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Determinant of Matrix by Order Condensation

Feng Cheng Chang[1*]

[1]*Allwave Corporation, Torrance, California, USA.*

Original Research Article

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Abstract

A simple and direct process is derived to compute the determinant of any square matrix of high order. The approach involves successive applying the matrix order condensation algorithm. A computer program listing in MATLAB is included and examples for finding the determinant of a given 7x7 matrix are given here for illustration.

Keywords: Matrix determinant, matrix inversion, co-factor expansion, QR decomposition, gauss elimination, polynomial division.

1 Introduction

The algorithm is developed for the matrix order condensation. By successive applying this algorithm, the determinant of any high order matrix may readily evaluated. It is very simple, efficient and direct, comparing to the familiar approaches, such as co-factor expansion, Gauss elimination, QR decomposition, etc.

2 Algorithm for Matrix Order Condensation

From a given matrix $\mathbf{A}^{(n)}$ of order n, after arbitrary drawing the selected l sets of cross-lines, the pivot matrix $\mathbf{A}^{(l)}$ of order l and the condensed matrix $\mathbf{A}^{(m)}$ or $\mathbf{A}_o^{(m)}$ of order m (= $n - l$) are generated, such that

$$\det(\mathbf{A}^{(n)}) = \det(\mathbf{A}^{(l)})^{1-(n-l)} * \det(\mathrm{A}^{(m)}), \quad \text{or}$$

$$\det(\mathbf{A}^{(n)}) = \det(\mathbf{A}^{(l)}) * \det(\mathbf{A}_o^{(m)}).$$

The pivot matrix $\mathbf{A}^{(l)}$ is directly obtained with elements at the l x l cross-points of the l sets of cross-lines, provided that $\det(\mathbf{A}^{(l)})$ is not equal to zero. The condensed matrix $\mathbf{A}^{(m)}$ is then created with entries at the m x m un-crossed out locations, where each entry must be replaced by a computed determinant formed by (l+1) x (l+1) cross-points of the (l+1) sets of cross-lines (l at the selected, plus 1 at its entry location). And finally the nominal condensed matrix $\mathbf{A}_o^{(m)}$ is computed:

*Corresponding author: fcchang007@yahoo.com;

$$\mathbf{A}_\mathbf{o}^{(m)} = \mathbf{A}^{(m)} / \det(\mathbf{A}^{(l)}), \qquad \text{or}$$

$$\det(\mathbf{A}_\mathbf{o}^{(m)}) = \det(\mathbf{A}^{(m)}) / \det(\mathbf{A}^{(l)})^{m}.$$

3 Typical Examples

The evaluation of the determinant of a given 7 x 7 matrix is given here for illustration. For comparison of feasibility, two computing schemes are presented: (1) in terms of 2x2 determinants, and (2) in terms of 3x3 determinants.

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(1) Find 7x7 determinant, in terms of 2x2 determinants. ($l = 1$)

$$\det(\mathbf{A}) = \begin{vmatrix} 2 & -1 & 5 & 8 & 3 & -4 & 5 \\ 0 & -2 & -3 & 4 & 3 & 0 & 8 \\ 1 & -3 & -2 & 5 & 0 & 7 & 1 \\ 4 & 6 & -4 & 1 & 2 & 9 & 0 \\ 3 & 5 & -2 & 4 & -7 & 7 & 0 \\ 2 & 0 & 1 & 6 & -3 & 2 & 8 \\ 3 & -5 & 7 & -9 & 1 & 6 & 1 \end{vmatrix}$$

Cross-lines:
row: 6; column: 3.

$$= |+1|^{1-(7-1)} \begin{vmatrix} \begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 5 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 5 & 8 \\ 1 & 6 \end{vmatrix} & \begin{vmatrix} 5 & 3 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} 5 & -4 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 5 & 5 \\ 1 & 8 \end{vmatrix} \\ \begin{vmatrix} 0 & -3 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} -2 & -3 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} -3 & 4 \\ 1 & 6 \end{vmatrix} & \begin{vmatrix} -3 & 3 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} -3 & 0 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} -3 & 8 \\ 1 & 8 \end{vmatrix} \\ \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} -3 & -2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} -2 & 5 \\ 1 & 6 \end{vmatrix} & \begin{vmatrix} -2 & 0 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} -2 & 7 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} -2 & 1 \\ 1 & 8 \end{vmatrix} \\ \begin{vmatrix} 4 & -4 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 6 & -4 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} -4 & 1 \\ 1 & 6 \end{vmatrix} & \begin{vmatrix} -4 & 2 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} -4 & 9 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} -4 & 0 \\ 1 & 8 \end{vmatrix} \\ \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 5 & -2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} -2 & 4 \\ 1 & 6 \end{vmatrix} & \begin{vmatrix} -2 & -7 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} -2 & 7 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} -2 & 0 \\ 1 & 8 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 3 & 7 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ -5 & 7 \end{vmatrix} & \begin{vmatrix} 1 & 6 \\ 7 & -9 \end{vmatrix} & \begin{vmatrix} 1 & -3 \\ 7 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 7 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 8 \\ 7 & 1 \end{vmatrix} \end{vmatrix}$$

$$= (+1)^{1-6} \begin{vmatrix} -8 & -1 & 22 & -18 & 14 & 35 \\ 6 & -2 & -22 & 6 & -6 & -32 \\ 5 & -3 & -17 & 6 & -11 & -17 \\ 12 & 6 & -25 & 10 & -17 & -32 \\ 7 & 5 & -16 & 13 & -11 & -16 \\ 11 & 5 & -51 & 22 & -8 & -55 \end{vmatrix}$$

$$= (+1) \times \begin{vmatrix} -8 & -1 & 22 & -18 & 14 & 35 \\ 6 & -2 & -22 & 6 & -6 & -32 \\ 5 & -3 & -17 & 6 & -11 & -17 \\ 12 & 6 & -25 & 10 & -17 & -32 \\ 7 & 5 & -16 & 13 & -11 & -16 \\ 11 & 5 & -51 & 22 & -8 & -55 \end{vmatrix}$$

Cross-lines:
row: 2; column: 2.

$$= (+1) \times \quad |-2|^{1-(6-1)} \begin{vmatrix} \begin{vmatrix} -8 & -1 \\ 6 & -2 \end{vmatrix} & \begin{vmatrix} -1 & 22 \\ -2 & -22 \end{vmatrix} & \begin{vmatrix} -1 & -18 \\ -2 & 6 \end{vmatrix} & \begin{vmatrix} -1 & 14 \\ -2 & -6 \end{vmatrix} & \begin{vmatrix} -1 & 35 \\ -2 & -32 \end{vmatrix} \\ \begin{vmatrix} 6 & -2 \\ 5 & -3 \end{vmatrix} & \begin{vmatrix} -2 & -22 \\ -3 & -17 \end{vmatrix} & \begin{vmatrix} -2 & 6 \\ -3 & 6 \end{vmatrix} & \begin{vmatrix} -2 & -6 \\ -3 & -11 \end{vmatrix} & \begin{vmatrix} -2 & -32 \\ -3 & -17 \end{vmatrix} \\ \begin{vmatrix} 6 & -2 \\ 12 & 6 \end{vmatrix} & \begin{vmatrix} -2 & -22 \\ 6 & -25 \end{vmatrix} & \begin{vmatrix} -2 & 6 \\ 6 & 10 \end{vmatrix} & \begin{vmatrix} -2 & -6 \\ 6 & -17 \end{vmatrix} & \begin{vmatrix} -2 & -32 \\ 6 & -32 \end{vmatrix} \\ \begin{vmatrix} 6 & -2 \\ 7 & 5 \end{vmatrix} & \begin{vmatrix} -2 & -22 \\ 5 & -16 \end{vmatrix} & \begin{vmatrix} -2 & 6 \\ 5 & 13 \end{vmatrix} & \begin{vmatrix} -2 & -6 \\ 5 & -11 \end{vmatrix} & \begin{vmatrix} -2 & -32 \\ 5 & -16 \end{vmatrix} \\ \begin{vmatrix} 6 & -2 \\ 11 & 5 \end{vmatrix} & \begin{vmatrix} -2 & -22 \\ 5 & -51 \end{vmatrix} & \begin{vmatrix} -2 & 6 \\ 5 & 22 \end{vmatrix} & \begin{vmatrix} -2 & -6 \\ 5 & -8 \end{vmatrix} & \begin{vmatrix} -2 & -32 \\ 5 & -55 \end{vmatrix} \end{vmatrix}$$

$$= (+1) \times \quad (-2)^{1-5} \begin{vmatrix} 22 & 66 & -42 & 34 & 102 \\ -8 & -32 & 6 & 4 & -62 \\ 60 & 182 & -56 & 70 & 256 \\ 44 & 142 & -56 & 52 & 192 \\ 52 & 212 & -74 & 46 & 270 \end{vmatrix}$$

$$= (+1)\times(-2)\times \begin{vmatrix} -11 & -33 & 21 & -17 & -51 \\ 4 & 16 & -3 & -2 & 31 \\ -30 & -91 & 28 & -35 & -128 \\ -22 & -71 & 28 & -26 & -96 \\ -26 & -106 & 37 & -23 & -135 \end{vmatrix}$$

Cross-lines: row: 2; column: 4.

$$= (+1)\times(-2)\times \quad |-2|^{1-(5-1)} \begin{vmatrix} \begin{vmatrix} -11 & -17 \\ 4 & -2 \end{vmatrix} & \begin{vmatrix} -33 & -17 \\ 16 & -2 \end{vmatrix} & \begin{vmatrix} 21 & -17 \\ -3 & -2 \end{vmatrix} & \begin{vmatrix} -17 & -51 \\ -2 & 31 \end{vmatrix} \\ \begin{vmatrix} 4 & -2 \\ -30 & -35 \end{vmatrix} & \begin{vmatrix} 16 & -2 \\ -91 & -35 \end{vmatrix} & \begin{vmatrix} -3 & -2 \\ 28 & -35 \end{vmatrix} & \begin{vmatrix} -2 & 31 \\ -35 & -128 \end{vmatrix} \\ \begin{vmatrix} 4 & -2 \\ -22 & -26 \end{vmatrix} & \begin{vmatrix} 16 & -2 \\ -71 & -26 \end{vmatrix} & \begin{vmatrix} -3 & -2 \\ 28 & -26 \end{vmatrix} & \begin{vmatrix} -2 & 31 \\ -26 & -96 \end{vmatrix} \\ \begin{vmatrix} 4 & -2 \\ -26 & -23 \end{vmatrix} & \begin{vmatrix} 16 & -2 \\ -106 & -23 \end{vmatrix} & \begin{vmatrix} -3 & -2 \\ 37 & -23 \end{vmatrix} & \begin{vmatrix} -2 & 31 \\ -23 & -135 \end{vmatrix} \end{vmatrix}$$

$$= (+1)\times(-2)\times \quad (-2)^{1-4} \begin{vmatrix} 90 & 338 & -93 & -629 \\ -200 & -742 & 161 & 1341 \\ -148 & -558 & 134 & 998 \\ -144 & -580 & 143 & 983 \end{vmatrix}$$

$$= (+1)\times(-2)\times(-2)\times \begin{vmatrix} -45.0 & -169.0 & 46.5 & 314.5 \\ 100.0 & 371.0 & -80.5 & -670.5 \\ 74.0 & 279.0 & -67.0 & -499.0 \\ 72.0 & 290.0 & -71.5 & -491.5 \end{vmatrix}$$

Cross-lines: row: 2; column: 1.

$$= (+1)\times(-2)\times(-2)\times \quad |+100|^{1-(4-1)} \begin{vmatrix} \begin{vmatrix} -45.0 & -169.0 \\ 100.0 & 371.0 \end{vmatrix} & \begin{vmatrix} -45.0 & 46.5 \\ 100.0 & -80.5 \end{vmatrix} & \begin{vmatrix} -45.0 & 314.5 \\ 100.0 & -670.5 \end{vmatrix} \\ \begin{vmatrix} 100.0 & 371.0 \\ 74.0 & 279.0 \end{vmatrix} & \begin{vmatrix} 100.0 & -80.5 \\ 74.0 & -67.0 \end{vmatrix} & \begin{vmatrix} 100.0 & -670.0 \\ 74.0 & -499.0 \end{vmatrix} \\ \begin{vmatrix} 100.0 & 371.0 \\ 72.0 & 290.0 \end{vmatrix} & \begin{vmatrix} 100.0 & -80.5 \\ 72.0 & -71.5 \end{vmatrix} & \begin{vmatrix} 100.0 & -670.0 \\ 72.0 & -491.5 \end{vmatrix} \end{vmatrix}$$

$$= (+1)\times(-2)\times(-2)\times \quad (+100)^{1-3} \begin{vmatrix} 205.0 & -1027.5 & -1277.5 \\ 446.0 & -743.0 & -283.0 \\ 2288.0 & -1354.0 & -874.0 \end{vmatrix}$$

$$= (+1)\times(-2)\times(-2)\times(+100)\times \begin{vmatrix} 2.050 & -10.275 & -12.775 \\ 4.460 & -7.430 & -2.830 \\ 22.880 & -13.540 & -8.740 \end{vmatrix}$$

Cross-lines: row: 1; column: 2.

$$= (+1)\times(-2)\times(-2)\times(+100)\times \quad |-10.275|^{1-(3-1)} \begin{vmatrix} \begin{vmatrix} 2.050 & -10.275 \\ 4.460 & -7.430 \end{vmatrix} & \begin{vmatrix} -10.275 & -12.775 \\ -7.430 & -2.830 \end{vmatrix} \\ \begin{vmatrix} 2.050 & -10.275 \\ 22.880 & -13.540 \end{vmatrix} & \begin{vmatrix} -10.275 & -12.775 \\ -13.540 & -8.740 \end{vmatrix} \end{vmatrix}$$

$$= (+1)\times(-2)\times(-2)\times(+100)\times \quad (-10.275)^{1-2} \begin{vmatrix} 30.595 & -65.840 \\ 207.335 & -83.170 \end{vmatrix}$$

$$= (+1)\times(-2)\times(-2)\times(+100)\times(-10.275)\times \begin{vmatrix} -2.9776 & 6.4078 \\ -20.1786 & 8.0944 \end{vmatrix}$$

$$= (+1)\times(-2)\times(-2)\times(+100)\times(-10.275)\times \ (+105.2)$$

$$= -432364. \quad \text{(Answer)}$$

(2)..Find 7x7 determinant, in terms of 3x3 determinants. ($l = 2$)

$$\det(\mathbf{A}) = \begin{vmatrix} 2 & -1 & 5 & 8 & 3 & -4 & 5 \\ 0 & -2 & -3 & 4 & 3 & 0 & 8 \\ 1 & -3 & -2 & 5 & 0 & 7 & 1 \\ 4 & 6 & -4 & 1 & 2 & 9 & 0 \\ 3 & 5 & -2 & 4 & -7 & 7 & 0 \\ 2 & 0 & 1 & 6 & -3 & 2 & 8 \\ 3 & -5 & 7 & -9 & 1 & 6 & 1 \end{vmatrix}$$

Cross-lines:
rows: 2, 5; columns: 2, 5.

$$= \begin{vmatrix} -2 & 3 \\ 5 & -7 \end{vmatrix}^{1-(7-2)} \begin{vmatrix}
\begin{vmatrix} 2 & -1 & 3 \\ 0 & -2 & 3 \\ 3 & 5 & -7 \end{vmatrix} & \begin{vmatrix} -1 & 5 & 3 \\ -2 & -3 & 3 \\ 5 & -2 & -7 \end{vmatrix} & \begin{vmatrix} -1 & 8 & 3 \\ -2 & 4 & 3 \\ 5 & 4 & -7 \end{vmatrix} & \begin{vmatrix} -1 & 3 & -4 \\ -2 & 3 & 0 \\ 5 & -7 & 7 \end{vmatrix} & \begin{vmatrix} -1 & 3 & 5 \\ -2 & 3 & 8 \\ 5 & -7 & 0 \end{vmatrix} \\
\begin{vmatrix} 0 & -2 & 3 \\ 1 & -3 & 0 \\ 3 & 5 & -7 \end{vmatrix} & \begin{vmatrix} -2 & -3 & 3 \\ -3 & -2 & 0 \\ 5 & -2 & -7 \end{vmatrix} & \begin{vmatrix} -2 & 4 & 3 \\ -3 & 5 & 0 \\ 5 & 4 & -7 \end{vmatrix} & \begin{vmatrix} -2 & 3 & 0 \\ -3 & 0 & 7 \\ 5 & -7 & 7 \end{vmatrix} & \begin{vmatrix} -2 & 3 & 8 \\ -3 & 0 & 1 \\ 5 & -7 & 0 \end{vmatrix} \\
\begin{vmatrix} 0 & -2 & 3 \\ 4 & 6 & 2 \\ 3 & 5 & -7 \end{vmatrix} & \begin{vmatrix} -2 & -3 & 3 \\ 6 & -4 & 2 \\ 5 & -2 & -7 \end{vmatrix} & \begin{vmatrix} -2 & 4 & 3 \\ 6 & 1 & 2 \\ 5 & 4 & -7 \end{vmatrix} & \begin{vmatrix} -2 & 3 & 0 \\ 6 & 2 & 9 \\ 5 & -7 & 7 \end{vmatrix} & \begin{vmatrix} -2 & 3 & 8 \\ 6 & 2 & 0 \\ 5 & -7 & 0 \end{vmatrix} \\
\begin{vmatrix} 0 & -2 & 3 \\ 3 & 5 & -7 \\ 2 & 0 & -3 \end{vmatrix} & \begin{vmatrix} -2 & -3 & 3 \\ 5 & -2 & -7 \\ 0 & 1 & -3 \end{vmatrix} & \begin{vmatrix} -2 & 4 & 3 \\ 5 & 4 & -7 \\ 0 & 6 & -3 \end{vmatrix} & \begin{vmatrix} -2 & 3 & 0 \\ 5 & -7 & 7 \\ 0 & -3 & 2 \end{vmatrix} & \begin{vmatrix} -2 & 3 & 8 \\ 5 & -7 & 0 \\ 0 & -3 & 8 \end{vmatrix} \\
\begin{vmatrix} 0 & -2 & 3 \\ 3 & 5 & -7 \\ 3 & -5 & 1 \end{vmatrix} & \begin{vmatrix} -2 & -3 & 3 \\ 5 & -2 & -7 \\ -5 & 7 & 1 \end{vmatrix} & \begin{vmatrix} -2 & 4 & 3 \\ 5 & 4 & -7 \\ -5 & -9 & 1 \end{vmatrix} & \begin{vmatrix} -2 & 3 & 0 \\ 5 & -7 & 7 \\ -5 & 1 & 6 \end{vmatrix} & \begin{vmatrix} -2 & 3 & 8 \\ 5 & -7 & 0 \\ -5 & 1 & 1 \end{vmatrix}
\end{vmatrix}$$

$$= (-1)^{1-5} \begin{vmatrix} 7 & 35 & -36 & 25 & 59 \\ 28 & 83 & -125 & 70 & 169 \\ -62 & -196 & 295 & -145 & -416 \\ -20 & -56 & 90 & -44 & -128 \\ -42 & -109 & 163 & -97 & -241 \end{vmatrix}$$

$$= (-1) \times \begin{vmatrix} -7 & -35 & 36 & -25 & -59 \\ -28 & -83 & 125 & -70 & -169 \\ 62 & 196 & -295 & 145 & 416 \\ 20 & 56 & -90 & 44 & 128 \\ 42 & 109 & -163 & 97 & 241 \end{vmatrix}$$

Cross-lines:
rows: 1, 4; columns: 2, 4.

$$= (-1) \times \begin{vmatrix} -35 & -25 \\ 56 & 44 \end{vmatrix}^{1-(5-2)} \begin{vmatrix}
\begin{vmatrix} -7 & -35 & -25 \\ -28 & -83 & -70 \\ 20 & 56 & 44 \end{vmatrix} & \begin{vmatrix} -35 & 36 & -25 \\ -83 & 125 & -70 \\ 56 & -90 & 44 \end{vmatrix} & \begin{vmatrix} -35 & -25 & -59 \\ -83 & -70 & -169 \\ 56 & 44 & 128 \end{vmatrix} \\
\begin{vmatrix} -7 & -35 & -25 \\ 62 & 196 & 145 \\ 20 & 56 & 44 \end{vmatrix} & \begin{vmatrix} -35 & 36 & -25 \\ 196 & -295 & 145 \\ 56 & -90 & 44 \end{vmatrix} & \begin{vmatrix} -35 & -25 & -59 \\ 196 & 145 & 416 \\ 56 & 44 & 128 \end{vmatrix} \\
\begin{vmatrix} -7 & -35 & -25 \\ 20 & 56 & 44 \\ 42 & 109 & 97 \end{vmatrix} & \begin{vmatrix} -35 & 36 & -25 \\ 56 & -90 & 44 \\ 109 & -163 & 97 \end{vmatrix} & \begin{vmatrix} -35 & -25 & -59 \\ 56 & 44 & 128 \\ 109 & 97 & 241 \end{vmatrix}
\end{vmatrix}$$

$$= (-1) \times (-140)^{1-3} \begin{vmatrix} 1704 & 6602 & 8528 \\ 1652 & 7406 & 6104 \\ 3068 & 14584 & 14496 \end{vmatrix}$$

$$= (-1) \times (-140)^{-2} \times (8474334400)$$

$$= -432364 \quad \text{(Answer)}$$

4 Computer Program Listing

A compact computer program listing in MATLAB is listed as follow:

```
function detM = det_L(M)
%       Finding determinant of square matrix by Condensation
%       F C Chang     updated   04/12/2014
        Z = [ ];  M,;
    for k = 1:size(M,1),
        m = size(M,1),;
        pq = input('select rows and cols :  [p;q] =  ');
        p = sort(pq(1,:));
        q = sort(pq(2,:));
        e = (-1)^sum(pq(:));
        L = length(p);
        W = M(setdiff(1:m,p),setdiff(1:m,q)),;
        V = M(setdiff(1:m,p),q),;
        U = M(p,setdiff(1:m,q)),;
        R = M(p,q),;
        IR = inv(R),;
        dR = det(R),;
        Z = [Z,dR*e];
      if m <= L,  break, end;
        M = W-V*IR*U,;
     end;
        detM = prod(Z); Z,;
```

5 Remarks and Conclusion

The determinant of any given high order matrix can be readily computed by successively applying the matrix order condensation algorithm. This approach is the extension of the method given by [1] and Chio's method [2] is the special case of the presented algorithm with $l = 1$. It is especially useful for hand computation.

Comparing the two schemes in the example, we note that, excluding any elementary arithmetic's operations, such as addition, subtraction, multiplication, and division, it needs 6 steps and 91 (= 36+25+16+9+4+1) 2x2 determinant computations for scheme 1, while it requires only 3 steps and 35 (= 25+9+1) 3x3 determinant computations for scheme 2. However, it is concluded that the scheme 1 is better choice, since the number of multiplications needed is just only 2 for every 2x2 determinant comparing to 12 for every 3x3 determinant.

The required multiplications for finding an n x n determinant are in the order of order of $n\,!$ by using the classical methods of Leibniz' formula or Laplace's formula. It is found, however, that the multiplication/division operations required for scheme 1 are merely 1/3 $n(n-1)(2n-1)$, that is much less then n^3 operations need for the product of any two n x n matrices.

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Competing Interests

Author has declared that no competing interests exist.

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