is illustrated in Fig. 1 [5].

(2) ***Equal Sharing Model***: With a fixed total quantity of objects, the task is to divide the objects equally into a certain number of shares and determine the quantity of objects in each share. For example, the context of the German WE textbook is illustrated in Fig. 2 [16].

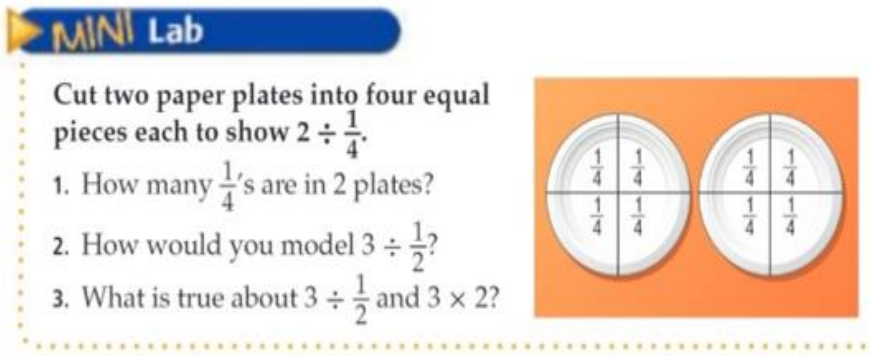


Fig. 1. Inclusion model

Alexander, Christine and Daniel share $\frac{6}{8}$ of the pizza equally. What portion of the pizza will each get?

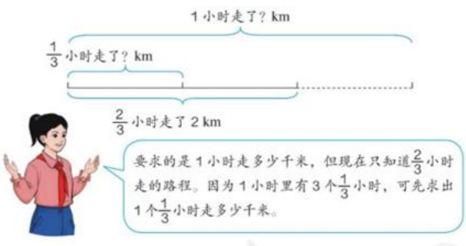
$$\frac{6}{8} \div 3 = \frac{2}{8}$$

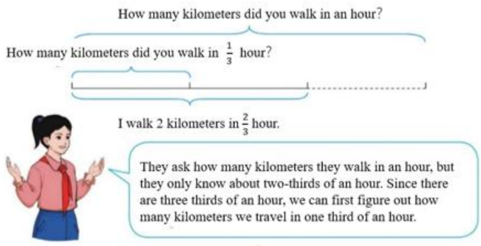
Read as six eighths divided by three equals two eighths, and each person gets two eighths of a pizza. Divide every six in the numerator by the divisor of three.

Fig. 2. Equal sharing model

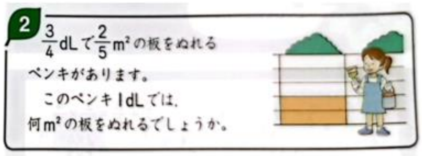
(3) ***Part-Whole Model***: The quantity of objects as a part of the whole is known, and the task is to determine the quantity of objects as the whole. It is a hybrid model relative to the equal sharing and inclusion models. For example, the contexts of the Chinese People's Education Press [4] and Japanese 636 textbooks [14] are illustrated in Fig. 3.

RJ





Math 636



If $\frac{3}{4}$dL paint can paint $\frac{2}{5}$ square meters of wall, how many square meters of wall can be painted with 1dL paint?

Fig. 3. Part-whole model

(4) ***Product-and-Factor Model***: The common representation of the product-and-factor model is "Area = Length × Width," where area corresponds to the product, and length or width corresponds to the other two factors. For example, the context of the Singapore MC textbook [9] is illustrated in Fig. 4.

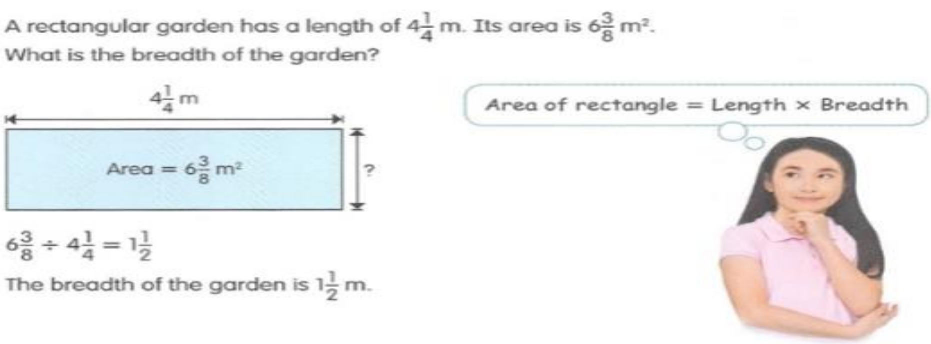


Fig. 4. Product-and-factor model

3.2 Revelation methods of arithmetic reasoning in fraction division

Intuitive geometry and deductive reasoning are the main ways to reveal the arithmetic reasoning in elementary school (a new perspective of procedural knowledge curriculum design: the integration of arithmetic reasoning and the integration of computational law) [19]. In the fraction division operation, there are four main ways to reveal the arithmetic reasoning in fraction division: intuitive geometry; Intuitive geometry combined with multiplication; Deductive reasoning based on intuitive geometry; Deductive reasoning supplemented by intuitive geometry; Deductive reasoning [2].

(1) **The intuitive geometry**

According to the meaning of fraction division, the results of fraction division problems are obtained directly by means of geometric figures or other intuitive geometrys. For example, in the South Korean TC textbook [15], 5/6÷1/6=5 is directly presented by dividing the number line. (See Fig. 5)

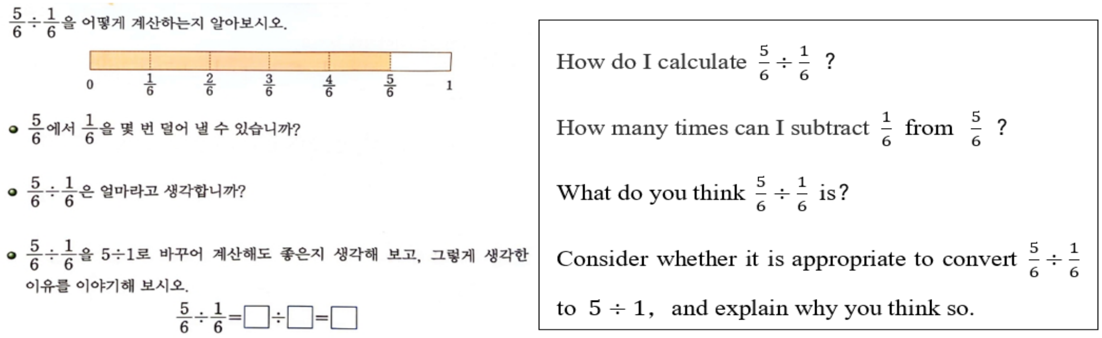


Fig. 5. The example of the intuitive geometry

(2) **Intuitive geometry Associated with Fraction Multiplication**

Obtain the result of fraction division through intuitive geometry, then express this result using fraction multiplication, thereby establishing the relationship between fraction division and multiplication operations [2]. For example, in the The United States's CM textbook [5], the example of 4 ÷ 1/3 = 12 involves first obtaining the result of 4 ÷ 1/3 through intuitive geometry as 12 and then connecting it with multiplication operations to discover that 4 × 3 equals 12. By associating intuitive geometry with fraction multiplication, a general arithmetic reasoning for fraction division can be derived: dividing by a fraction is equivalent to multiplying by the reciprocal of that fraction (see Fig. 6).

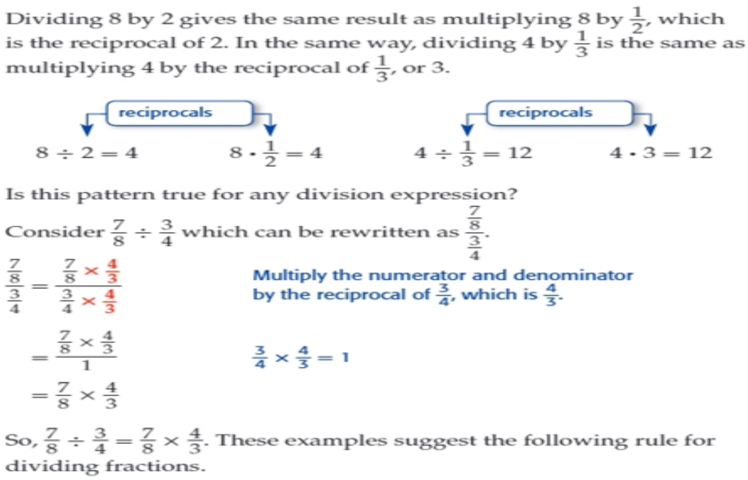


Fig. 6. The example of intuitive geometry associated with fraction multiplication

(3) **Deductive Reasoning Based on Intuitive geometry**

Utilize intuitive geometry to establish a mathematical conclusion, and then, starting from this conclusion, employ deductive reasoning to discover a new mathematical result [2]. For instance, in the Singapore MC textbook [9], when solving 5 ÷ 2/3, intuitive geometry reveals that there are three occurrences of 2/3 in 2. This becomes a crucial mathematical conclusion. Based on this conclusion, we can determine that 1 contains 3/2 occurrences of 2/3, and subsequently, 5 contains 5 × 3/2 occurrences of 2/3. Through deductive reasoning, a new mathematical result is obtained (see Fig. 7).

This approach differs from " The intuitive geometry " and " intuitive geometry associated with fraction multiplication" in two main aspects. Firstly, " The intuitive geometry " relies solely on intuitive geometry to derive mathematical conclusions without presenting any additional explanations or reasoning. Secondly, "deductive reasoning based on intuitive geometry " primarily depends on explanations and reasoning to obtain mathematical conclusions, with intuitive geometry serving as the starting point for explanations and reasoning [2]. Secondly, "deductive reasoning based on intuitive geometry " reveals the intrinsic connection between fraction division and multiplication, answering the question of "why dividing by a fraction is equivalent to multiplying by its reciprocal." Although " Intuitive geometry associated with fraction multiplication" also establishes a connection between fraction division and multiplication, the mechanism of this "intrinsic connection" remains a black-box problem and has not been answered positively (see Fig. 7).

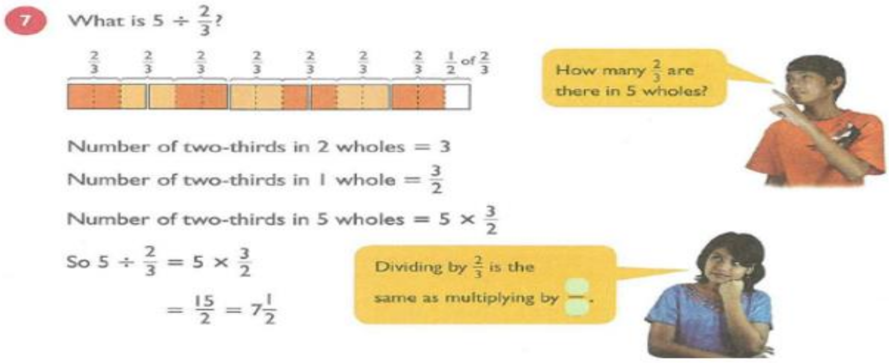


Fig. 7. The example of deductive reasoning based on intuitive geometry

(4) **Deductive reasoning supplemented by intuitive geometry**

The fraction division rule is constructed by deductive reasoning mainly in virtue of fraction meaning and fraction multiplication meaning. The intuitive geometry only plays an auxiliary role, and the extraction of intuitive geometry has little influence on the reasoning process.

The Japanese arithmetic 634 textbook [10] contains deductive reasoning supplemented by intuitive geometry (see Fig. 8). The problem to be solved in Fig. 8 is: 3/4 liter of paint can paint 2/5 square meters of wall surface, so how many square meters of wall surface can be painted with 1 liter of paint? According to the meaning of the score, 3/4 is 3 1/4, then the wall painted by 1/4 liter is 2/5÷3=2/5×1/3=2/15. According to the meaning of the score, 1 contains 4 1/4, then the wall painted by 1 liter of paint is 2/15×4=8/15.

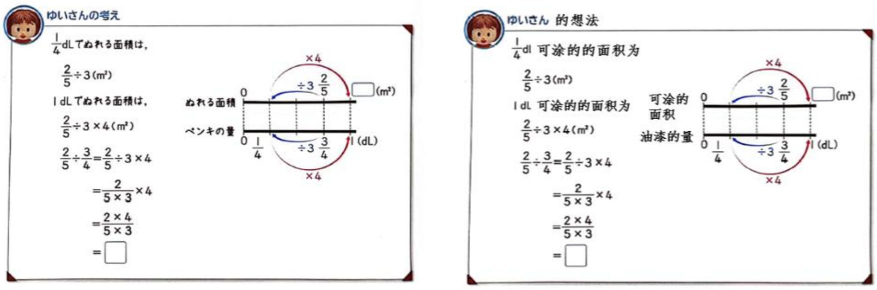


Fig. 8. The example of deductive reasoning supplemented by intuitive geometry

(5) **Deductive reasoning**

The arithmetic reasoning of fraction division is revealed by deductive reasoning, mainly by means of invariant properties of quotient and fraction multiplication law. In this process, there is no penetration of intuitive geometry. For example, in the Japanese arithmetic 634 textbook [10], 2/5÷3/4 is used as an example to reveal the arithmetic reasoning of fraction division. (See Fig. 9)

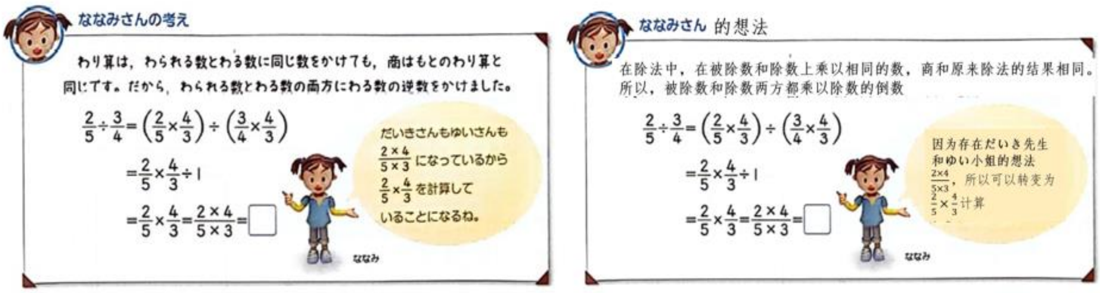


Fig. 9. The example of deductive reasoning

3 Results

3.1 Significance of fraction division

The models of the significance of fraction division in each teaching material are illustrated in Table 2.

Table 2. Models that introduce the meaning of fraction division

Nation	Textbook version	Using models
China	People's Education Press<	Part-Whole Model
		Equal Sharing Model
The United States	CM	Inclusion Model
	M5	Inclusion Model
	HM	Inclusion Model
Singaporee	TM	Equal Sharing Model
		Part-Whole Model
	MC	Part-Whole Model
		Inclusion Model
		Product-and-Factor Model
		Equal Sharing Model
Japan	Math 634	Part-Whole Model
	Math 631	Part-Whole Model
	Math 638	Part-Whole Model
	Math 633	Part-Whole Model
	Math 636	Part-Whole Model
Germany	WE	Equal Sharing Model
		Inclusion Model
	NA	Equal Sharing Model
		Inclusion Model
	MB	Equal Sharing Model
South Korea	TC	Inclusion Model

Fraction division is the inverse operation of fraction multiplication. The inclusion model and the equal division model are based on the concept of "a certain number of a certain quantity" in the multiplication of fraction, the former emphasizes how many parts, that is, the "a certain quantity" in the front, the latter emphasizes how many parts, that is, the "a certain number" in the back, and students can understand the rationality of the fraction division operation from the cognitive basis of the fraction multiplication.

Part-Whole Model emphasizes the concept of "unit" and focuses on students' reasoning and analysis ability of reverse thinking. Product and factor models combined with intuitive geometry make it easier for students to understand and relate to reality, and have a deeper understanding of fraction division.

The model the significance of fraction division used in the United States and South Korea is relatively simple. The inclusion model is used in the United States and South Korea, and the reason is related to the curriculum standards in the United States and South Korea. The new curriculum standards of the two countries not only focus on letting students master the arithmetic reasoning of fraction division, but also emphasize the application of practical problems, so as to cultivate students' ability to flexibly use mathematical knowledge to solve complex problems, rather than just staying in the mechanical calculation level.

Japanese curriculum standards focus on students' better understanding of the concept and operation methods of fractions as well as their application in practical problems, and put forward higher requirements for the application of arithmetic reasoning. Therefore, Japanese textbooks use part-whole model in the significance model of fraction division, and design multiple methods for one model to help students understand the consistency of the significance of fraction division.

German curriculum standards focus on cultivating students' multidimensional understanding and application ability of fraction division, and the equal division model adopted by them emphasizes the concept of equal distribution and part-whole, which is applicable to many problems involving equal distribution. The inclusion model, on the other hand, emphasizes partial inclusion and unequal distribution, and is suitable for more complex problems, such as allocating different amounts of resources or calculating unequal ratios. Through the introduction of these two types of models, the new curriculum standards in Germany aim to enable students to cope with different types of fraction division problems in a more comprehensive way. This kind of relatively balanced method is helpful to satisfy the learning needs of different students, help students better understand mathematical concepts, solve practical problems, and promote the overall development of mathematics education.

Singapore's curriculum standards emphasize not only the development of multidimensional application skills, but also a deep understanding of fraction division. In the TM and MC versions of the textbook, the four models of equal division, inclusion, part-whole, product and factor are involved, so that students can deal with different types of fraction division problems more comprehensively, promote the development of mathematical thinking and problem solving ability, and cultivate their mathematical literacy and creative thinking.

Chinese curriculum standards emphasize the effectiveness and depth of mathematics education. The introduction of the equal division model and part-whole model provides students with an intuitive and effective way of learning, which helps students better grasp the knowledge and skills of fraction division, so as to help students deeply understand the concept, establish a solid mathematical foundation, cultivate mathematical thinking and the ability to deal with practical problems.

To sum up, curriculum standards are not only a guide for the preparation of textbooks, but also a basis for evaluation. As the main carrier of curriculum standards, textbook carries the core value of education. Through in-depth analysis of curriculum standards in various countries, we find that: in the selection of materials, the arrangement of each chapter and detail of the textbook is tightly tied to the requirements of the national curriculum standards, reflecting the training requirements of the core literacy of the country, which is conducive to the guidance of teachers and the exploration of students.

3.2 Revelation methods of arithmetic reasoning in fraction division

See Table 3 for the detailed models of revelation methods of arithmetic reasoning in fraction division in each textbook.

The models used in The United States’s textbooks are basically geometric and intuitive, with a small part associated with fraction multiplication and less deduction. Germany and South Korea are similar to the United States in that they basically focus on complete intuitive geometry, which is partly related to the emphasis on practical problem solving in the curriculum standards of these countries.

In Singapore, there are three types of models for explaining the arithmetic reasoning of fraction division, and there are also different ways to explain the revelation methods of arithmetic reasoning in fraction division, which is relatively comprehensive and complete, echoing the Singapore curriculum standards that emphasizes students' multidimensional understanding of fraction division.

Fraction division in the teaching material of Peoples Education Press presents intuitive geometry, but the teaching material is more inclined to reveal the arithmetic reasoning through deductive reasoning, and intuitive geometry is only an auxiliary means of deductive reasoning [2]. In the process of reasoning, two types of reasoning are used, intuitive geometry and deductive reasoning supplemented by intuitive geometry, and more attention is paid to the level of reasoning, which is closely related to the Chinese curriculum standard that emphasizes the effectiveness of mathematics education and the depth of mathematics learning [20].

Table 3. Shows the model of revelation methods of arithmetic reasoning in fraction division

Nation	**Textbook version**	**Using models**
China	Peoples Education Press	The intuitive geometry
		Deductive reasoning supplemented by intuitive geometry
The United States	CM	Intuitive geometry Associated with Fraction Multiplication
	M5	The intuitive geometry
	HM	The intuitive geometry
Singapore	TM	The intuitive geometry
		Deductive Reasoning Based on Intuitive geometry
	MC	The intuitive geometry
		Deductive Reasoning Based on Intuitive geometry
		Deductive reasoning supplemented by intuitive geometry
Japan	Math 634	Deductive reasoning
		Deductive reasoning supplemented by intuitive geometry
		Deductive Reasoning Based on Intuitive geometry
	Math 631	Deductive reasoning
		Intuitive geometry Associated with Fraction Multiplication
		Deductive reasoning supplemented by intuitive geometry
	Math 638	Deductive reasoning
		Intuitive geometry Associated with Fraction Multiplication
		Deductive reasoning supplemented by intuitive geometry
	Math 636	Deductive reasoning
		Intuitive geometry Associated with Fraction Multiplication
		Deductive reasoning supplemented by intuitive geometry
	Math 633	Deductive reasoning
		Intuitive geometry Associated with Fraction Multiplication
		Deductive reasoning supplemented by intuitive geometry
Germany	WE	The intuitive geometry
	NA	The intuitive geometry
	MB	The intuitive geometry
South Korea	TC	The intuitive geometry

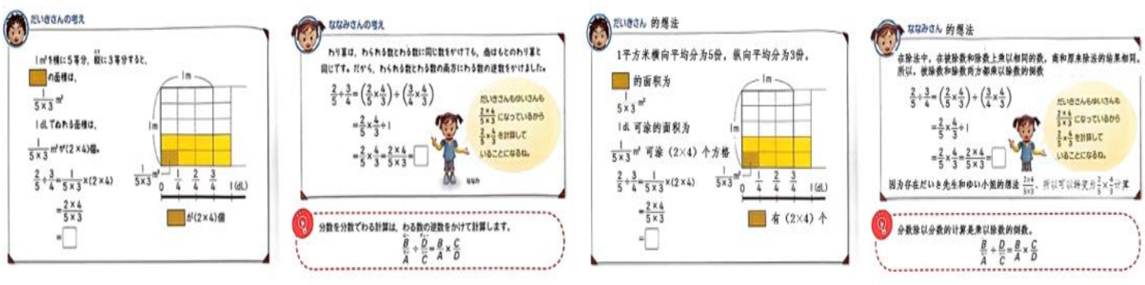


Fig. 10. The revelation methods of arithmetic reasoning in fraction division of Japanese textbooks

Comparing the curriculum standards of China and South Korea, it is found that both countries pay attention to arithmetic reasoning. China focuses on intuitive geometry so as to help students understand arithmetic reasoning in fraction division, which is more abstract and general; South Korea puts more emphasis on the diversity of arithmetic reasoning and the use of "learning materials" in textbooks.

The revelation methods of arithmetic reasoning of Japanese textbooks is more special, and the author will take Japanese arithmetic 634 textbook [10] as an example to illustrate it.

The arrangement of Japanese textbooks has reached a high degree of consistency, there are more than three ways to explain the same topic, and the exploration of a topic is relatively profound. In the revelation methods of arithmetic reasoning, deductive reasoning supplemented by intuitive geometry and the reasoning associated with intuitive geometry and fraction multiplication are mainly used. Through role play to show different reasoning methods, from intuitive to abstract, progressive presentation, it is more conducive to students' deep understanding of arithmetic reasoning. Its design is also very novel.

Based on the curriculum standards, the research on the revelation methods of arithmetic reasoning in textbooks of various countries is conducive to the understanding and implementation of the meaning of fraction division, and is conducive to teachers' effective teaching according to students' cognitive level. The grade who learn fraction division is senior elementary school students, and the core quality of elementary school includes reasoning consciousness, while junior middle school students include reasoning ability. The development of mathematical ability of senior primary school students requires that reasoning consciousness be connected with junior middle school reasoning ability, but senior primary school students still stay in the concrete operation stage, and need geometric and intuitive assistance in the process of revealing and reasonably explaining arithmetic reasoning. However, the overall adoption of intuitive geometry will cause students to be afraid of difficulties when they encounter problems requiring high logical thinking such as reasoning proof, which hinders the development of students' deductive ability. Therefore, in the mathematics textbooks of senior primary schools, based on the connection between psychological development stage and elementary school, how to deal with the proportion of intuitive geometry and deductive reasoning needs to be explored.

4 Discussion

After studying 15 textbooks, the author found that most countries often demonstrate the meaning of fraction division from a single dimension. This narrow interpretation simplifies the understanding difficulty for beginners to a certain extent. However, from the perspective of knowledge construction, it fails to fully reveal the deep meaning and multiple applications of fraction division, making students' understanding of this concept tend to be one-sided. This one-sided revealing process not only limits students' deeper understanding of fraction division, but also hinders them from building a complete and three-dimensional mathematical knowledge network. Therefore, we need to find a more comprehensive and profound way to reveal the meaning of fraction division, so as to promote students to better understand and master this important mathematical concept.

After a thorough exploration, the author found that the singularity of the revealing process of fractional division lies in the narrow dimension of its meaning. The meaning of each fraction division often corresponds to only one or two ways of revelation, which not only hinders students' progress in understanding deeply, but also makes it difficult for them to build a complete knowledge network. Therefore, we need to enrich the meaning dimension of fraction division. By revealing the connotation and application of fraction division from multiple dimensions and perspectives, we can broaden the path of its revelation process, provide students with more diversified learning experiences, and help them establish a complete and profound knowledge network.

To overcome this limitation, we need to tap into the power of reasoning and explore the inherent connections between fraction division and other related knowledge, such as the meaning of fractions and fraction multiplication. By gradually reasoning from fraction multiplication and the meaning of fractions to fraction division, we can deeply reveal the underlying significance of fraction division and achieve effective knowledge transfer. In this process, we can not only closely link knowledge points such as the meaning of fractions, fraction multiplication, and fraction division to form a complete knowledge network, but also deepen students' conceptual understanding of fraction division, cultivate their logical thinking and reasoning abilities, and lay a solid foundation for future learning and research. Therefore, we should pay attention to the application of reasoning in mathematics teaching, guide students to think deeply and explore, so that they can truly grasp the core meaning of fraction division and achieve internalization and sublimation of knowledge.

5 Suggestions

In the field of elementary mathematics education, fraction division is a crucial topic. By comparing the meaning of fraction division and arithmetic in textbooks of different countries, we provide the following beneficial suggestions for elementary school mathematics teachers to improve the teaching method of fraction division, so as to better satisfy the learning needs of students.

Firstly, we suggest that teachers should focus on two aspects in the teaching of fraction division: one is the meaning of fraction division, and the other is the revelation methods of arithmetic reasoning in fraction division. The meaning of fraction division is the foundation for students to understand this concept, while the revelation methods of arithmetic reasoning is the key to helping students master operational skills. Therefore, when designing teaching activities, teachers should ensure that both aspects are fully attended to and explained, thus ensuring that students can comprehensively and deeply understand the essence of fraction division.

Secondly, we suggest that textbooks should present multiple meanings when explaining the meaning of fraction division. A single meaning may limit students' understanding depth and breadth, while multiple meanings can help students recognize fraction division from multiple perspectives and form a more complete and three-dimensional knowledge network. Such textbook design will help students better grasp the core meaning of fraction division and flexibly apply it in actual calculations.

These two suggestions are aimed at improving the teaching effect of fraction division and meeting the learning needs of students, focusing on teaching and textbooks respectively. We hope that these suggestions can provide useful references for elementary school mathematics teachers and promote the continuous improvement and development of fraction division teaching.

6 Conclusion

Through the comparative study of 15 elementary school mathematics textbooks "fraction division" from "China, Japan, South Korea, Singapore, The United States and Germany", this study analyzes that there are great differences in the representation Models of Fraction Division Meaning and the revealing process of arithmetic reasoning.

Based on the curriculum standards of different countries, the author explains the reasons for the differences, and expounds the meaning model of fraction division in different countries and the relationship between the proportion of intuitive geometry and deductive reasoning according to the connection between the psychological development of students in the upper grades of elementary school , clarify the context so as to find a suitable learning path for students and a teaching method in line with the national conditions. At the same time, the author will select practical schools to carry out action research, and put the above theoretical results into practice, so as to effectively implement the new curriculum standards and teaching material requirements, and truly achieve the purpose of "light burden and high quality".

Competing Interests

Authors have declared that no competing interests exist.

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