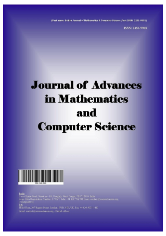

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Conservation Laws and Travelling Wave Solutions for System of Ion Sound and Langmuir Waves

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this paper, system of equations for ion sound and Langmuir waves (ISLWs) is studied to construct novel exact solutions of the coupled nonlinear system. The extended F-expansion method is applied to get exact solutions of ISLWs model. These solutions include many different expressions in hyperbolic, trigonometric, rational and Jacobi elliptic function solutions, dark and bright solitary wave solutions. Geometrical shape for some of the obtained results are plotted under the selection of proper parameters. Furthermore, we employed the Lie point symmetry to investigate the conservation laws for the system.

Keywords: Ion sound and Langmuir waves; Jacobi elliptic function; exact solutions.

PACS numbers: 04.20.Jb, 02.30.Jr, 05.45.Yv.

1 Introduction

Many physical phenomena that arising in various fields of science can be described by nonlinear evolution equations (NLEEs) for instance optics, plasma wave, solid state physics, chemical physics, and mathematical physics. To understand these nonlinear phenomena, many physicist and mathemati-cians have made efforts to

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get various exact solutions of them. The investigation of the exact solutions of NLEEs are important to provide better information, know the mechanism and their applications. With the aid of symbolic computation software such as Mathematica or Maple abundant methods are extensively studied to obtain exact solutions, for example the Bäcklund transform [1, 2], the extended tanh-function method [3], the F-expansion method [4], sine-cosine method [5], the extended F-expansion method [6], Jacobi elliptic function method [7], the Kudryashov method [8], the extended Kudryashov method [9, 10], the lie point symmetry method [11]- [13] and other methods [14]-[17]. The dynamical behavior for a famous class of NLEEs is presented in [18]. Two-step modified natural decomposition method is proposed to determine the approximate closed form solutions or rather exact solutions for the nonlinear Klein- Gordon equation [19].

In this study, we construct several kinds of exact solutions of the ISLWs model by applying the extended F-expansion method. The ISLWs model [20]-[26] has the form as:

$$\begin{aligned} &i\,E_t + \tfrac{1}{2}\,E_{xx} - n\,E = 0, \\ &n_{tt} - n_{xx} - 2\,(|E|^2)_{xx} = 0, \end{aligned} \tag{1.1}$$

where the normalized electric field of the Langmuir oscillation is $E\,e^{-iw_p t}$ and the normalized density perturbation is n, x denotes the spatial and t denotes the time variables. The physical natures of ISLWs model is useful to seek plasma physics and the effect on incoherent structures. The above system of Eq. (1.1) was formulated by Zakharov [27] in 1972. Recently, many researchers used different techniques to find exact solutions of ISLWs model. Demiray and Bulut [21] applied generalized Kudryashov method to get travelling wave solutions of the ISLWs model. Also, soliton solutions of this system was considered by Seadawy et. al.[22], Alam and Osman [23] and Mohammed et. al. [24]. The graphical of some specific solutions are useful to understand the physical phenomena of Eq. (1.1). Moreover, conservation laws are great important in physics and mathematics. Mathematical expressions of physical laws are the coservation laws, such as coservation of mass, energy and momentum. The coservation laws can be used to study the properties of the existance, uniqueness and stability of solutions.

The outline of this paper is as the following : Firstly, we summarized the analytical method that we will use to construct novel exact solutions of the ISLWs model in section 2. In section 3, we get the solutions of the studied model with Jacobi elliptic functions (JEFs) via the extended F-expansion method. The geometrical shape of some solutions in the form of two-dimentional and three-dimentional have been plotted. Furthermore, the Lie point symmetry and the conservation laws for (1.1) are obtained in section 4. Section 5 is results and discussion. Finally, conclusions of the paper are presented in the latest section.

2 Summary of the Extended F-expansion Method

In this section, the extended F-expansion method [6], will summarize as follows:
Consider NLEEs

$$P(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0. \tag{2.1}$$

1- Suppose that $u(x,t) = u(\xi)$ and $\xi = c\,t + k\,x + \xi_0$, such that c and k are constants to be evaluated later and ξ_0 is an arbitrary constant. Then (2.1) is transformed to the following equation:

$$G(u,\, u',\, u'', ...) = 0, \tag{2.2}$$

2- Suppose that the solutions of (2.2) in the form

$$u(x,t) = u(\xi) = A_0 + \sum_{i=1}^{N} [A_i F^i(\xi) + B_i\, F^{-i}(\xi)], \tag{2.3}$$

where N is a positive integer and $A_0, A_i,\, B_i\, (i = 1,\, 2, ...,\, N)$ are constants to be determined. The function $F(\xi)$ satisfies the following ordinary differential equation (ODE):

$$(F'(\xi))^2 = q_0 + q_2 F^2(\xi) + q_4 F^4(\xi), \tag{2.4}$$

and the values q_0, q_2 and q_4 are constants.

3- Putting (2.3) with (2.4) in (2.2), we obtain a polynomial in $F(\xi)$. Setting all coefficients of it to zero, we get an algebraic equations for A_0, k, A_i, $B_i (i = 1, 2, ..., N)$ and c.
4- Setting the values of q_0, q_2, q_4 and the corresponding JEFs $F(\xi)$, we get many exact JEF solutions of Eq. (2.1).

The JEFs can be written as $\text{sn}\xi = \text{sn}(\xi, m)$, $\text{cn}\xi = \text{cn}(\xi, m)$ and $\text{dn}\xi = \text{dn}(\xi, m)$, where $m\,(0 < m < 1)$ is the modulus of the elliptic function. The functions $\text{sn}\xi$, $\text{cn}\xi$ and $\text{dn}\xi$ become $\tanh\xi$, $\text{sech}\xi$ and $\text{sech}\xi$, respectively when $m \longrightarrow 1$. Also, $\text{sn}\xi$, $\text{cn}\xi$ and $\text{dn}\xi$ become $\sin\xi$, $\cos\xi$, 1, respectively when $m \longrightarrow 0$.

3 The Exact Solution of the ISLWs Model by the Extended F-expansion Method

Suppose that the solution of (1.1) as the following:

$$E(x,t) = U(\xi)\, e^{i\,\theta}, \quad n(x,t) = V(\xi), \quad \xi = c\,t + k\,x + \xi_0, \quad \theta = \Omega\, x + \mu\, t, \tag{3.1}$$

where c, Ω, k and μ are constants. Putting (3.1) in (1.1) and splitting the imaginary and the real parts, we get

$$(c + \Omega\, k)\, U' = 0 \quad \Rightarrow c = -\Omega\, k, \tag{3.2}$$

$$k^2\, U'' - (2\,\mu + \Omega^2)\, U - 2\, U\, V = 0, \tag{3.3}$$

$$(c^2 - k^2)\, V'' - 2\, k^2\, (U^2)'' = 0. \tag{3.4}$$

By integrating (3.4) twise and putting the integration constant to zero, we obtain

$$V(\xi) = \frac{2\,k^2}{c^2 - k^2}\, U^2(\xi). \tag{3.5}$$

Substituting (3.5) into (3.3), we obtain

$$k^2(\Omega^2 - 1)\, U'' - (\Omega^2 - 1)(\Omega^2 + 2\,\mu)\, U - 4\, U^3 = 0. \tag{3.6}$$

By using balancing procedure, we have $N = 1$. Then (3.6) has solution as

$$U(\xi) = A_0 + A_1\, F(\xi) + \frac{B_1}{F(\xi)}, \tag{3.7}$$

where A_0, A_1, B_1 are undetermined constants. Putting (3.7) into (3.6) and using (2.4), we get a polynomial in the function $F(\xi)$. Setting all coefficients of $F(\xi)$ to zero, we have cases as follows:

Case 1

$$k = \pm\sqrt{\frac{\Omega^2 + 2\,\mu}{q_2}},\, \Omega = \Omega,\, \mu = \mu,\, A_0 = 0,\, A_1 = \pm\sqrt{\frac{q_4\,(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1)}{2\,q_2}},\, B_1 = 0. \tag{3.8}$$

Substituting (3.8) into (3.7), we have

$$U(\xi) = \pm\sqrt{\frac{q_4\,(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1)}{2\,q_2}}\, F(\xi), \quad \xi = \pm\sqrt{\frac{\Omega^2 + 2\,\mu}{q_2}}\, x + c\,t + \xi_0. \tag{3.9}$$

Putting q_0, q_2, q_4 and the JEFs solution for $F(\xi)$ in (3.9) with (3.5) and (3.1). Therefore, we construct the exact solutions of (1.1) as follows:

Case 1.1: When $q_0 = 1$, $q_2 = -1 - m^2$, $q_4 = m^2$ and $F(\xi) = \text{sn}\xi$. So, periodic wave solutions of (1.1) given as,

$$E(x,t) = \pm m\sqrt{\frac{(\Omega^2 + 2\mu)(1-\Omega^2)}{2(1+m^2)}}\, e^{i\Theta}\,\text{sn}\xi,\;\; n(x,t) = \frac{-m^2(\Omega^2+2\mu)}{(1+m^2)}\,\text{sn}^2\xi, \tag{3.10}$$

Case 1.2: Setting $q_0 = 1 - m^2$, $q_2 = 2m^2 - 1$, $q_4 = -m^2$ and $F(\xi) = \text{cn}\xi$, we have the JEFs solutions of (1.1)

$$E(x,t) = \pm m\sqrt{\frac{(\Omega^2 + 2\mu)(\Omega^2-1)}{2(1-2m^2)}}\, e^{i\Theta}\,\text{cn}\xi, \quad n(x,t) = \frac{m^2(\Omega^2+2\mu)}{(1-2m^2)}\,\text{cn}^2\xi, \tag{3.11}$$

Case 1.3: Putting $q_0 = m^2 - 1$, $q_2 = 2 - m^2$, $q_4 = -1$ and $F(\xi) = \text{dn}\xi$. Thus, the periodic wave solutions of (1.1) obtain as follows:

$$E(x,t) = \pm\sqrt{\frac{(\Omega^2 + 2\mu)(\Omega^2-1)}{2(m^2-2)}}\, e^{i\Theta}\,\text{dn}\xi, \qquad n(x,t) = \frac{(\Omega^2+2\mu)}{(m^2-2)}\,\text{dn}^2\xi, \tag{3.12}$$

Case 1.4: Putting $q_0 = \frac{1}{4}$, $q_2 = \frac{m^2-2}{2}$, $q_4 = \frac{m^4}{4}$, we have $F(\xi) = \frac{\text{sn}\xi}{1\pm\text{dn}\xi}$. Thus, we construct solutions of (1.1) expressed by rational expressions of JEFs:

$$E(x,t) = \pm\frac{m^2}{2}\sqrt{\frac{(\Omega^2 + 2\mu)(\Omega^2-1)}{(m^2-2)}}\,\frac{\text{sn}\xi}{1\pm\text{dn}\xi}\, e^{i\Theta}, \quad n(x,t) = \frac{m^4(\Omega^2+2\mu)}{2(m^2-2)}\left(\frac{\text{sn}\xi}{1\pm\text{dn}\xi}\right)^2, \tag{3.13}$$

Case 1.5: If $q_0 = \frac{m^2-1}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{m^2-1}{4}$ we get $F(\xi) = \frac{\text{dn}\xi}{1\pm m\,\text{sn}\xi}$. Then, we have the solutions of (1.1) as

$$\begin{aligned} E(x,t) &= \pm\tfrac{1}{2}\sqrt{\tfrac{(m^2-1)(\Omega^2+2\mu)(\Omega^2-1)}{(m^2+1)}}\,\tfrac{\text{dn}\xi}{1\pm m\,\text{sn}\xi}\, e^{i\Theta}, \\ n(x,t) &= \tfrac{(m^2-1)(\Omega^2+2\mu)}{2(m^2+1)}\left(\tfrac{\text{dn}\xi}{1\pm m\,\text{sn}\xi}\right)^2, \end{aligned} \tag{3.14}$$

Case 1.6: When $q_0 = \frac{1-m^2}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{1-m^2}{4}$ and $F(\xi) = \frac{\text{cn}\xi}{1\pm\text{sn}\xi}$, we obtain

$$\begin{aligned} E(x,t) &= \pm\tfrac{1}{2}\sqrt{\tfrac{(1-m^2)(\Omega^2+2\mu)(\Omega^2-1)}{(m^2+1)}}\,\tfrac{\text{cn}\xi}{1\pm\text{sn}\xi}\, e^{i\Theta}, \\ n(x,t) &= \tfrac{(1-m^2)(\Omega^2+2\mu)}{2(m^2+1)}\left(\tfrac{\text{cn}\xi}{1\pm\text{sn}\xi}\right)^2, \end{aligned} \tag{3.15}$$

Case 1.7: When $q_0 = \frac{-(1-m^2)^2}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{-1}{4}$ and $F(\xi) = (m\,\text{cn}\xi \pm \text{dn}\xi)$, we have

$$\begin{aligned} E(x,t) &= \pm\tfrac{1}{2}\sqrt{\tfrac{(\Omega^2+2\mu)(1-\Omega^2)}{(m^2+1)}}\,(m\,\text{cn}\xi\pm\text{dn}\xi)\, e^{i\Theta}, \\ n(x,t) &= \tfrac{-(\Omega^2+2\mu)}{2(m^2+1)}\left(m\,\text{cn}\xi\pm\text{dn}\xi\right)^2, \end{aligned} \tag{3.16}$$

Case 1.8: When $q_0 = \frac{1}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{(1-m^2)^2}{4}$ and $F(\xi) = \frac{\text{sn}\xi}{\text{cn}\xi\pm\text{dn}\xi}$, thus the solutions of (1.1) are

$$\begin{aligned} E(x,t) &= \pm\tfrac{1-m^2}{2}\sqrt{\tfrac{(\Omega^2+2\mu)(\Omega^2-1)}{(m^2+1)}}\,\tfrac{\text{sn}\xi}{\text{cn}\xi\pm\text{dn}\xi}\, e^{i\Theta}, \\ n(x,t) &= \tfrac{(1-m^2)^2(\Omega^2+2\mu)}{2(m^2+1)}\left(\tfrac{\text{sn}\xi}{\text{cn}\xi\pm\text{dn}\xi}\right)^2, \end{aligned} \tag{3.17}$$

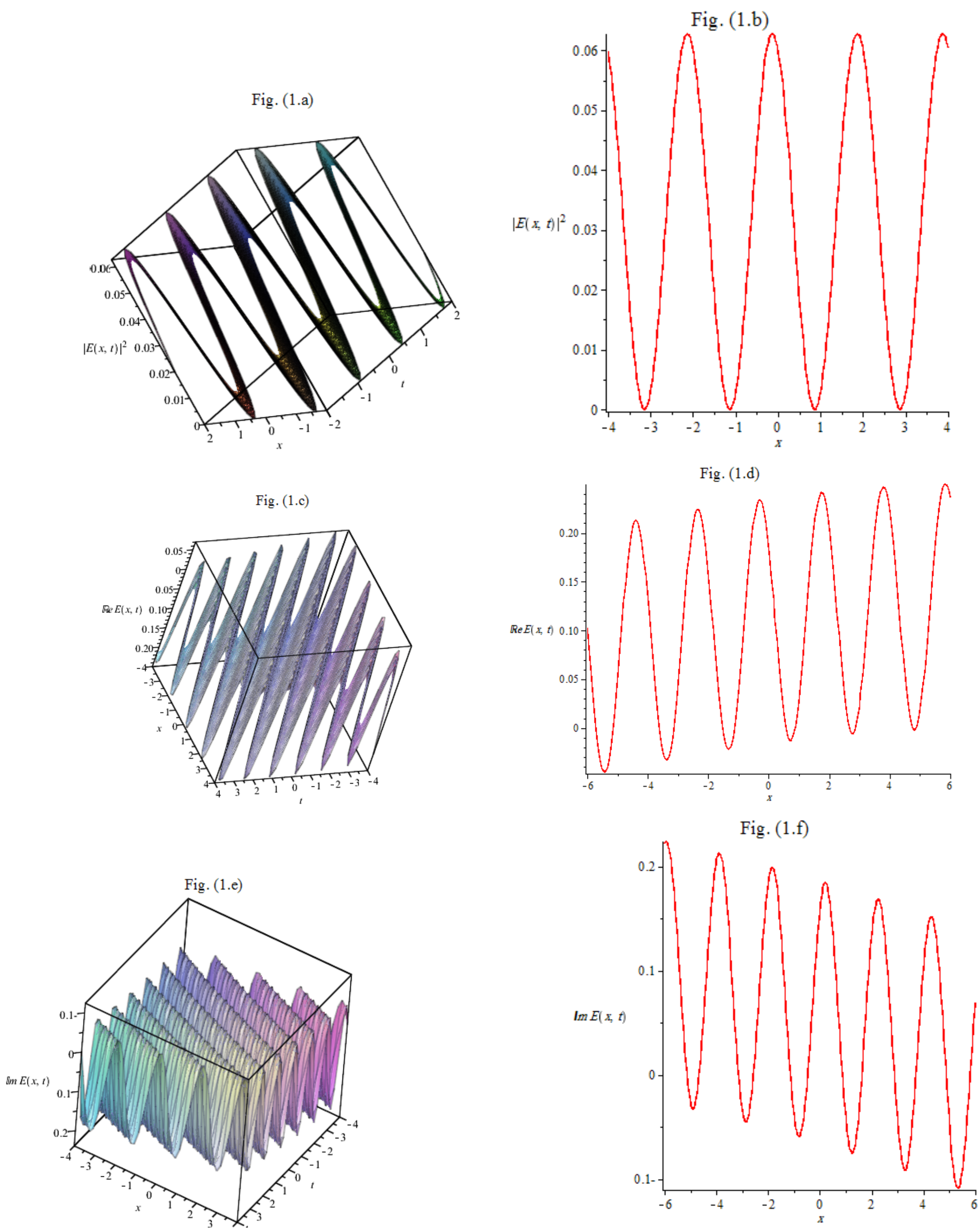

Fig. 1. (a-f) 3D and 2D the periodic wave solution (3.10) are plotted when (+) sign is taken with the parameters $\Omega = 1.5, m = 0.2, \mu = -2.43$ and $\xi_0 = 1$ for 3D figure and $t = 1$ for 2D figure

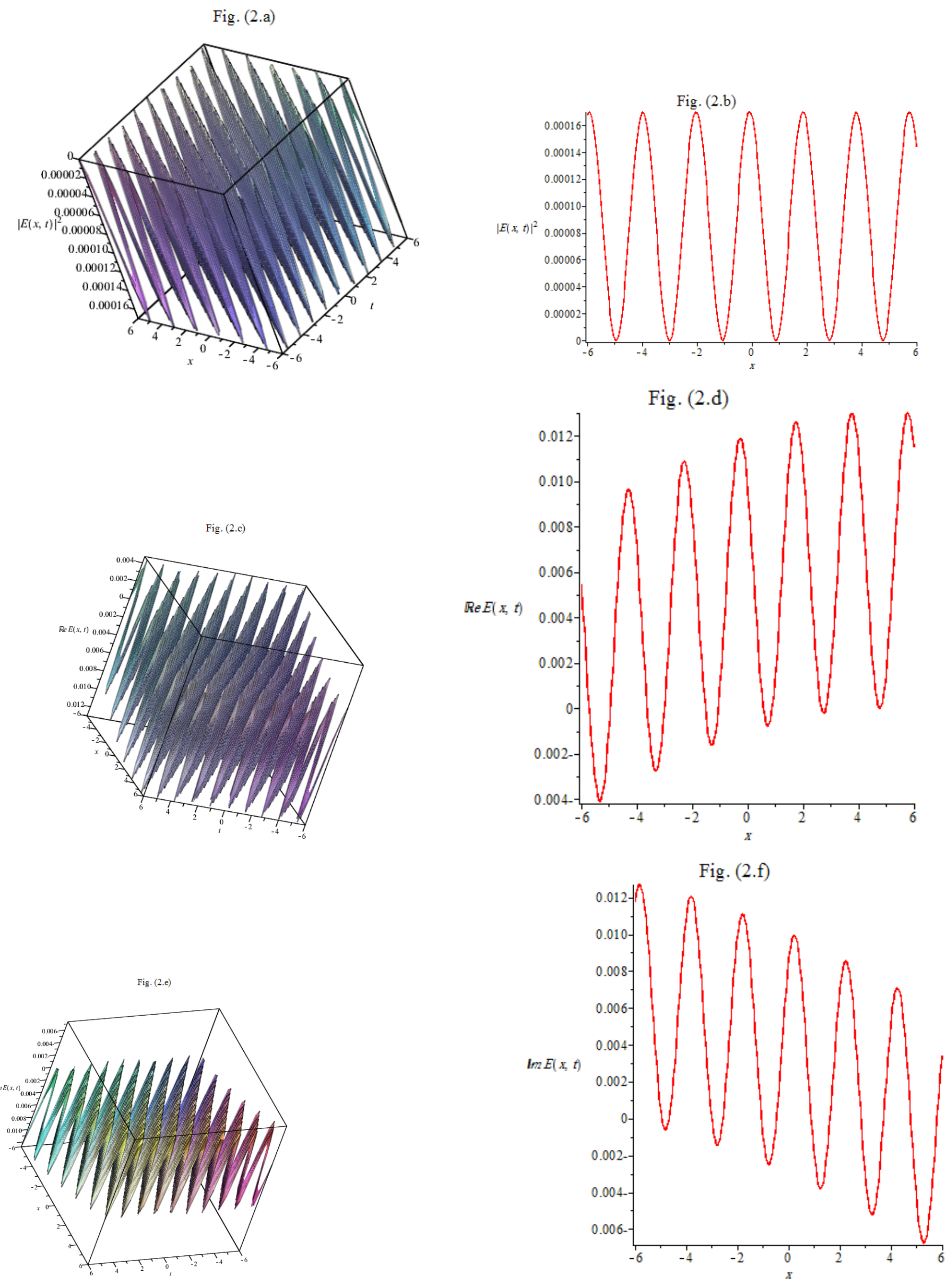

Fig. 2. The periodic wave solution (3.13) are plotted when (+) sign is taken for (a-f) 3D and 2D with the choice of parameters $\Omega = 1.5, m = 0.2, \mu = -2.43$ and $\xi_0 = 1$ for 3D figure and $t = 1$ for 2D figure

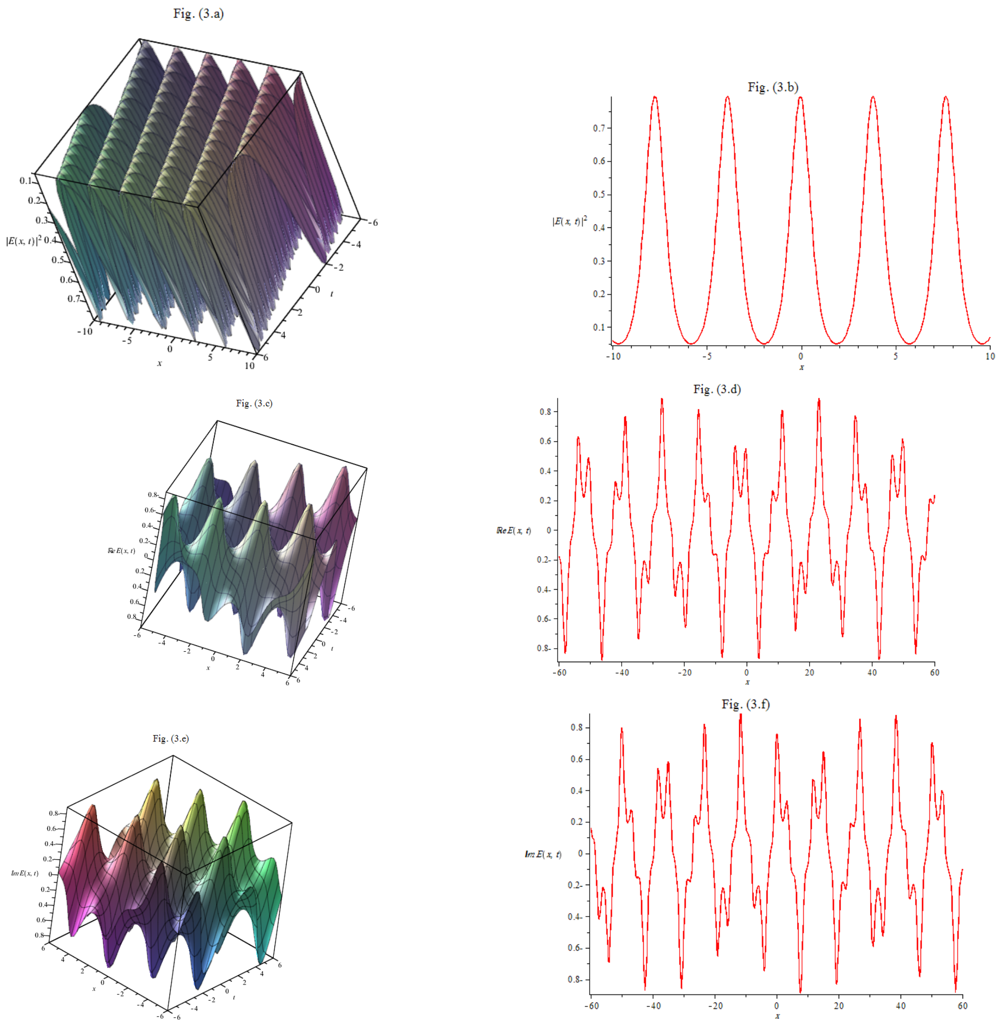

Fig. 3. The periodic wave solution (3.16) are plotted when (+) sign is taken for (a-f) 3D and 2D with the choice of parameters $\Omega = 0.5, m = 0.6, \mu = 1$ and $\xi_0 = 1$ for 3D figure and $t = 1$ for 2D figure

Case 2

$$k = \pm\sqrt{\frac{\Omega^2 + 2\,\mu}{q_2}},\ \Omega = \Omega,\ \mu = \mu,\ A_0 = 0,\ A_1 = 0,\ B_1 = \sqrt{\frac{q_0\,(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1)}{2\,q_2}}. \tag{3.18}$$

Substituting (3.18) into (3.7), we obtain the general solutions as,

$$U(\xi) = \pm\sqrt{\frac{q_0\,(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1)}{2\,q_2}}\,\frac{1}{F(\xi)},\quad \xi = \pm\sqrt{\frac{\Omega^2 + 2\,\mu}{q_2}}\,x + c\,t + \xi_0. \tag{3.19}$$

Putting q_0, q_2, q_4 and the JEFs solution for $F(\xi)$ in (3.19) with (3.5) and (3.1). Therefore, we have the exact solutions of (1.1) as

Case 2.1: When $q_0 = 1$, $q_2 = -1 - m^2$, $q_4 = m^2$, and $F(\xi) = \text{sn}\xi$. So, the periodic wave solutions of (1.1) given as

$$E(x,t) = \pm \frac{\sqrt{(\Omega^2 + 2\,\mu)\,(1 - \Omega^2)}}{\sqrt{2\,(1 + m^2)}}\, e^{i\Theta}\ \text{ns}\xi,\ n(x,t) = \frac{-(\Omega^2 + 2\,\mu)}{(1 + m^2)}\, \text{ns}^2\xi, \tag{3.20}$$

Case 2.2: When $q_0 = 1 - m^2$, $q_2 = 2m^2 - 1$, $q_4 = -m^2$ and $F(\xi) = \text{cn}\xi$, then, the exact JEFs solutions of Eq. (1.1) are

$$E(x,t) = \pm \frac{\sqrt{(1 - m^2)(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1)}}{\sqrt{2\,(2\,m^2 - 1)}}\, e^{i\Theta}\, \text{nc}\xi,\ n(x,t) = \frac{(1 - m^2)(\Omega^2 + 2\,\mu)}{(2\,m^2 - 1)}\, \text{nc}^2\xi, \tag{3.21}$$

Case 2.3: If $q_0 = m^2 - 1$, $q_2 = 2 - m^2$, $q_4 = -1$ and $F(\xi) = \text{dn}\xi$, thus yeilds the periodic wave solutions of Eq. (1.1) as follows:

$$E(x,t) = \pm \frac{\sqrt{(m^2 - 1)(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1)}}{\sqrt{2\,(2 - m^2)}}\, e^{i\Theta}\ \text{nd}\xi,\ n(x,t) = \frac{(m^2 - 1)(\Omega^2 + 2\,\mu)}{(2 - m^2)}\ \text{nd}^2\xi, \tag{3.22}$$

Case 2.4: Putting $q_0 = \frac{1}{4}$, $q_2 = \frac{m^2-2}{2}$, $q_4 = \frac{m^4}{4}$ and $F(\xi) = \frac{\text{sn}\xi}{1\pm\text{dn}\xi}$. We construct solutions of (1.1) expressed by rational expressions of JEFs

$$E(x,t) = \pm \sqrt{\frac{(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1)}{m^2 - 2}}\, \frac{1 \pm \text{dn}\xi}{2\,\text{sn}\xi}\, e^{i\Theta}, \quad n(x,t) = \frac{(\Omega^2 + 2\,\mu)}{2\,(m^2 - 2)} \left(\frac{1 \pm \text{dn}\xi}{\text{sn}\xi}\right)^2, \tag{3.23}$$

Case 2.5: If $q_0 = \frac{m^2-1}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{m^2-1}{4}$, then $F(\xi) = \frac{\text{dn}\xi}{1\pm m\,\text{sn}\xi}$. Thus, we have the solutions of (1.1) as

$$\begin{aligned} E(x,t) &= \pm \sqrt{\tfrac{(m^2-1)(\Omega^2+2\,\mu)\,(\Omega^2-1)}{m^2+1}}\, \tfrac{(1\pm m\,\text{sn}\xi)}{2\,\text{dn}\xi}\, e^{i\Theta}, \\ n(x,t) &= \tfrac{(m^2-1)(\Omega^2+2\,\mu)}{2\,(m^2+1)} \left(\tfrac{1\pm m\,\text{sn}\xi}{\text{dn}\xi}\right)^2, \end{aligned} \tag{3.24}$$

Case 2.6: When $q_0 = \frac{1-m^2}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{1-m^2}{4}$ and $F(\xi) = \frac{\text{cn}\xi}{1\pm\text{sn}\xi}$, we obtain

$$\begin{aligned} E(x,t) &= \pm \sqrt{\tfrac{(1-m^2)(\Omega^2+2\,\mu)\,(\Omega^2-1)}{m^2+1}}\, \tfrac{(1\pm\text{sn}\xi)}{2\,\text{cn}\xi}\, e^{i\Theta}, \\ n(x,t) &= \tfrac{(1-m^2)(\Omega^2+2\,\mu)}{2\,(m^2+1)} \left(\tfrac{(1\pm\text{sn}\xi)}{\text{cn}\xi}\right)^2, \end{aligned} \tag{3.25}$$

Case 2.7: If $q_0 = \frac{-(1-m^2)^2}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{-1}{4}$, and $F(\xi) = (m\,\text{cn}\xi \pm \text{dn}\xi)$. Then, the exact solutions of Eq. (1.1) given as,

$$\begin{aligned} E(x,t) &= \pm \tfrac{(1-m^2)\sqrt{(\Omega^2+2\,\mu)\,(1-\Omega^2)}}{2\,\sqrt{(m^2+1)}\,(m\,\text{cn}\xi\pm\text{dn}\xi)}\, e^{i\Theta}, \\ n(x,t) &= \tfrac{-(1-m^2)^2\,(\Omega^2+2\,\mu)}{2\,(m^2+1)\,(m\,\text{cn}\xi\pm\text{dn}\xi)^2}, \end{aligned} \tag{3.26}$$

Case 3

$$k = \pm\sqrt{\frac{\Omega^2 + 2\,\mu}{q_2 \pm 6\,\sqrt{q_0\,q_4}}},\ \Omega = \Omega,\ \mu = \mu,\ A_0 = 0,\ A_1 = \pm\sqrt{\frac{q_4\,(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1) - *}{2\,(q_2 \pm 6\,\sqrt{q_0\,q_4})}}$$

,

$$B_1 = \mp\sqrt{\frac{q_0\,(\Omega^2 + 2\,\mu)\,(\Omega^2 - 1)}{2\,(q_2 \pm 6\,\sqrt{q_0\,q_4})}}. \tag{3.27}$$

Substituting (3.27) into (3.7), we obtain the general solutions in the form,

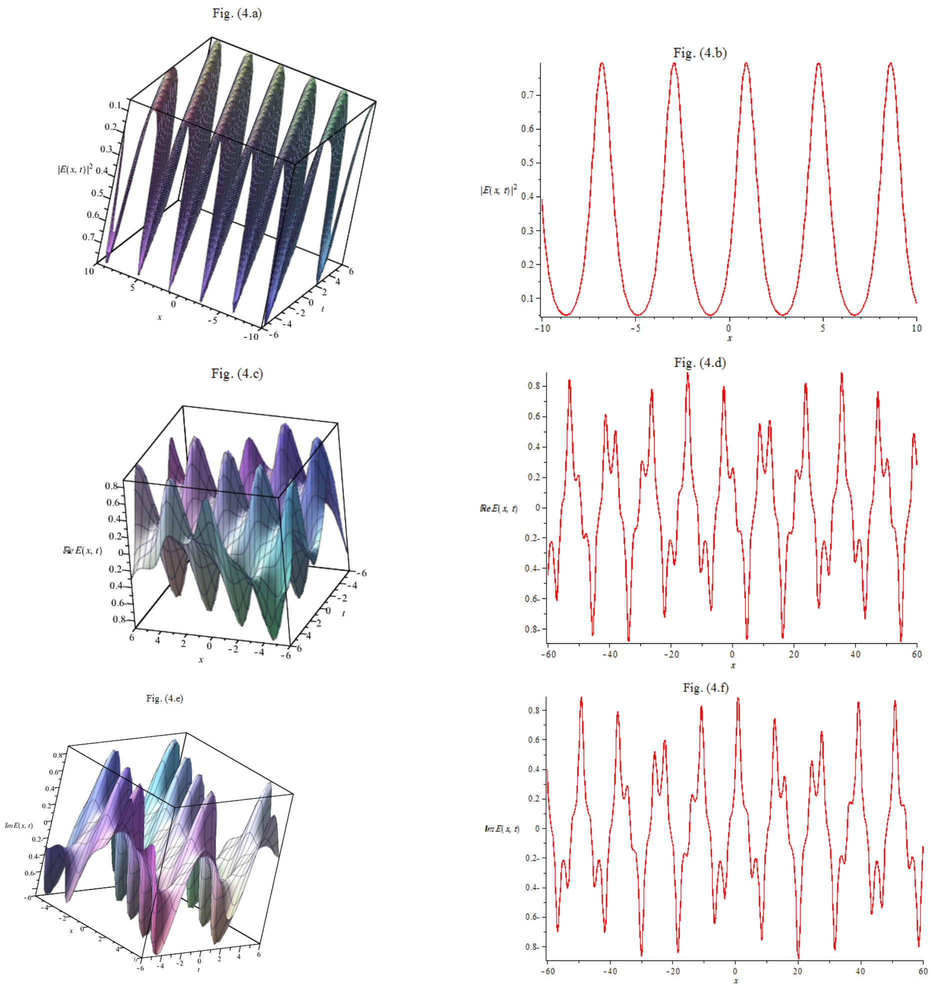

Fig. 4. (a-f) 3D and 2D are plotted for the periodic wave solution (3.24) when (+) sign is taken with the choice of parameters $\Omega = 0.5$, $m = 0.6$, $\mu = 1$ and $\xi_0 = 1$ for 3D plots and $t = 1$ for 2D plots

$$U(\xi) = \pm\sqrt{\frac{q_4\,(\Omega^2+2\,\mu)\,(\Omega^2-1)}{2\,(q_2 \pm 6\,\sqrt{q_0\,q_4})}}\,F(\xi) \mp \sqrt{\frac{q_0\,(\Omega^2+2\,\mu)\,(\Omega^2-1)}{2\,(q_2 \pm 6\,\sqrt{q_0\,q_4})}}\,\frac{1}{F(\xi)},$$

$$\xi = \pm\sqrt{\frac{\Omega^2+2\,\mu}{q_2 \pm 6\,\sqrt{q_0\,q_4}}}\,x + c\,t + \xi_0. \tag{3.28}$$

We can find many JEF solutions of Eq. (3.28) by setting q_0, q_2, q_4 and the JEFs solution for $F(\xi)$. Hence, the solutions of Eq. (1.1) written as the following:

Case 3.1: When $q_0 = 1$, $q_2 = -1 - m^2$, $q_4 = m^2$ and $F(\xi) = \text{sn}\xi$. So, we obtain the following combined JEFs solutions of (1.1):

$$\begin{aligned} E(x,t) &= \sqrt{\frac{(\Omega^2+2\mu)\,(\Omega^2-1)}{2\left[-(1+m^2)\pm 6\,m\right]}}\left(\pm\, m\,\text{sn}\xi \mp \text{ns}\xi\right)e^{i\Theta}, \\ n(x,t) &= \frac{(\Omega^2+2\mu)}{\left[-(1+m^2)\pm 6\,m\right]}\left(m\,\text{sn}\xi - \text{ns}\xi\right)^2, \end{aligned} \tag{3.29}$$

Case 3.2: Putting $q_0 = 1 - m^2$, $q_2 = 2m^2 - 1$, $q_4 = -m^2$ and $F(\xi) = \text{cn}\xi$. Thus, the exact JEFs solutions of (1.1) are

$$\begin{aligned} E(x,t) &= \sqrt{\frac{(\Omega^2+2\mu)\,(1-\Omega^2)}{2\left(2\,m^2-1\pm 6\,m\,\sqrt{m^2-1}\right)}}\left(\pm\, m\,\text{cn}\xi \mp \sqrt{m^2-1}\;\text{nc}\xi\right)e^{i\Theta}, \\ n(x,t) &= \frac{-(\Omega^2+2\mu)}{\left(2\,m^2-1\pm 6\,m\,\sqrt{m^2-1}\right)}\left(m\,\text{cn}\xi - \sqrt{m^2-1}\;\text{nc}\xi\right)^2, \end{aligned} \tag{3.30}$$

Case 3.3: If $q_0 = m^2 - 1$, $q_2 = 2 - m^2$, $q_4 = -1$ and $F(\xi) = \text{dn}\xi$, we obtain the combined solutions of (1.1) as

$$\begin{aligned} E(x,t) &= \sqrt{\frac{(\Omega^2+2\mu)\,(1-\Omega^2)}{2\left(2-m^2\pm 6\,\sqrt{1-m^2}\right)}}\left(\pm\,\text{dn}\xi \mp \sqrt{1-m^2}\;\text{nd}\xi\right)e^{i\Theta}, \\ n(x,t) &= \frac{-(\Omega^2+2\mu)}{\left(2-m^2\pm 6\,\sqrt{1-m^2}\right)}\left(\text{dn}\xi - \sqrt{1-m^2}\;\text{nd}\xi\right)^2, \end{aligned} \tag{3.31}$$

Case 3.4: If $q_0 = \frac{1}{4}$, $q_2 = \frac{m^2-2}{2}$, $q_4 = \frac{m^4}{4}$, we get $F(\xi) = \frac{\text{sn}\xi}{1\pm\text{dn}\xi}$, this yeilds the exact solutions of (1.1) as

$$\begin{aligned} E(x,t) &= \frac{1}{2}\sqrt{\frac{(\Omega^2+2\mu)\,(\Omega^2-1)}{\left(m^2-2\pm 3\,m^2\right)}}\left(\pm\,\frac{m^2\text{sn}\xi}{1\pm\text{dn}\xi} \mp \frac{1\pm\text{dn}\xi}{\text{sn}\xi}\right)e^{i\Theta}, \\ n(x,t) &= \frac{(\Omega^2+2\mu)}{2\left(m^2-2\pm 3\,m^2\right)}\left(\frac{m^2\text{sn}\xi}{1\pm\text{dn}\xi} - \frac{1\pm\text{dn}\xi}{\text{sn}\xi}\right)^2, \end{aligned} \tag{3.32}$$

Case 3.5: When $q_0 = \frac{m^2-1}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{m^2-1}{4}$, we have $F(\xi) = \frac{\text{dn}\xi}{1\pm m\,\text{sn}\xi}$. Thus, the rational JEFs solutions of (1.1) are

$$\begin{aligned} E(x,t) &= \frac{1}{2}\sqrt{\frac{(m^2-1)(\Omega^2+2\mu)\,(\Omega^2-1)}{[m^2+1\pm 3\,(m^2-1)]}}\left(\pm\,\frac{\text{dn}\xi}{1\pm m\,\text{sn}\xi} \mp \frac{1\pm m\,\text{sn}\xi}{\text{dn}\xi}\right)e^{i\Theta}, \\ n(x,t) &= \frac{(m^2-1)(\Omega^2+2\mu)}{2\,[m^2+1\pm 3\,(m^2-1)]}\left(\frac{\text{dn}\xi}{1\pm m\,\text{sn}\xi} - \frac{1\pm m\,\text{sn}\xi}{\text{dn}\xi}\right)^2, \end{aligned} \tag{3.33}$$

Case 3.6: When $q_0 = \frac{1-m^2}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{1-m^2}{4}$ and $F(\xi) = \frac{\text{cn}\xi}{1\pm\text{sn}\xi}$, we obtain

$$\begin{aligned} E(x,t) &= \frac{1}{2}\sqrt{\frac{(1-m^2)(\Omega^2+2\mu)\,(\Omega^2-1)}{[m^2+1\pm 3\,(1-m^2)]}}\left(\frac{\text{cn}\xi}{1\pm\text{sn}\xi} \mp \frac{1\pm\text{sn}\xi}{\text{cn}\xi}\right)e^{i\Theta}, \\ n(x,t) &= \frac{(1-m^2)(\Omega^2+2\mu)}{2\,[m^2+1\pm 3\,(1-m^2)]}\left(\frac{\text{cn}\xi}{1\pm\text{sn}\xi} - \frac{1\pm\text{sn}\xi}{\text{cn}\xi}\right)^2 \end{aligned} \tag{3.34}$$

Case 3.7: If $q_0 = \frac{-(1-m^2)^2}{4}$, $q_2 = \frac{m^2+1}{2}$, $q_4 = \frac{-1}{4}$ and $F(\xi) = (m\,\text{cn}\xi \pm \text{dn}\xi)$. Then, the exact solutions of Eq. (1.1) are

$$\begin{aligned} E(x,t) &= \frac{1}{2}\sqrt{\frac{(\Omega^2+2\mu)\,(1-\Omega^2)}{[m^2+1\pm 3\,(1-m^2)]}}\left(\pm\,(m\,\text{cn}\xi \pm \text{dn}\xi) \mp \frac{1-m^2}{m\,\text{cn}\xi\pm\text{dn}\xi}\right)e^{i\Theta}, \\ n(x,t) &= \frac{-(\Omega^2+2\mu)}{2\,[m^2+1\pm 3\,(1-m^2)]}\left((m\,\text{cn}\xi \pm \text{dn}\xi) - \frac{1-m^2}{m\,\text{cn}\xi\pm\text{dn}\xi}\right)^2. \end{aligned} \tag{3.35}$$

We can obtain other JEFs solutions, but we ignored here for simplicity. If $m \to 1$, then the travelling wave solutions given as

$$E(x,t) = \pm\frac{1}{2}\sqrt{(\Omega^2+2\,\mu)\,(1-\Omega^2)}\,\frac{\tanh\xi}{1\pm\mathrm{sech}\xi}\,e^{i\Theta}, \quad n(x,t) = \frac{-(\Omega^2+2\,\mu)}{2}\left(\frac{\tanh\xi}{1\pm\mathrm{sech}\xi}\right)^2, \tag{3.36}$$

$$E(x,t) = \pm\frac{1}{2}\sqrt{(\Omega^2+2\,\mu)\,(1-\Omega^2)}\,\frac{1\pm\mathrm{sech}\xi}{\tanh\xi}\,e^{i\Theta}, \quad n(x,t) = \frac{-(\Omega^2+2\,\mu)}{2}\left(\frac{1\pm\mathrm{sech}\xi}{\tanh\xi}\right)^2, \tag{3.37}$$

$$E(x,t) = \pm\sqrt{\frac{(\Omega^2+2\,\mu)\,(1-\Omega^2)}{2}}\,\mathrm{sech}\xi\,e^{i\Theta}, \qquad n(x,t) = -(\Omega^2+2\,\mu)\,\mathrm{sech}^2\xi, \tag{3.38}$$

$$E(x,t) = \pm\frac{1}{2}\sqrt{(\Omega^2+2\,\mu)\,(1-\Omega^2)}\,\tanh\xi\,e^{i\Theta}, \qquad n(x,t) = \frac{-(\Omega^2+2\,\mu)}{2}\,\tanh^2\xi, \tag{3.39}$$

$$E(x,t) = \pm\frac{1}{2}\sqrt{(\Omega^2+2\,\mu)\,(1-\Omega^2)}\,\coth\xi\,e^{i\Theta}, \qquad n(x,t) = \frac{-(\Omega^2+2\,\mu)}{2}\,\coth^2\xi, \tag{3.40}$$

$$E(x,t) = \pm\sqrt{\frac{(\Omega^2+2\,\mu)\,(\Omega^2-1)}{2}}\,\mathrm{csch}\xi\,e^{i\Theta}, \qquad n(x,t) = (\Omega^2+2\,\mu)\,\mathrm{csch}^2\xi. \tag{3.41}$$

Also, if $m \to 0$, then we can obtain the triangular function solutions for (1.1) , but we omitted these solutions for simplicity. The solutions (3.38) and (3.39) are called bright and dark soliton solutions, respectively. The bright solitary wave solution (3.38) are plotted in Fig. 5 with the parameters $\Omega = 0.1$, $\mu = 2.43$ and $\xi_0 = 1$ for 3D figure and $t = 1$ for 2D figure. Moreover, in Fig. (5.b), we have studied the intensity profile at different values of the parameter Ω. We get that with the increase of Ω the width increases and the amplitude decreases. In addition, multiplying Eq. (3.6) by U' and integrating once, we get

$$\frac{1}{2}\,(U')^2 = C + \frac{(\Omega^2+2\,\mu)}{2\,k^2}\;U^2 + \frac{U^4}{k^2\,(\Omega^2-1)},$$

where C is an arbitrary constant. We can be written this equation like as an energy equation of classical particle as $\frac{1}{2}\,(U')^2 + f(U) = 0$, where $f(U)$ is the potential energy and it is given by $f(U) = -[\,C + \frac{(\Omega^2+2\,\mu)}{2\,k^2}\;U^2 + \frac{U^4}{k^2\,(\Omega^2-1)}\,]$. The physical solution to exist it must satisfy $f(U) = 0$ and $\frac{df(U)}{dU} = 0$ at $U = 0$. It is clear that $(U')^2 \geq 0$, this means $f(U) \leq 0$. So, there exist a point U_c such that $f(U_c) = 0$, i.e.,

$$C + \frac{(\Omega^2+2\,\mu)}{2\,k^2}\;U_c^2 + \frac{U_c^4}{k^2\,(\Omega^2-1)} = 0.$$

When $C = 0$, the amplitude of solitary wave $f(U_c) = 0$ is $U_c = \pm\sqrt{\frac{(\Omega^2+2\,\mu)\,(1-\Omega^2)}{2}}$.

4 Lie Symmetry Analysis and Conservation Laws

In this section, we derive Lie symmetry analysis of the model (1.1) and investigate the conservation laws by Ibragimove's theorem [28]. For this, we consider the transformation

$$E(x,t) = u(x,t) + i\,v(x,t). \tag{4.1}$$

Substituting (4.1) in (1.1) and splitting the result into imaginary and real parts, we get the following system:

$$\begin{cases} F^1 = u_t + \frac{1}{2}\,v_{xx} - n\,v = 0, \\ F^2 = v_t - \frac{1}{2}\,u_{xx} + n\,u = 0, \\ F^3 = n_{tt} - n_{xx} - 4\left(u_x^2 + v_x^2 + u\,u_{xx} + v\,v_{xx}\right) = 0. \end{cases} \tag{4.2}$$

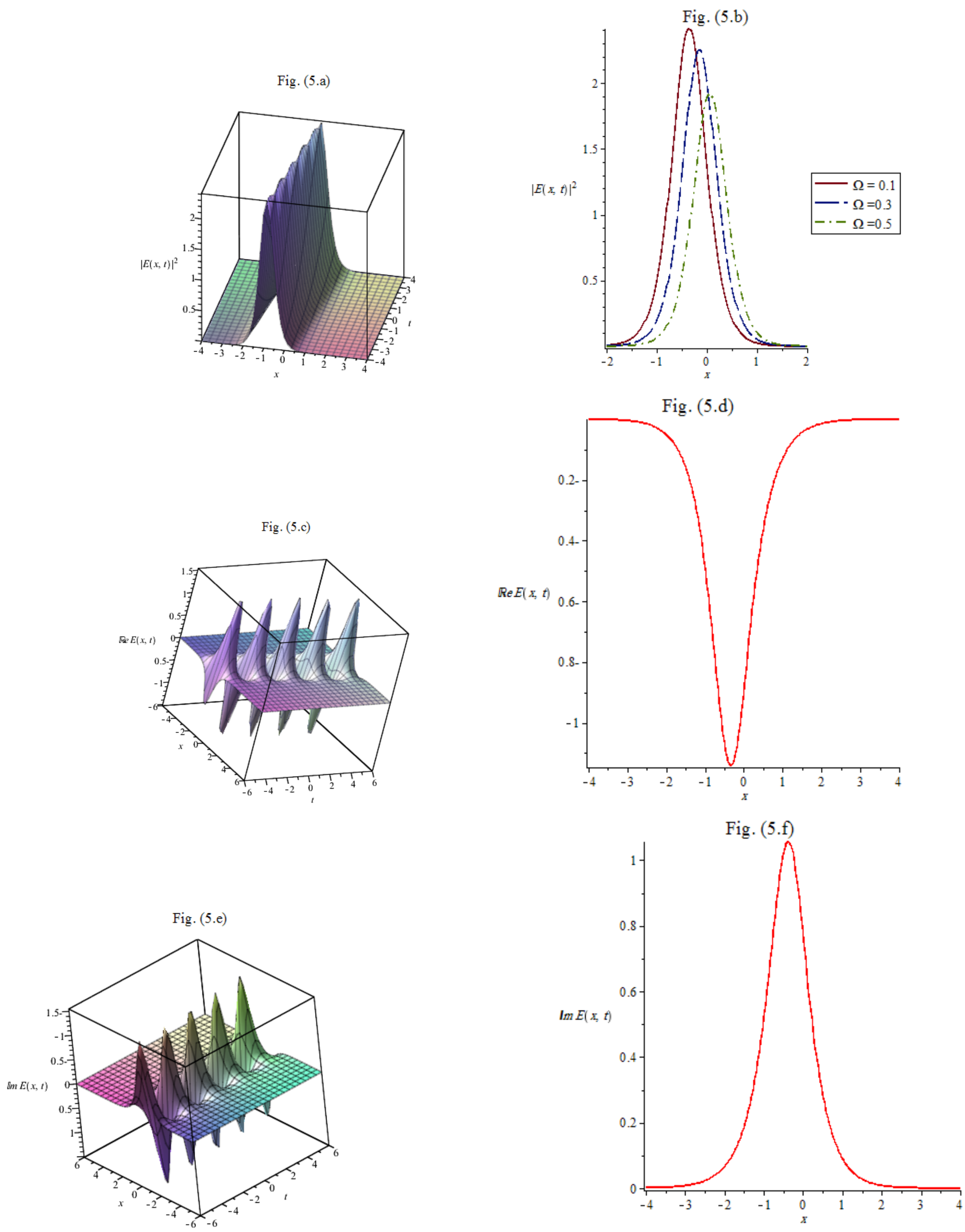

Fig. 5. The bright solitary wave solution (3.38) are plotted in (a-f) 3D and 2D with the parameters $\Omega = 0.1$, $\mu = 2.43$ and $\xi_0 = 1$ for 3D figure and $t = 1$ for 2D figure. In Fig. (5.b), we discuss the intensity profile at different values of Ω

We can be written this equation in the form

$$n_{tt} - n_{xx} - 4\left[u_x^2 + v_x^2 + 2\left(n\,(u^2+v^2) + u\,v_t - v\,u_t\right)\right] = 0. \tag{4.3}$$

The Lie point symmetries for (4.3) is generated by a vector field in the form

$$\begin{aligned} X &= \xi^1(x,t,u,v,n)\,\partial_x + \xi^2(x,t,u,v,n)\,\partial_t + \eta^1(x,t,u,v,n)\,\partial_u + \eta^2(x,t,u,v,n)\,\partial_v + \\ &\eta^3(x,t,u,v,n)\,\partial_n. \end{aligned} \tag{4.4}$$

Appling the prolongation $\mathrm{Pr}^{(2)}X$ to (4.3), we have system of linear partial differential equations(PDEs). Solving it by Maple, we get the infinitesmals as follows:

$$\xi^1 = c_3,\ \xi^2 = c_4,\ \eta^1 = c_1\,t + c_2,\ \eta^2 = \frac{1}{2}\,v\,t^2\,c_1 + v\,t\,c_2 + v\,c_5,\ \eta^3 = -\frac{1}{2}\,u\,t^2\,c_1 - u\,t\,c_2 - u\,c_5. \tag{4.5}$$

where c_1, c_2, c_3, c_4 and c_5 are constants. Eq. (4.3) admits the algebra of Lie point symmetries generated as

$$\begin{aligned} &X_1 = t\,\partial_u + \tfrac{1}{2}\,v\,t^2\partial_v - \tfrac{1}{2}\,u\,t^2\partial_n,\ \ X_2 = \partial_u + v\,t\,\partial_v - u\,t\,\partial_n,\ \ X_3 = \partial_x,\ \ X_4 = \partial_t, \\ &X_5 = v\,\partial_v - u\,\partial_n. \end{aligned} \tag{4.6}$$

For simplicity, suppose that a sth-order system of PDEs of r dependent variables $u = (u^1, u^2, ..., u^r)$ and k independent variables $x = (x^1, x^2, ..., x^k)$, define as

$$F_\alpha(x, u, u_{(1)}, ..., u_{(s)}) = 0,\ \ \alpha = 1, 2, ..., r, \tag{4.7}$$

where, $u_{(1)}, u_{(2)}, ..., u_{(s)}$ denote the collections of all first, second,...,sth-order partial derivatives. This means that, $u_i^\alpha = D_i(u^\alpha)$, $u_{ij}^\alpha = D_j D_i(u^\alpha), ...$, respectively, where the total derivative operator with respect to x^i given as

$$D_i = \frac{\partial}{\partial x^i} + u_i^\alpha\,\frac{\partial}{\partial u^\alpha} + u_{ij}^\alpha\,\frac{\partial}{\partial u_j^\alpha} + ...,\ \ i = 1, 2, ..., k. \tag{4.8}$$

Also, we can define the symmetry operator and the adjoint equation for the system (4.7), respectively as

$$X = \xi^i\,\frac{\partial}{\partial x^i} + \eta^\alpha\,\frac{\partial}{\partial u^\alpha}, \tag{4.9}$$

$$F_\alpha^*(x, u, v, u_{(1)}, v_{(1)}, ..., u_{(s)}, v_{(s)}) = \frac{\delta(v^i F^i)}{\delta u^\alpha} = 0,\ \ \alpha = 1, 2, ..., r. \tag{4.10}$$

Theorem [28]: Any Lie point, Lie-Bäcklund and non-local symmetry X, that is define in (4.9) admitted by the system (4.7) provides a conservation law for (4.7) and its adjoint (4.10), then T^i that is called the conserved vector are calculated by

$$\begin{aligned} T^i = \xi^i\,L + W^\alpha&\left[\frac{\partial L}{\partial u_i^\alpha} - D_j\left(\frac{\partial L}{\partial u_{ij}^\alpha}\right) + D_j D_k\left(\frac{\partial L}{\partial u_{ijk}^\alpha}\right) - ...\right] + \\ D_j(W^\alpha)&\left[\frac{\partial L}{\partial u_{ij}^\alpha} - D_k\left(\frac{\partial L}{\partial u_{ijk}^\alpha}\right) + D_k D_r\left(\frac{\partial L}{\partial u_{ijkl}^\alpha}\right) - ...\right] + \\ D_j D_k(W^\alpha)&\left[\frac{\partial L}{\partial u_{ijk}^\alpha} - D_r\left(\frac{\partial L}{\partial u_{ijkl}^\alpha}\right) + ...\right] + ..., \end{aligned} \tag{4.11}$$

where, $W^\alpha = \eta^\alpha - \xi^i u_i^\alpha$ and $L = \sum_{i=1}^r v^i F^i$ are the Lie characteristic function and the formal lagrangian, respectively. Now we will obtain the conservation laws for (1.1), first we can define the Lagrangian formal for the system (1.1) as

$$L = \bar{u}\left(u_t + \frac{1}{2}\,v_{xx} - n\,v\right) + \bar{v}\left(v_t - \frac{1}{2}\,u_{xx} + n\,u\right) + \bar{n}\left[n_{tt} - n_{xx} - 4\left(u_x^2 + v_x^2 + u\,u_{xx} + v\,v_{xx}\right)\right], \tag{4.12}$$

where $\bar{u}$, $\bar{v}$ and $\bar{n}$ are new dependent variables. By using (4.11) and (4.12), we get

$$\begin{cases} T^1 = \xi^1 L + W^1(-8\,\bar{n}\,u_x + \frac{1}{2}\bar{v}_x + 4\,\bar{n}_x\,u + 4\,\bar{n}\,u_x) - (\frac{1}{2}\,\bar{v} + 4\,\bar{n}\,u)\,D_x(W^1) + (\frac{1}{2}\,\bar{u} - 4\,\bar{n}\,v) \\ D_x(W^2) + W^2(-8\,\bar{n}\,v_x - \frac{1}{2}\bar{u}_x + 4\,\bar{n}_x\,v + 4\,\bar{n}\,v_x) + \bar{n}_x\,W^3 - \bar{n}\,D_x(W^3), \\ T^2 = \xi^2 L + W^1\,\bar{u} + W^2\,\bar{v} - \bar{n}_t\,W^3 + \bar{n}\,D_t(W^3). \end{cases} \tag{4.13}$$

From the symmetry operators given in (4.6) with (4.13), we get the following cases for the conservation laws:

Case 1: We consider the symmetry operator $X_1 = t\,\partial_u + \frac{1}{2}\,v\,t^2\partial_v - \frac{1}{2}\,u\,t^2\partial_n$, we have $\xi^1 = \xi^2 = 0$, $\eta^1 = t$, $\eta^2 = \frac{1}{2}\,v\,t^2$, $\eta^3 = -\frac{1}{2}\,u\,t^2$ and the Lie characteristic functions corresponding to this symmetry are $W^1 = t$, $W^2 = \frac{1}{2}\,v\,t^2$ and $W^3 = \frac{1}{2}\,u\,t^2$. Thus, the associated conserved vectors are

$$\begin{cases} T^1 = \bar{n}\,u_x\,(\frac{1}{2}\,t^2 - 4\,t) + t^2\,v_x(\frac{1}{4}\,\bar{u} - 4\,\bar{n}\,v) + \frac{1}{2}\,t(\bar{v}_x - \frac{1}{2}\,t\,v\,\bar{u}_x) + \bar{n}_x[4\,u + t^2\,(2\,v^2 - \frac{1}{2}\,u)], \\ T^2 = \frac{1}{2}t^2\,(v\,\bar{v} + u\,\bar{n}_t) + t\,(\bar{u} - \bar{n}\,u), \end{cases} \tag{4.14}$$

Case 2: Using the symmetry operator $X_2 = \partial_u + v\,t\,\partial_v - u\,t\,\partial_n$, we have $\xi^1 = \xi^2 = 0$, $\eta^1 = 1$, $\eta^2 = v\,t$, $\eta^3 = -u\,t$. Then $W^1 = 1$, $W^2 = v\,t$ and $W^3 = -u\,t$. Thus, the associated conserved vectors given as

$$\begin{cases} T^1 = \bar{n}\,u_x\,(t-4) + t\,v_x\,(\frac{1}{2}\,\bar{u} - 8\,v\,\bar{n}) + \frac{1}{2}\,\bar{v}_x + \frac{1}{2}\,v\,t\,\bar{u}_x + \bar{n}_x[4\,u + t\,(4\,v^2 - u)], \\ T^2 = \bar{u} + t\,[v\,\bar{v} + u\,\bar{n}_t - \bar{n}\,u_t] - \bar{n}\,u, \end{cases} \tag{4.15}$$

Case 3: For the symmetry operator $X_3 = \partial_x$, we have $\xi^1 = 1$, $\xi^2 = \eta^1 = \eta^2 = \eta^3 = 0$ and $W^1 = -u_x$, $W^2 = -v_x$ and $W^3 = -n_x$. So, we obtain

$$\begin{cases} T^1 = \bar{u}(u_t - n\,v) + \bar{v}\,(v_t + n\,u) + \bar{n}\,n_{tt} + \frac{1}{2}(u_x\,\bar{v}_x + v_x\,\bar{u}_x) - \bar{n}_x(4\,u\,u_x + 4\,v\,v_x + n_x), \\ T^2 = \bar{n}_t\,n_x - \bar{n}\,n_{xt} - \bar{u}\,u_x - \bar{v}\,v_x, \end{cases} \tag{4.16}$$

Case 4: Using the symmetry operator $X_4 = \partial_t$, we have $\xi^2 = 1$, $\xi^1 = \eta^1 = \eta^2 = \eta^3 = 0$ with $W^1 = -u_t$, $W^2 = -v_t$ and $W^3 = -n_t$. So, we obtain the conserved vectors as

$$\begin{cases} T^1 = 4\,\bar{n}\,u_x u_t + u_{xt}\,(\frac{1}{2}\bar{v} + 4\,u\,\bar{n}) - v_{xt}\,(\frac{1}{2}\bar{u} - 4\,v\,\bar{n}) + \frac{1}{2}(v_t\bar{u}_x - u_t\bar{v}_x) + \bar{n}\,(4\,v_x\,v_t + n_{xt}), \\ -\bar{n}_x\,[\,4\,(u\,u_t + v\,v_t) + n_t], \\ T^2 = \bar{u}(\frac{1}{2}\,v_{xx} - n\,v) - \bar{v}(\frac{1}{2}\,u_{xx} - n\,u) - \bar{n}[n_{xx} + 4\,(u_x^2 + v_x^2 + u\,u_{xx} + v\,v_{xx})] + \bar{n}_t\,n_t, \end{cases} \tag{4.17}$$

Case 5: Using the symmetry operator $X_5 = v\,\partial_v - u\,\partial_n$, we have $\xi^1 = \xi^2 = \eta^1 = 0$, $\eta^2 = v$, $\eta^3 = -u$ and $W^1 = 0$, $W^2 = v$ and $W^3 = -u$. So, we obtain the conserved vectors as

$$\begin{cases} T^1 = \frac{1}{2}\,(\bar{u}\,v_x - v\,\bar{u}_x) + \bar{n}\,(u_x - 8\,v\,v_x) + \bar{n}_x\,(4\,v^2 - u), \\ T^2 = v\,\bar{v} + u\,\bar{n}_t - \bar{n}\,u_t. \end{cases} \tag{4.18}$$

5 Results and Discussion

In this section, we have explained the results of the ISLWs model by drawing some 3D and 2D figures of the obtained solutions with the support of the symbolic calculation software Maple. Also, we have compared our constructed results with other results in different papers. The 3D and 2D are ploted the absolute, real and imaginary parts to illustrate the abundant soliton and periodic wave solutions. The graphical illustrations of the abundant periodic wave solutions and soliton solutions are plotted by taking suitable values of involved unknown parameters to visualize the mechanism of the ISLWs model that given in Fig. 1 - Fig. 5. The behaviors of the periodic wave solutions (3.10) and (3.13) are presented in Fig. 1 and Fig. 2, respectively with the same parameters $\Omega = 1.5, m = 0.2, \mu = -2.43$ and $\xi_0 = 1$ for 3D figure and $t = 1$ for 2D figure. We can find that the absolute, real and imaginary parts are periodic wave solutions. The graphical representation of the periodic wave solutions (3.16) and (3.24) are drawn in Fig. 3 and Fig. 4, respectively under the same choice of parameters $\Omega = 0.5$, $m = 0.6$, $\mu = 1$ and $\xi_0 = 1$ for 3D plots and $t = 1$ for 2D plots. As expected, the absolute, real and imaginary parts are periodic wave solutions. Fig. 5 plots the behavior of the bright soliton solution (3.38). We observe that the absolute, real and imaginary parts are bright soliton solutions. Also, we have discussed the effect of the parameter Ω at different values on the intensity profile of the bright soliton solution (3.38) in Fig. (5.b). Moreover, many our results are novel and some of them are obtained in research literature such as the solutions (3.10), (3.11), (3.38) and (3.39) are the same as the results obtained in [24] and the solutions (3.39) and (3.40) are similar to the solutions given in [20, 25]. Furthermore, we investigate the conservation laws by the Lie point symmetry. Noteworthy that all obtained solutions are checked by using Maple software program.

6 Conclusion

In this paper, we considered the ISLWs model and we succeeded implementing the extended F-expansion method in the NLEEs for getting exact traveling wave solutions. As results, several kinds of solutions of the underlying model including periodic wave solutions with JEFs, hyperbolic function solutions dark and bright solutions have been obtained in the study, in which many are novel. The computer systems like as Maple is used to solve the complecated algebraic equations to get these solutions. To the best of our knowledge, the obtained solutions of ISLWs model contain the known result in [20, 22, 24, 25] and other traveling wave solutions are new. The geometrical shape for some of the obtained results are plotted for various choices of the parameters that appear in the results which may help researchers to known some physical meaning of this model. Graphical simulation of some solutions in the form of two-dimentional and three-dimentional are helpfull to see the behaviour of these solutions. In addition, the conservation laws for the (1.1) are constructed. We hope that the obtained solutions are useful in the study of plasma physics and other important equations of mathematical physics.

Competing Interests

Authors have declared that no competing interests exist.

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