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Second Order Runge-Kutta Method in Solving Renewable Natural Resources Model Case Study: Fishes and Water Resources Management

R. Oguara [a*], C. A. Nse [a] and F. K. Namah [b]

[a] Department of Mathematics, Federal University of Technology, Owerri, Imo State, Nigeria.
[b] Department of Mathematics and Statistics, Federal Polytechnic of Oil and Gas, Bonny, Rivers State, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This research presents a study of a dynamical system which models natural resources. Using fishes and water resources as the case study. In this work a model equation was used to determine the density of the resources. We also introduced second stage Runge Kutta method, which was used to obtain the result for harvesting terms $h(p)$ from 10% to 50% producing the population of life in the pond after each harvesting.

In this research work, the solution of a dynamic system that can model natural resources using the second stage Runge-Kutta method was reported. To actualize this result, a model, a first order ordinary differential equation and the famous Runge-Kutta second stage method is used. Harvesting terms $H(p)$ from 10% to 50% is used for the iteration to demonstrate the validity of the result. The result of this study was applied to the population of fish in a pond, which is a renewable natural resource, with great success.

**Corresponding author: Email: oguararichard02@gmail.com;*

In line with the objectives stated at the beginning of the work, the results of this research has shown that by applying the second order Runge-Kutta method, the solution of a dynamical system can be obtained and applied to model natural resources with great success.

Keywords: Natural resources; 2nd order Runge Kutta; harvesting term; model equation.

1 Introduction

The sustainable use of natural resources is of utmost importance for every community. In particular, it is important for every given generation to plan in such a way that proper provision is made for future generations. The scientific understanding of resources used and appreciation for its life-supporting capacity is therefore essential [1]. Mathematical modeling has proved useful to inform the planning and management of strategies for sustainable use of natural resources [2]. Some specific topics in resource management have been studied intensively through many decades. In particular, mining, fisheries, forestry and water resources are among these. Instead of presenting a study of the latter topics, this dissertation presents a variety of cases of mathematical modeling in resource management. The aim is to improve the general understanding of the relevant problems [1]. A dynamical system is all about the evolution of something over time. To create a dynamical system, we simply need to decide what is the "something" that will evolve over time and what is the rule that specifies how that something evolves with time. In this way, a dynamical system is simply a model describing the temporal evolution of a system [3]. To study a dynamic system of natural resources we deploy the Runge-kutta method which has proven effective in solving problems of this kind [4]. Runge–Kutta method is an effective and widely used method for solving the initial-value problems of differential equations. Runge–Kutta method can be used to construct high order accurate numerical method by functions' self without needing the high order derivatives of functions [5-10]. The Runge-Kutta method attempts to overcome the problem of the Euler's method [5]. Amidst this background, our work employed the second order Runge Kutta method to establish the population of fishes in a pond with varying harvesting degree for a five-month period. Another objective was to develop a model for the population using different harvesting terms. This will pave the way for effective use if resources and appreciating the life support capacity of the pond.

2 Preliminaries

Basic terms related the modeling of natural resources using the runge kutta second stage method.

2.1 "Classical second order Runge Kutta Method"

A conventional one step method for the IVP which is given by

$$(y' = f(x, y), y(a) = \eta, a \le x \le b) \tag{2.1}$$

can be written as;

$$(y_{n+1} = y_n + h\phi(x_n, y_n, h)), \tag{2.2}$$

where $\phi(x_n, y_n, h)$ is called the increment function and it is given by

$$\phi(x_n, y_n, h) = \sum_{r=0}^{h^r} \frac{h^r}{(r+1)!} f^r(x, y) \tag{2.3}$$

When $r = 0$ we have

$$\phi(x_n, y_n, h) = +\frac{h^0}{1!} f^0(x, y) = f \tag{2.4}$$

When $r = 1$ we have

$$\phi(x_n, y_n, h) = \frac{h^1}{(1+1)!} f^1(x, y) = \frac{h}{2} f^1(x, y) \quad (2.5)$$

When $r = 2$ we have

$$\phi(x_n, y_n, h) = \frac{h^2}{(2+1)!} f(x, y) = \frac{h^2}{3!} f^I(x, y) = \frac{h^2}{6} f^{II}(x, y) \quad (2.6)$$

Summing (2.4), (2.5) and (2.6) we obtain

$$\phi(x_n, y_n, h) = f + \frac{h}{2} f'(x, y) + \frac{h^2}{6} f''(x, y) + O(h^3) \quad (2.7)$$

Next is to find the parameter f, $f'(x, y)$ and $f''(x, y)$

From equation (2.3) $f^r(x, y)$

where $r = 0(1)P - 1$ denotes the derivative of $f(x, y)$ and is given by

$$f^r(x, y(x)) = y^{r+1}(x) = \left[\frac{\partial}{\partial x} + f\frac{\partial}{\partial y}\right]^r \quad (2.8)$$

When $r = 0$ we obtain

$$f\ (x, y) = y^1(x) = f \quad (2.9)$$

When $r = 1$ we obtain

$$f'(x, y) = y^2(x) = \left[\frac{\partial}{\partial x} + f\frac{\partial}{\partial y}\right]^1 = \frac{\partial}{\partial x} + f\frac{\partial}{\partial y}, \quad (2.10)$$

where $\frac{\partial}{\partial x} = f_x$, $\frac{\partial}{\partial y} = f_y$

by substituting in (2.10) we obtain

$$f_x + ff_y = M \quad (2.11)$$

When $r = 2$ we obtain

$$f''(x, y) = y^3(x) = \left[\frac{\partial}{\partial x} + f\frac{\partial}{\partial y}\right]^2$$

By expansion we have

$$\left(\frac{\partial}{\partial x}+f\frac{\partial}{\partial y}\right)^2=\left(\frac{\partial}{\partial x}+f\frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial x}+f\frac{\partial}{\partial y}\right)+\left(\frac{\partial}{\partial x}+f\frac{\partial}{\partial y}\right)$$

$$\left(\frac{\partial}{\partial x}+f\frac{\partial}{\partial y}\right)^2=\frac{\partial^2 f}{\partial x^2}+f\frac{\partial^2 f}{\partial x\partial y}+f\frac{\partial^2 f}{\partial y\partial x}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial f\partial f}{\partial x\partial y}+f\left(\frac{\partial f}{\partial y}\right)$$

$$\left(\frac{\partial}{\partial x}+f\frac{\partial}{\partial y}\right)^2=\frac{\partial^2 f}{\partial x^2}+2f\frac{\partial^2 f}{\partial x\partial y}+f^2\frac{\partial^2 f}{\partial y^2}+\frac{\partial f\partial f}{\partial x\partial y}+f\left(\frac{\partial f}{\partial y}\right)$$

where,

$$\frac{\partial^2 f}{\partial x^2}=f_{xx},\ \frac{\partial^2 f}{\partial x\partial y}=f_{xy},\ \frac{\partial f\partial f}{\partial x\partial y}=f_x f_y \text{ and } \frac{\partial f}{\partial y}=f_y$$

$$f''(x,y)=f_{xx}+2ff_{xy}+f^2 f_{xy}+f_y\left[f_x+ff_y\right]+0(h^3) \tag{2.12}$$

Put $N=f_{xx}+2ff_{xy}+f^2 f_{xy}$ and $M=f_x+ff_y$

Substituting in equation (2.12) we obtain;

$$f''(x,y)=N+Mf_y \tag{2.13}$$

Substituting equation (2.9), (2.11) and (2.13) into equation (2.7)

$$\phi(x_n,y_n,h)=f+\frac{h}{2}(f_x+ff_y)+\frac{h^2}{6}(f_{xx}+2ff_{xy}+f_{yy}^2+f_x f_y+f(f_y)^2)+0(h^3) \tag{2.14}$$

$$\phi(x_n,y_n,h)=f+\frac{h}{2}M+\frac{h^2}{6}(N+Mf_y)+0(h^3)$$

Recall that the increment function is given by

$$\phi(x_n,y_n,h)=\sum_{r=1}^{R}c_r k_r \text{ which is also known as the R-stage} \tag{2.15}$$

For consistency we have $\sum_{r=1}^{R}c_r=1$

For stage two $R=2$

$$\phi(x_n,y_n,h)=\sum_{r=1}^{R}c_r k_r=c_1k_1+c_2k_2 \tag{2.16}$$

The general form of RungeKutta of stage 2 is given by

$$y_{n+1}=y_n+h(c_1+k_1+c_2k_2) \tag{2.17}$$

2.2 Model equation for harvesting of renewable natural resources

If $p(t)$ represent the population at time (t) and $\frac{dp}{dt}$ is the rate of change at which the population grow at a certain time. Then the logistic equation becomes;

$\frac{dp}{dt} = pr - \frac{prt}{k} - h(p)$ this can be written as;

$$\frac{dp}{dt} = p\left(r - \frac{r}{k}t\right) - h(p), \qquad (3.1)$$

where $\frac{dp}{dt}$ is the rate of change in population with time.

p , is the animal population
r , is the growth rate
k , is the carrying capacity which is also known as the saturated level
$h(p)$ is the harvesting term

3 Analysis and Interpretation of Result

In this chapter, we present the numerical solution of dynamical system that model dynamical solution of fish population in a pond over a period of time, using second stage Runge-Kutta Methods.

Definition 1. Renewable natural resources are natural resources that can reproduce and grow while non-renewable resources are resources in which a fixed stock is depleted overtime. Some of the renewable natural resources are fishes in the ocean and sea. We introduce a mathematical model providing some insight into management of renewable resources.

Using the model equation stated in 3.1 above. We will assume that humans will be harvesting from the animal population. The effect of harvesting a renewable natural resources such as fish can be model.

Suppose $r = 0.5, h(p) = 10\%$, given that $k = 100$ then from the model by putting the given values we have;

$$\frac{dp}{dt} = p\left(r - \frac{r}{k}t\right) - h(p)$$

$$\frac{dp}{dt} = p\left(0.5 - \frac{0.5}{100}t\right) - 0.1$$

$$\frac{dp}{dt} = p\left(0.5 - 0.005t\right) - 0.1$$

$$\frac{dp}{dt} = 0.5p - 0.005pt - 0.1,$$

which is the required model

Applying Runge-Kutta Second Stage Method which is given by $p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)$

where;

$k_1 = hf(p_n t_n)$ and $k_2 = hf\left(p_n + \frac{h}{2}, t_n + \frac{1}{2}k_1\right)$ with the initial condition $p(t) = 0 \Rightarrow p_0 = 0, t_0 = 0, h = 0.5$ [mesh size]

Then $k_1 = hf(p_0 t_n)$ when $n = 0$

$$k_1 = hf(p_0 t_0) \text{ when } n = 0$$

$$k_1 = hf(p_0 t_0) = hf(0,0)$$

$$k_1 = 0.5 f(0,0)$$

$$k_1 = 0.5\left[0.5 p_n - 0.005 p_n t_n - 0.1\right] \quad n = 0$$

$$k_1 = 0.5\left[0.5 p_0 - 0.005 p_0 t_0 - 0.1\right]$$

$$k_1 = 0.5\left[0.5(0) - 0.005(0)(0) - 0.1\right]$$

$$k_1 = 0.5\left[0 - 1\right] = 0.05$$

For k_2

$$k_2 = hf\left(p_n + h/2, t_n + \frac{1}{2}k_1\right) \text{ at } n = 0$$

$$k_2 = hf\left(p_0 + 0.5/2, 0 + 0.025\right)$$

$$k_2 = hf\left(0.25, 0, 025\right)$$

$$k_2 = 0.5 f(0.25, 0.025)$$

$$k_2 = 0.5\left[0.5 p_0 - 0.005 p_0 t_0 - 0.1\right]$$

$$k_2 = 0.5\left[0.5(0.25) - 0.005(0.25)\right]$$

$$k_2 = 0.5\left[0.125 - 0.00003125 - 0.1\right]$$

$$k_2 = 0.5\left[0.125 - 0.10003125\right]$$

$$k_2 = 0.5\left[0.02496875\right]$$

$$k_2 = 0.012484375$$

Putting K_1 and K_2 in the equation below,

$$p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right) \text{ at } n = 0$$

$$p_1 = p_0 = 0.5\left[-\frac{0.05}{2} + \frac{0.012484375}{2}\right]$$

$$p_1 = p_0 = 0.5\left[-0.05 + 0.00624218751\right]$$

$$p_1 = 0 + 0.5\left[-0.010757812\right]$$

$$p_1 = 0.5\left[-0.018757812\right]$$

$$p_1 = -0.0094$$

When the harvesting term $H(p) = 20\% = 0.2$ from the model which is given by

$$\frac{dp}{dt} = p\left(r - \frac{r}{k}t\right) - h(p)$$

$$\frac{dp}{dt} = p\left[0.5 - 0.005t\right] - 0.2$$

$$\frac{dp}{dt} = \left[0.5p - 0.0005pt\right] - 0.2$$

Applying the Runge-Kutta formula

$$p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)$$

$$k_1 = hf\left(p_n + t_n\right) \text{ and } k_2 = hf\left(p_n + \frac{h}{2}, t_n + \frac{1}{2}k_1\right)$$

$$k_1 = hf\left(p_n, t_n\right) \text{ when } n = 1$$

$$k_1 = hf(p_1, t_1) \text{ where } p_1 = -0.0094$$

$$t_1 = t_0 + h = 0 + 0.5 = 0.5$$

$$\text{Then } k_1 = 0.5f(-0.0094, 0.5)$$

$$k_1 = 0.5\left[0.5p_n - 0.005p_nt_n - 0.2\right] \quad at\ n = 1$$

$$k_1 = 0.5\left[0.5p_1 - 0.005p_1t_1 - 0.2\right]$$

$$k_1 = 0.5\left[0.5(-0.0094) - 0.005(-0.0094)(0.5) - 0.2\right]$$

$$k_1 = 0.5\left[-0.0047 + 0.0000235 - 0.2\right]$$

$$k_1 = 0.5\left[-0.0047 - 0.1999765\right]$$

$$k_1 = 0.5\left[-0.2046765\right]$$

$$k_1 = 0.10233825 = -0.1023$$

Then for k_2 we have

$$k_2 = hf\left(p_n + \frac{h}{2}, t_n + \frac{1}{2}k_1\right) \quad at\ \ n = 1$$

$$k_2 = 0.5f\left(p_1 + \frac{0.5}{2}, t_1 + \left(\frac{0.1023}{2}\right)\right)$$

$$k_2 = 0.5f\left[-0.0094 + 0.25, 0.5 - 0.05115\right]$$

$$k_2 = 0.5f\left[0.2406, 0.44885\right]$$

$$k_2 = 0.5f\left[0.5p_n - 0.005p_nt_n - 0.2\right]$$

$$k_2 = 0.5f\left[0.5p_1 - 0.005p_1t_1 - 0.2\right]$$

$$k_2 = 0.5f\left[0.5(0.2406) - 0.005(0.2406)(0.4488...)\right]$$

$$k_2 = 0.5\left[0.1203 - 0.005399665 - 0.2\right]$$

$$k_2 = 0.5\left[0.1203 - 0.2005399665\right]$$

$$k_2 = 0.5\left[-0.080239966\right]$$

$$k_2 = -0.40119983$$

Putting k_1 and k_2 in the equation below,

$$p_{n+1} = p_n + h\left(\tfrac{1}{2}k_1 + \tfrac{1}{2}k_2\right) at\ n = 1$$

$$p_2 = p_1 + 0.5\left[\frac{0.1023}{2} + \left(\frac{-0.40119983}{2}\right)\right]$$

$$p_2 = -0.0094 + 0.5[-0.05115 - 0.200599915]$$

$$p_2 = -0.0094 + 0.5[-0.25174915]$$

$$p_2 = -0.0094 - 0.125874575 = -0.1353$$

Then the harvesting term $h(p) = 30\% = 0.3$

From the model

$$\frac{dp}{dt} = p\left(r - \frac{r}{k}t\right) - h(p)$$

$$\frac{dp}{dt} = p[0.5 - 0.005t] - 0.3$$

$$\frac{dp}{dt} = 0.5p - 0.005pt - 0.3$$

Applying the formula which is given by

$$p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)$$

where

$$k_1 = hf(p_n, t_n) \text{ and } k_2 = hf\left(p_n + \frac{h}{2}, t_n + \frac{1}{2}k_1\right) \text{ for } n = 2$$

$$k_1 = hf(p_2, t_2) \text{ where } p_2 = -0.1353$$

$$t_2 = t_0 + 2h = 0 + 2(0.5) = 1$$

Then;

$$k_1 = hf(-0.1353, 1)$$

$$k_1 = 0.5f(-0.1353, 1)$$

$$k_1 = 0.5[0.5p_n - 0.005p_nt_n - 0.3]$$

$$k_1 = 0.5[0.5p_2 - 0.005p_2t_2 - 0.3]$$

$$k_1 = 0.5[0.5(-0.1353) - 0.005(-0.1353)(1) - 0.3]$$

$$k_1 = 0.5[-0.06765 + 0.0006765 - 0.3]$$

$$k_1 = 0.5[-0.06765 - 0.2993235]$$

$$k_1 = 0.5[-03669735]$$

$$k_1 = -0.1835$$

For k_2

$$k_2 = hf\left(p_n \frac{h}{2}, t_n + \frac{1}{2}k_1\right) \text{at } n = 2$$

$$k_2 = hf\left[p_2 + \frac{100}{2}, t_2 + \frac{-0.1835}{2}\right]$$

$$k_2 = 0.5 f\left(-0.1353 + 0.25, 1 - 0.0919\right)$$

$$k_2 = 0.5 f\left(0.1147, 0.9082\right)$$

$$k_2 = 0.5\left[05 p_n - 0.005 p_n, t_n - 0.3\right]$$

$$k_2 = 0.5\left[05 p_2 - 0.005 p_2, t_2 - 0.3\right]$$

$$k_2 = 0.5\left[0.5(0.1147) - 0.005(0.1147), (0.9082) - 0.3\right]$$

$$k_2 = 0.5\left[0.05735 - 0.0005208507 - 0.3\right]$$

$$k_2 = 0.5\left[0.05735 - 0.3005208527\right]$$

$$k_2 = 0.5\left[-0.243170852\right]$$

$$k_2 = -0.1216$$

Putting k_1 and k_2 in the equation below,

$$p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right) \text{ at } n = 2$$

$$p_3 = p_2 + 0.5\left[-\frac{0.1842}{2} + \left(\frac{-0.1216}{2}\right)\right]$$

$$p_3 = -0.1353 + 0.5\left[-0.0921 - 0.0608\right]$$

$$p_3 = 0.1353 + 0.5\left[-0.1529\right]$$

$$p_3 = 0.1353 - 0.07645$$

$$p_3 = -0.2112$$

When the harvesting term $h(p) = 40\% = 0.4$ from the model

$$\frac{dp}{dt} = p\left(r - \frac{r}{k}t\right) - h(p)$$

$$\frac{dp}{dt} = p\left[0.5 - 0.005t\right] - 0.4$$

$$\frac{dp}{dt} = 0.5 - 0.0005\,pt - 0.4$$

Applying the classical Runge-Kutta Method

$$p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)$$

where,

$$k_1 = hf\left(p_n, t_n\right) \text{ and } k_2 = hf\left(p_n + \frac{h}{2}, t_n + \frac{1}{2}k_1\right) \text{ for } n = 3$$

$$k_1 = hf\left(p_3, t_3\right) \text{ where } p_3 = -0.2112$$

$$t_3 = t_0 + 3h = 0 + 3(0.5) = 1.5$$

$$\text{Then } k_1 = hf\left(0.2112, 1.5\right)$$

$$k_1 = 0.5 f\left(0.2112, 1.5\right)$$

$$k_1 = 0.5\left[05 p_n - 0.005 p_n, t_n - 0.4\right] n = 3$$

$$k_1 = 0.5\left[0.5 p_3 - 0.005 p_3 t_3 - 0.4\right]$$

$$k_1 = 0.5\left[0.5(0.2112) - 0.005(0.2112)(1.5) - 0.4\right]$$

$$k_1 = 0.5\left[-0.1056 + 0.001584 - 0.4\right]$$

$$k_1 = 0.5\left[-0.1056 - 0.398416\right]$$

$$k_1 = 0.5\left[-0.504016\right]$$

$$k_1 = -0.2520$$

To find k_2

$$k_2 = hf\left(p_n + h/2, t_n + 1/2\, k_1\right) \text{at } n = 3$$

$$k_2 = hf\left[p_3 + 0.5/2, t_3 + \left(\frac{-0.2536}{2}\right)\right]$$

$$k_2 = hf\left[-0.2112 + 0.25, 1.5 - 0.126\right]$$

$$k_2 = 0.5 f\left[0.0388, 1.374\right]$$

$$k_2 = 0.5\left[05 p_n - 0.005 p_n, t_n - 0.4\right] n = 3$$

$$k_2 = 0.5\left[0.5 p_3 - 0.005 p_3 t_3 - 0.4\right]$$

$$k_2 = 0.5\left[0.5(0.0388) - 0.005(0.0388)(1.374) - 0.4\right]$$

$$k_2 = 0.5\left[0.0194 - 0.000266556 - 0.4\right]$$

$$k_2 = 0.5\left[0.0194 - 0.400266556\right]$$

$$k_2 = 0.5\left[-0.380866556\right] = -0.1904$$

Putting k_1 and k_2 in the equation below,

$$p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right) \text{ at } n = 3$$

$$p_4 = p_3 + 0.5\left[-\frac{0.2580}{2} + \left(\frac{-0.1904}{2}\right)\right]$$

$$p_4 = p_3 + 0.5\left[-0.126 - 0.0952\right]$$

$$p_4 = -0.2112 + 0.5\left[-0.2212\right]$$

$$p_4 = -0.2122 - 0.1106$$

$$p_4 = -0.3218$$

When the harvesting term $h(p) = 50\% = 0.5$ from the model

$$\frac{dp}{dt} = p\left(r - \frac{r}{k}t\right) - h(p)$$

$$\frac{dp}{dt} = p\left[0.5 - 0.005t\right] - 0.5$$

$$\frac{dp}{dt} = 0.5 - 0.0005\,pt - 0.5$$

Applying the classical Runge-Kutta Method

$$p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)$$

where,

$$k_1 = hf\left(p_n, t_n\right) \text{ and } k_2 = hf\left(p_n + \frac{h}{2}, t_n + \frac{1}{2}k_1\right) \text{ for } n = 4$$

$$k_1 = hf\left(p_2, t_2\right) \text{ where } p_4 = -0.3218$$

$$t_4 = t_0 + 4h = 0 + 4\left(0.5\right) = 2$$

Then $k_1 = 0.5\,f\left(-0.3218, 2\right)$

$$k_1 = 0.5\left[05\,p_n - 0.005\,p_n, t_n - 0.5\right]$$

$$k_1 = 0.5\left[0.5\,p_4 - 0.005\,p_4 t_4 - 0.5\right]$$

$$k_1 = 0.5\left[0.5\left(0.3218\right) - 0.005\left(-0.3218\right)\left(2\right) - 0.5\right]$$

$$k_1 = 0.5\left[-0.1609 + 0.003218 - 0.5\right]$$

$$k_1 = 0.5\left[-0.1609 - 0.496782\right]$$

$$k_1 = 0.5\left[-0.657682\right]$$

$$k_1 = -0.3285$$

for k_2

$$k_2 = hf\left(p_n + h/2, t_n + \frac{1}{2}k_1\right) \text{ at } n = 4$$

$$k_2 = hf\left[p_4 + 0.5/2, t_4 + \left(\frac{-0.3288}{2}\right)\right]$$

$$k_2 = 0.5 f\left[-0.3218 + 0.25, 2 - 0.1644\right]$$

$$k_2 = 0.5 f\left[-0.0718, 18356\right]$$

$$k_2 = 0.5\left[05 p_n - 0.005 p_n, t_n - 0.5\right]$$

$$k_2 = 0.5\left[05 p_4 - 0.005 p_4, t_4 - 0.5\right]$$

$$k_2 = 0.5\left[(-0.0718) - 0.005(0.0718)(1.8356) - 0.5\right]$$

$$k_2 = 0.5\left[-0.0359 + 0.000658984 - 0.5\right]$$

$$k_2 = 0.5\left[-0.0559 - 0.499341019\right]$$

$$k_2 = 0.5\left[-0.535641019\right]$$

$$k_2 = -0.2676$$

putting k_1 and k_2 in the equation below,

$$p_{n+1} = p_n + h\left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right) \text{ at } n = 4$$

$$p_5 = p_4 + 0.5\left[-\frac{0.3288}{2} + \left(\frac{-0.2676}{2}\right)\right]$$

$$p_5 = -0.3218 + 0.5\left[-0.1622 - 1.338\right]$$

$$p_5 = -0.3218 + 0.5\left[-1.5002\right]$$

$$p_5 = -0.3218 - 0.7501$$

$$p_5 = -1.0719$$

Table 1. Summary of results

Number (N)	**Harvesting Term** $h(p)$	p_{n+1}
0	$10\% = 0.1$	$p_1 = -0.0094$
1	$20\% = 0.2$	$p_2 = -0.1353$
2	$30\% = 0.3$	$p_3 = -0.2112$
3	$40\% = 0.4$	$p_4 = -0.3218$
4	$50\% = 0.5$	$p_5 = -1.0519$

Where $n = 0, 1, 2, 3, 4$ and 5.

4 Summary

In this research work, the solution of a dynamic system that can model natural resources using the second stage Runge-Kutta method was reported. To actualize this result, a model, a first order ordinary differential equation and the famous Runge-Kutta second stage method is used. Harvesting terms $H(p)$ from 10% to 50% is used for the iteration to demonstrate the validity of the result. The result of this study was applied to the population of fish in a pond, which is a renewable natural resource, with great success.

5 Conclusion

In line with the objectives stated at the beginning of the work, the results of this research has shown that by applying the second order Runge-Kutta method, the solution of a dynamical system can be obtained and applied to model natural resources with great success. Thus, the full objectives of the study have been achieved. The population in the pond at each harvesting determined the remain life in the pond. It is observed that after the first harvesting, one fish is taken out of the pond which is 0.94. The second harvesting, 13.5 is taken out of the pond. The third harvesting, 21.12. after the fourth harvesting, 32.18 is taken out of the pond. While the last harvesting (fifth) 105.19 was taken out of the pond which is above 100 meaning that all fishes in the ponds are exhausted. For continue existence of live in the pond, we may not exceed harvesting limit of 40%

6 Recommendations

From the experiences gathered during the course of this study, it is recommended that further studies on dynamical systems should be encouraged using the Runge-Kutta method, specifically, extending this study to the third stage of the method. Furthermore, other methods should be explored for the developing models for the natural resources that are abundant within the country as this will give a better picture on the economic prospects of these resources and serve as a benchmark for further research.

Competing Interests

Authors have declared that no competing interests exist.

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