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Research Article

Significance of Broken $\mu-\tau$ Symmetry in Correlating δ_{CP}, θ_{13}, Lightest Neutrino Mass, and Neutrinoless Double Beta Decay $0\nu\beta\beta$

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Leptonic CP violating phase δ_{CP} in the light neutrino sector and leptogenesis via present matter-antimatter asymmetry of the Universe entails each other. Probing CP violation in light neutrino oscillation is one of the challenging tasks today. The reactor mixing angleθ_{13}measured in reactor experiments, LBL, and DUNE with high precision in neutrino experiments indicates towards the vast dimensions of scope to detect δ_{CP}. The correlation between leptonic Dirac CPV phase δ_{CP}, reactor mixing angle θ_{13}, lightest neutrino mass m_1, and matter-antimatter asymmetry of the Universe within the framework of $\mu-\tau$ symmetry breaking assuming the type I seesaw dominance is extensively studied here. Here, a SO(10) GUT model with flavor $\mu-\tau$ symmetry is considered. In this work, the idea is to link baryogenesis through leptogenesis and the hint of CP violation in the neutrino oscillation data to a breaking of the mu-tau symmetry. Small tiny breaking of the$\mu-\tau$ symmetry allows a large Dirac CP violating phase in neutrino oscillation which in turn is characterized by awareness of measured value of θ_{13} and to provide a hint towards a better understanding of the experimentally observed near-maximal value of$\nu_\mu-\nu_\tau$mixing angle$\theta_{23}\simeq(\pi/4)$. Precise breaking of the $\mu-\tau$ symmetry is achieved by adding a 120-plet Higgs to the$10+1\bar{2}6$-dimensional representation of Higgs. The estimated three-dimensional density parameter space of the lightest neutrino mass m_1, δ_{CP}, and reactor mixing angle θ_{13} is constrained here for the requirement of producing the observed value of baryon asymmetry of the Universe through the mechanism of leptogenesis. Carrying out numerical analysis, the allowed parameter space of m_1, δ_{CP}, and θ_{13} is found out which can produce the observed baryon to photon density ratio of the Universe.

1. Introduction

In 1950, Bruno Pontecorvo for the first time emphasized the idea of neutrino oscillations which resembled $K^0-\bar{K}^0$ oscillations. In neutrino oscillations, a neutrino originated with a definite flavor, (ν_e,ν_μ,ν_τ) oscillates to a distinct contrasting lepton flavor. Neutrino oscillation reveals that each of the three states of neutrino ν_α in flavor basis is a superposition of three mass eigen states (m_1,m_2,m_3) [1]. Neutrinos are massive, and they mix with each other. The massive neutrinos are formed in their gauge eigen states (ν_α) which are linked to their mass eigen states ν_i. Gauge eigen states participate in gauge interactions as

$$|\nu_a\rangle=\sum U_{a_i}|\nu_i\rangle, \tag{1}$$

where $\alpha=e,\mu,\tau$, ν_i is the neutrino of distinct mass m_i.U is parameterised as

$$U=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e_{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2}$$

where $\theta_{12} = 33^{\circ}, \theta_{23} = 38^{\circ} - 53^{\circ}$, and $\theta_{13} = 8^{\circ}$ [2] are the solar, atmospheric, and reactor angles according to the global fits, respectively. The Majorana phases α and β dwell in P, where

$$P = \text{diag}\left(1 \quad e^{i\alpha} \quad e^{i(\beta+\delta)}\right). \qquad (3)$$

$U * P$ is known as the Pontecorvo-Maki-Nakagawa-Sakata U_{PMNS} matrix [3]. Since a ν of a given flavor α is a mixed state of at least three ν with distinct masses, this three-generation mixing could result into the flavor mixing mass matrix or PMNS matrix possessing an irreducible imaginary component. This irreducible imaginary component is responsible for CP asymmetry. CP violation interchanges every particle into its antiparticle. δ_{CP} in PMNS matrix can induce CP violation. CP asymmetry can be observed in neutrino oscillations. δ_{CP} phase measures the amount of asymmetries between lepton oscillations and antilepton oscillations. Neutrinos are massive, and they mix with each other. This may be a source of CP violation if $\text{Sin}\delta_{\text{CP}} \neq 0$. The amount of δ_{CP} violation phase in this case is estimated by the Jarkslog invariant [4].

$$J_{\text{CP}} = \frac{1}{8}\text{Cos}\theta_{13}\text{Sin}2\theta_{12}\text{Sin}2\theta_{23}\text{Sin}2\theta_{13}\text{Sin}\delta_{\text{CP}}, \qquad (4)$$

when $\text{Sin}\delta_{\text{CP}} \neq 0$. In leptogenesis, lepton-antilepton asymmetry is explained if there are complex imaginary irreducible terms in the Yukawa couplings of lepton mass matrices. The lepton number generation of the early Universe can be estimated by the complex CPV phase term in the fermion mass matrices. The paper gives the impression that the neutrino Dirac CP phase is automatically connected to the baryon asymmetry of the universe; in some special cases, e.g., in flavored leptogenesis, such a connection exist. Also, softly broken mu-tau symmetry provides also another way (in addition to flavored leptogenesis) to connect the two important observables (Dirac CP phase and baryon asymmetry). The ongoing T2K experiment has reported that CP violating phase,δ_{CP}, which excludes the value$\delta_{\text{CP}} = 0, \pi$ [5] at the 2σ confidence interval for either of the mass orderings, normal ordering, or inverted ordering. The value of Dirac CPV phase, $\delta_{\text{CP}} = 276.5^{\circ}$ is preferred in [6]. The neutrino mass matrix is invariant under $\mu - \tau$ exchange symmetry, in a basis where the charged leptons are mass eigen states. Under the $\mu - \tau$ exchange symmetry, the 2-3 mixing is maximal, i.e, $\theta_{23} = \pi/4$ and the 1-3 mixing is zero, i.e, $\theta_{13} = 0$. The deviation of $\delta\theta_{23}$ from the maximal angle θ_{23}, the explanation of reactor angle θ_{13}, and the existence of CP violating phase necessitate the spontaneous breaking of the $\mu - \tau$ exchange symmetry in the neutrino sector. The measurement of the neutrino mixing angle θ_{13} in concurrence with a measurement of the departure from maximality of the atmospheric mixing angle can be a very strong way to probe any possible $\nu \leftrightarrow \tau$ symmetry present in the neutrino mass matrix. Different types of plausible mechanism of breaking of $\mu - \tau$ symmetry and the possible types of resultant gauge symmetry for generation of nonzero θ_{13} are introduced in [7].

$\mu - \tau$ symmetry is an important idea in neutrino physics in view of the near-maximal atmospheric mixing angle. The original papers which introduced the concept of $\mu - \tau$ symmetry are cited in [8–10].

Here, in this work, an explicit form of the Dirac neutrino mass matrix in broken $\mu - \tau$ [11] symmetry framework in type I seesaw mechanism is used in our calculation for generating baryon asymmetry of the Universe via leptogenesis. This scenario is characterized by small divergence of $\delta\theta_{23}$ from the maximal angle θ_{23}, which is consistent with a liberal size of $\theta_{13} \sim 8^{\circ} - 9^{\circ}$ and a large δ_{CP} phase in the neutrino sector. The renormalisable Dirac neutrino Yukawa couplings of the Dirac mass matrices are determined from the fermion Yukawa couplings to the 10, $1\bar{2}6$, and 120 dimensional fields of Higgs multiplet in the SO(10) group. Higgs field under the 10 and $1\bar{2}6$ representations is symmetrical under the generalized $\mu - \tau$ symmetry, while the 120-dimensional representation changes sign. This spontaneously breaks the $\mu - \tau$ invariant symmetry, which in turn allows a generalized δ_{CP} phase in the PMNS matrix.

Here, we made an effort for correlating or constraining the values of δ_{CP} phase, nonzero reactor angle θ_{13}, and the lightest neutrino mass space for both the hierarchies in the context of leptogenesis and current ratio of baryon to photon density of the Universe. Both CPV phase δ_{CP} and reactor angle θ_{13} have good vibes between each other. A precise value of θ_{13} plays an imperative role in its CP violation phase measurements. On the basis of this fact, nonzero values of θ_{13} are predicted here in consistency with the δ_{CP} phase. Taking into account constraints from the global fit values of ν oscillation parameters and cosmology, a density plot of the favourable values of the δ_{CP} phase, lightest neutrino mass, and θ_{13} is being initiated, which is compatible with the contraints on the sum of the absolute neutrino masses, $\sum_i m(\nu_i) < 0.23\,\text{eV}$ from CMB, Planck 2015 data ($\text{CMB15} + \text{LRG} + \text{lensing} + H_0$) [12]. Constraints from the leptonic asymmetry of the Universe are also considered for further restricting the δ_{CPV} phase space and lightest neutrino mass. The leptonic CP asymmetry is being deliberated via leptogenesis in terms of baryon density to photon density ratio of the Universe η_{B} accessible as $5.8 \times 10^{-10} < \eta_{\text{B}} < 6.6 \times 10^{-10}$ [13]. We also calculate the effective mass spectrum for neutrinoless double beta decay, $0\nu\beta\beta$ decay given by

$$m_{ee} = \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta_{\text{CP}})} \right|, \qquad (5)$$

for favourable values of the δ_{CP} phase and lightest ν mass explored here in this work. In this paper, we apply the broken $\mu - \tau$ symmetry to the Dirac neutrino Yukawa couplings in type I seesaw mechanism in the SO(10) model in predicting favourable values of the δ_{CP} phase, lightest neutrino mass, and θ_{13}. We then scan free parameters in these models and search for the allowed region in which the neutrino oscillation data can be fitted. For the allowed parameter sets, we show the predictions of observables like the δ_{CP} phase, lightest neutrino mass, and θ_{13} in the neutrino sector. Finally, we show our predictions for the effective mass spectrum for neutrinoless double beta, $0\nu\beta\beta$, decay for favourable values of the δ_{CP} phase.

This paper deals with an important aspect of neutrino physics, i.e., its CP violating Dirac phase and its possible connection to the matter-antimatter asymmetry of the universe. In this work, we have used user-defined Dirac Neutrino Yukawa couplings [11] for the Yukawa interactions associated with the broken $\mu-\tau$ symmetry model for the generation of nonzero reactor mixing angle θ_{13} and leptonic CP phase δ_{CP} in type I seesaw mechanism in the light of leptogenesis; there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B+L$ violating (sphaleron, Sakharov conditions) processes as discussed in [6]. A small explicit breaking of $\mu-\tau$ symmetry [11] by hand with the specific numerical value in Equation (30) inherits the property of generating nonzero CP violation in U_{PMNS} matrices and δ_{CP} phase and results in θ_{13} being nonzero. Here, we consider the type I seesaw as the main donor to neutrino mass. We also take into account both inverted and normal ordering of neutrino mass spectrum as well as two different types of the lightest neutrino mass m_1($m_3 = 0.07118 eV(0.0657\ \mathrm{eV})$) to visualise the results of hierarchical ν mass spectrum. In the case of normal ordering of ν masses, the dependance of leptonic CPV phase, δ_{CP}, on the lightest ν mass is predicted in Figures 1–3 (in the light of recent ratio of the baryon to photon density bounds, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$). The favoured values of δ_{CP} phase is found to lie between $\delta_{CP} \in [304^0, 307^0]$ for best fit values of $\theta_{13} = 8.41$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as a result of contribution of type I seesaw mechanism to neutrino mass matrix. The favoured values of the lightest ν mass, m_1, in this case come out to be $\in$[0.085,0.1] eV. In case of inverted hierarchy, the variation of δ_{CP} phase is found to be very intense with the best fit values of $\theta_{13} = 8.49$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]. Values of the δ_{CP} phase favoured are $\delta_{CP} = 220°, 223°, 252°, 268°, 293°, 309°, 345°$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as is evident from Figure 4. The allowed spectrum of the lightest ν mass is m_3, $\in$[0.02,0.055] eV.

The paper is organized as follows. In Section 2, we introduce our broken $\mu-\tau$ symmetry models. In Section 3, we perform parameter scan to fit neutrino oscillation data and provide some information in predicting observables like the δ_{CP} phase in the neutrino sector. Also, we show our predictions for the effective mass spectrum for neutrinoless double beta, $0\nu\beta\beta$, decay for favourable values of the δ_{CP} phase. Section 4 is our results and conclusions for the Yukawa interactions associated with broken $\mu-\tau$ symmetry model discussed here.

2. Broken $\mu-\tau$ Symmetry with Type I Seesaw Mechanism

When $\mu-\tau$ symmetry is unified with grand unification subsequently, a more general symmetry results that interchanges the second and third generations of fermions. This generalized symmetry is aimed at explaining why the Cabibbo angle is greater than the other two angles, and a soft explicit breaking of this $\mu-\tau$ symmetry leads to correct explanation of the quark mixing angles and masses. In this work, we consider a model based on the SO(10) $\otimes Z_2^{\mu-\tau} \otimes Z_2^P$. First, Z_2 symmetry represents the generalized $\mu-\tau$ symmetry. Second, Z_2 symmetry indicates parity [15] transformation between two components of the 16-dimensional representation of the field acting as (4, 2, 1) and (4, 1, 2) under the Pati-Salam group decomposition of SO(10). Sixteen-dimensional representation of fermions gets their masses from coupling to three Higgs multiplets which transform as 10, $1\overline{2}6$ and 120 representations under SO(10). The SO(10) breaking is fulfilled with 210 plets. In a supersymmetric realm, an additional 126 plets of Higgs conserve the supersymmetry at the GUT breaking scale. These Higgs multiplets have in them altogether six doublets which match with the quantum numbers of the minimal supersymmetric standard model (MSSM) field H_d and six other quantum numbers which complement the quantum field numbers of H_u. Two linear superpositions of these Higgs doublets stay light and play the role of the H_d and H_u fields. This is controlled by the fine-tuning conditions [16, 17]. After this fine-tuning results, the fermion masses can be written as

$$-L_{\mathrm{mass}} = \bar{f}_{\mathrm{L}} M_f f_{\mathrm{R}} + \bar{\nu}_{\mathrm{L}} M_{\mathrm{D}} \nu_{\mathrm{R}} + \frac{1}{2}\bar{\nu}_{\mathrm{L}} M_{\mathrm{L}} \nu_{\mathrm{L}}^c + \frac{1}{2}\bar{\nu}_{\mathrm{R}}^c M_{\mathrm{R}} \nu_{\mathrm{R}} + h.c., \tag{6}$$

where $f = u, d, l$ represents the up and down quarks and the charged leptons, respectively. The mass matrices representing different scalar representations to obtain the flavor texture of interest in the paper appearing in the above Lagrangian can be [18, 19] written as

$$M_d = H + F + iG, \tag{7}$$

$$M_u = rH + sF + itG, \tag{8}$$

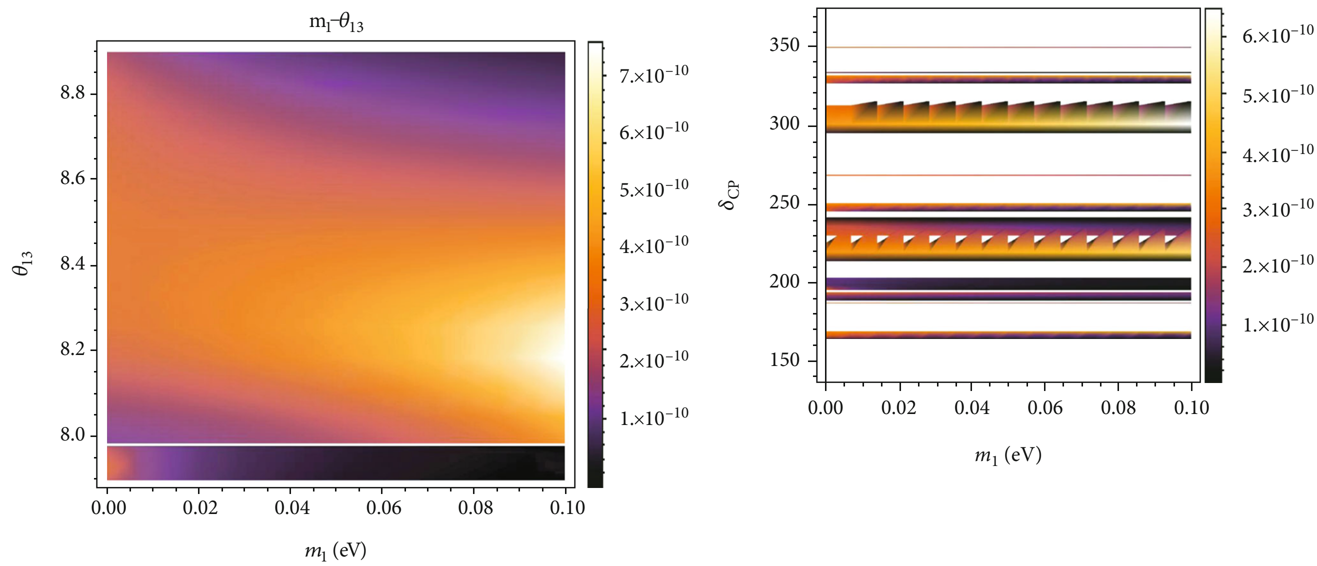
$$M_l = H - 3F + ipG, \tag{9}$$

$$M_{\mathrm{D}} = rH - 3sF + iqG, \tag{10}$$

$$M_{\mathrm{L}} = r_{\mathrm{L}} F, \tag{11}$$

$$M_{\mathrm{R}} = r_{\mathrm{R}}^{-1} F. \tag{12}$$

Here, M_{D} is the Dirac ν mass matrix. In the basis where $M_{\mathrm{L}}(M_{\mathrm{R}})$, the Majorana mass matrix for the left(right)-handed neutrinos, gets benefaction only from the vacuum expectation value (vev) of the $1\overline{2}6$ Higgs field, gauge coupling unification in the minimal model needs that the vacuum expectation value (vev) contributing to M_{R} is adjacent to the GUT scale. $r, s, t, p, q, r_{\mathrm{L}}$, and r_{R} are the dimensionless parameters which are determined by the Clebsch-Gordan coefficients, ratios of vevs, and mixing among the Higgs fields [18]. The matrices H, F, and G rise from the fermion couplings to the 10, $1\overline{2}6$, and 120 Higgs fields, respectively. Normally, $(G)H, F$ are complex (anti) symmetric matrices. Nonetheless, generalized parity symmetry keeps $(G)H, F$ real. Moreover, when all vevs and thus $r, s, t, p, q, r_{\mathrm{L}}$, and r_{R} are real, then the Dirac masses defined in Equation (10) are Hermitian and M_{L} and M_{R} are



(a) (b)

Figure 1: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering. (a) Predicted favoured values of the (m_1, θ_{13}) plane for the best fit values of $\delta_{CP} = 222^{\circ}$ with $\Delta\chi^2 = 6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, (b) shows the predicted favoured values of the (m_1, δ_{CP}) plane, for the best fit value of $\theta_{13} = 8.41$ with $\Delta\chi^2 = 9.5$ [14].

real. The 10- and $1\overline{2}6$-dimensional Higgs field representations are invariant under the generalized $\mu-\tau$ symmetry while the 120-dimensional representation changes sign. This allows spontaneous explicit breaking of the $\mu-\tau$ symmetry.

Let H, F (G) be any complex symmetric (anti) matrices in general, which are a measure of the fermion Yukawa couplings to the 10, $1\overline{2}6$, and 120 Higgs field, respectively. Here,

$$H = \begin{pmatrix} h_{11} & h_{12} & h_{12} \\ h_{12} & h_{22} & h_{23} \\ h_{12} & h_{23} & h_{22} \end{pmatrix}, \qquad (13)$$

Similarly,

$$F = \begin{pmatrix} f_{11} & f_{12} & f_{12} \\ f_{12} & f_{22} & f_{23} \\ f_{12} & f_{23} & f_{22} \end{pmatrix}, \qquad (14)$$

Similarly,

$$G = \begin{pmatrix} 0 & g_{12} & -g_{12} \\ -g_{12} & 0 & g_{23} \\ g_{12} & -g_{23} & 0 \end{pmatrix}. \qquad (15)$$

The matrices H, F, and G originate from the fermion couplings to the 10, 126, and 120 fields, respectively. (G) H, F are complex (anti) symmetric matrices in general. However, generalized parity makes them real. In addition, if all vevs and hence r, s, t, p, q, r_L, and r_R are real, then all the Dirac masses in Equations (7), (8), (9), (10), (11), and (12) are Hermitian and M_L and M_R are real. Here, the Higgs field in the 10 and 126 representations are symmetric and invariant under the generalized $\mu-\tau$ symmetry while the 120-dimensional representation changes sign. This assumption in turn allows spontaneous breaking of the $\mu-\tau$ symmetry. In the $\mu-\tau$ symmetric mass matrices in Equations (13), (14), and (15), Higgs field 120 induces antisymmetry of the Yukawa matrix. If that matrices as well as the 10 and 126 couplings are complex, it is well known that if there are no extra symmetries, they will induce a Dirac phase. Explicit breaking of $\mu-\tau$ symmetry leads to nonzero θ_{13}. Small explicit tiny breaking of the $\mu-\tau$ symmetry allows a large Dirac CP violating phase in neutrino oscillation. Fermion mass spectrum can be explained by Hermitian mass matrices derived from the renormalizable Yukawa couplings of the 16 plets of fermions with the Higgs fields transforming as 10, $1\overline{2}6$, and 120 representations of the SO(10) group. The $\mu-\tau$ symmetry upon spontaneously broken down through the 120 plets leads to nonzero reactor angle θ_{13}, which in turn induces leptonic dirac CPV phase in the U_{PMNS} matrix. Tiny explicit breaking of $\mu-\tau$ symmetry leads to nonzero θ_{13}. This scenario implies a generalized CP invariance of the fermion mass matrices and vanishing CP violating phases if the Yukawa couplings are symmetric under the $\mu-\tau$ symmetry. Small tiny breaking of the $\mu-\tau$ symmetry allows a large Dirac CP violating phase in neutrino oscillation. Explicit breaking of the $\mu-\tau$ symmetry by hand as evident from Equation (30) provides a nice spectrum of all the fermion masses and mixing and leads to nonzero θ_{13}, which in turn induces the δ_{CP} phase and allows a large required Dirac CP violating phase in neutrino oscillation. Detailed fits to the fermion spectrum are presented in several scenarios in [11].

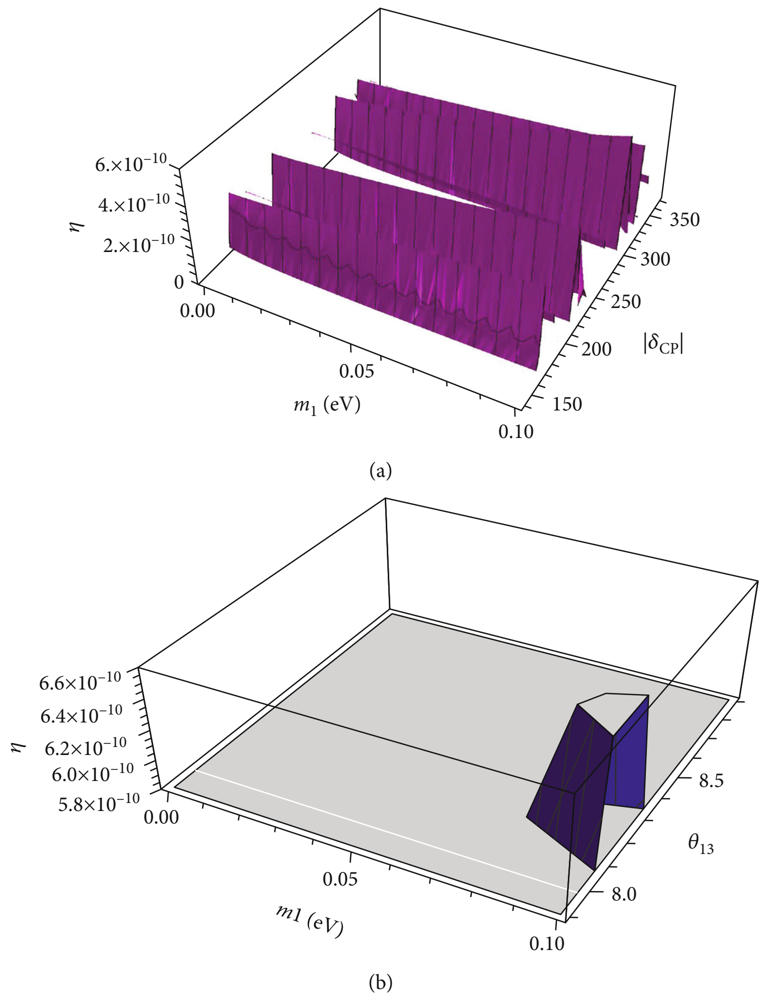


Figure 2: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering: (a) the three-dimensional plot of preferred values of the (m_1, δ_{CP}) plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, (b) shows the three-dimensional plot of favourable values of the (m_1, θ_{13}) plane, for the best fit value of $\delta_{CP} = 222^\circ$ of $\Delta\chi^2 = 6.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$).

h, f, and g are all real. $\mu-\tau$ symmetry is invariant under the exchange of second- and third-generation fermions. When $\mu-\tau$ symmetry is added with SO(10) grand unified theory, then a general symmetry results which satisfies

$$S^T(H, F, G)S = (H, F, -G), \quad (16)$$

where

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (17)$$

The implicit neutrino mass matrix M_ν can be written as

$$M_\nu = r_R M_D F^{-1} M_D^T. \quad (18)$$

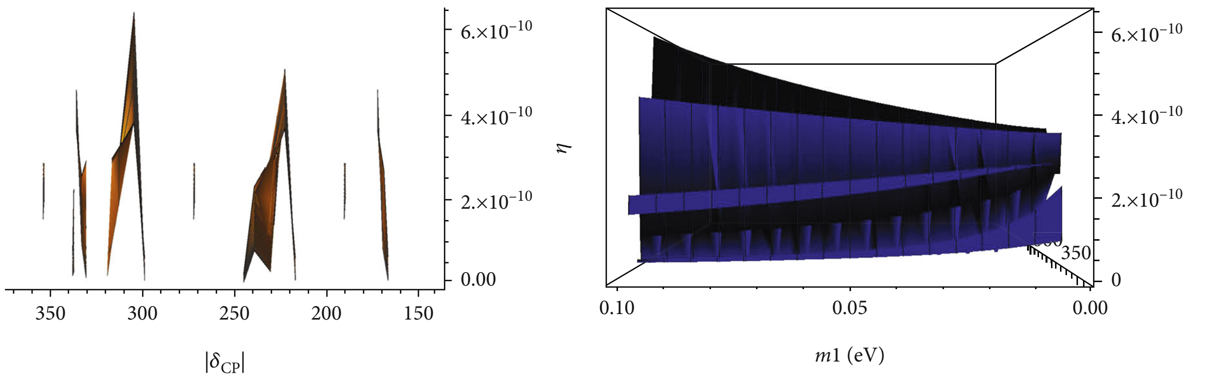
$r_{L,R}$ are inversely proportional to the vev of the RH triplet component in $1\overline{2}6$ dimensional Higgs field.

The Lagrangian of the type I seesaw model is [20, 21]

$$L_N^M = \frac{i}{2}\bar{N}_I^R \delta N_I^R - y_{I\alpha}\bar{N}_I^R \tilde{\phi}^\dagger L_\alpha - \frac{1}{2}\bar{N}_I^R M_{IJ}\left(N_J^R\right)^C + h.c. \quad (19)$$

Here, $y_{I\alpha}$ is the complex Yukawa coupling matrix; $L_\alpha = (\nu_\alpha^L, L_\alpha^l)^T$ is the standard model left-handed lepton doublet of flavor α, when $\alpha = e, \mu, \tau$; and $\tilde{\phi}$ is the hypercharge-conjugated Higgs doublet, $(\phi_0^*, -\phi_-)^T$.

The Lagrangian describes the scenario of generation of ν masses via Higgs mechanism. Electroweak symmetry breaking process allows neutral part of the Higgs field to acquire a VEV, $v = 246$ GeV, and $\sqrt{2}\langle\phi_0\rangle = v$ so that the left-handed and right-handed neutrinos form massive Dirac fermions.



(a) (b)

Figure 3: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering: (a) the preferred values of δ_{CP} for the best fit values of $\theta_{13}=8.41$ of $\Delta\chi^2=9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds 5.8 times $10^{-10}<\eta<6.6\times 1$ 0^{-10}) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, (b) shows the favoured values of the lightest neutrino mass, m_1, for the best fit value of $\delta_{\mathrm{CP}}=222^\circ$ of $\Delta\chi^2=6.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10}<\eta<6.6\times10^{-10}$).

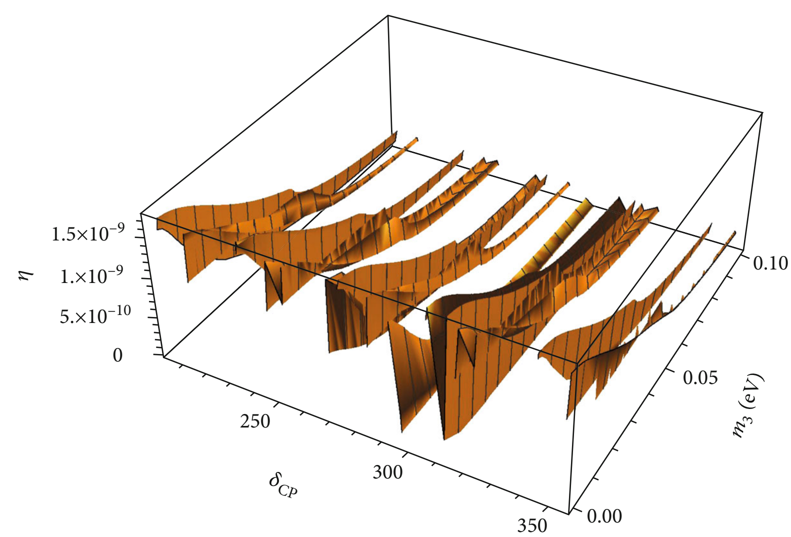


Figure 4: Predictions in the broken $\mu-\tau$ symmetry model for inverted ordering: three-dimensional plot for predicted favoured values of the $(m_3,\delta_{\mathrm{CP}},\eta)$ plane for the best fit values of $\theta_{13}=8.49^\circ$ of $\Delta\chi^2=9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

In Equation (6), M_{IJ} is a symmetric matrix of right-handed violating Majorana masses, where $M_{IJ}=M_I\delta_{IJ}$ is real and diagonal. Here, right-handed neutrino masses are larger than the electroweak scale. The ν masses are then suppressed by right-handed neutrino Yukawa couplings and also by

$$\frac{v}{M_I}<1. \tag{20}$$

In type I seesaw, the baryon asymmetry of the Universe (BAU) occurs via leptogenesis mechanism via out of equilibrium decay of heavy RH Majorana neutrinos in the early Universe via electroweak sphaleron processes [22]. The resulting Majorana mass matrix of light SM neutrinos is

$$M_{\alpha\beta}=-\left(m_{\mathrm{D}}^{T}\right)_{\alpha I}\left(M_{IJ}^{-1}\right)\left(m_{\mathrm{D}}\right)_{J\beta}, \tag{21}$$

where $(m_{\mathrm{D}})_{\alpha I}$ is the Dirac mass matrix. Equation (8) shows that in the type I seesaw mechanism, SM ν masses are suppressed by the combination of small Yukawa couplings and large RH ν masses. Neutrino mass matrix on diagonalision gives two eigen values—light neutrino $\sim m_{\mathrm{D}}^2/M_{\mathrm{R}}$ and a heavy neutrino state $\sim M_{\mathrm{R}}$. This is known as type I seesaw mechanism.

In SO(10), heavy right-handed Majorana neutrino couples to the left-handed ν via Dirac mass matrix m_{D}. Out of the decay of the lightest of the RH Majorana neutrinos, M_1, i.e., $M_3, M_2 \gg M_1$, will contribute to CP asymmetry [6, 23] (for leptogenesis), i.e., ε_l^{CP} and leptogenesis [24–30].

At the end of inflation [31], a certain number density of right-handed neutrinos, n_{ν_R}, was created, which is linked to the present cosmological scenario. Right-handed neutrinos decayed, with a decay rate that reads, at tree level,

$$\Gamma_{D_i} = \Gamma\left(\nu_{R_i \longrightarrow l_i + H_u}\right) + \Gamma\left(\nu_{R_i \longrightarrow \tilde{l}_i + \tilde{H}_u}\right) = \frac{1}{8}\left(Y_\nu Y_\nu^\dagger\right)_{ii} M_i. \quad (22)$$

It is convenient to work in a basis of right-handed neutrinos, where the RH ν mass matrix is diagonal, the type I seesaw mechanism contribution to ε_l^{CP} is given by decay of M_1, or the CP violating parameter is given as

$$\varepsilon_l^{CP} = \frac{\Gamma_{D_i} - \overline{\Gamma}_{D_i}}{\Gamma_{D_i} + \overline{\Gamma}_{D_i}}, \quad (23)$$

where

$$\Gamma_{D_i} = \Gamma\left(\nu_{R_i} \longrightarrow l_i H_u\right) + \Gamma\left(\nu_{R_i} \longrightarrow \tilde{L}_i \tilde{h}_u\right) = \frac{1}{8}\left(Y_\nu Y_\nu^\dagger\right) M_i, \quad (24)$$

where $\Gamma(\nu_{R_i} \longrightarrow l_i H_u)$ means decay rate of heavy Majorana RH ν of mass M_1 to a lepton and Higgs. In the electroweak sphaleron process, asymmetries produced by the out of equilibrium decay of M_2 and M_3 get washed out by lepton number violating interactions after ν_R or M_1 decay. In lepton number violating interactions, decays, inverse decays, and scatterings must be out of equilibrium when the right-handed neutrinos decay. In the basis where the RH ν mass matrix is diagonal, the type I seesaw mehanism contribution to ε_l^{CP} is given by [32]

$$\varepsilon_{CP} = -\frac{3M_1}{8\pi} \frac{Im\left[\Delta m_\odot^2 Q_{12}^2 + \Delta m_A^2 Q_{13}^2\right]}{v^2 \sum \left|Q_{1j}\right|^2 m_j}, \quad (25)$$

where v is the Higg's vev. Q is a complex unitary orthogonal matrix where Q is parameterized as [33] $Q = D_{\sqrt{M^{-1}}} Y_\nu U D_{\sqrt{K^{-1}}}$, where Y_ν is the Dirac neutrino Yukawa couplings. To reproduce the physical, low-energy, parameters, i.e., the light neutrino masses (contained in D_K) and mixing angles and CP phases (contained in U_{PMNS}), we have taken the most general Dirac neutrino mass matrix in broken $\mu - \tau$ symmetry framework as [11]

$$Y_\nu = \frac{M_\nu}{246\,\text{GeV}} = \frac{1}{246\,\text{GeV}} \begin{pmatrix} 11353.7 & -11193.7 + 12692.2i & -11193.7 - 12692.2i \\ -11193.7 - 12692.2i & 62440.4 & 62279.4 - 14582.3i \\ -11193.7 + 12692.2i & 62279.4 + 14582.3i & 62021.3 \end{pmatrix}. \quad (26)$$

The numerical values of the above Dirac Neutrino Yukawa coupling matrix come from the best fit values of the parameters of H, F, G, and Equation (10) by performing a fit using the best fit solutions for fermion masses and mixing obtained assuming the type I seesaw dominance in [11]. The Dirac ν mass matrix is expressed in megaelectron volt units.

In the flavor basis, where the charged-lepton Yukawa matrix Y_e and gauge interactions are flavor-diagonal,

$$D_K = U^T Y_\nu^T D_M^{-1} Y_\nu U. \quad (27)$$

In terms of user-defined Dirac neutrino mass matrices, [11]

$$K = Y_\nu^T M_R^{-1} Y_\nu. \quad (28)$$

U is the PMNS matrix and M_R is the RH neutrino Majorana scale. We can always choose to work in a basis of right neutrinos where M is diagonal $D_M = \text{Diag}(M_1, M_2, M_3)$ where $M_3, M_2 \gg M_1$. Equation (12) expresses ε_{CP} in terms of both the solar (Δm_{21}^2) and atmospheric (Δm_A^2) mass squared differences. Equation (12) also reveals that CP asymmetry is linked to the Dirac CPV phase. Here, we utilise this fact to generate the allowed region of the δ_{CP} phase in the context of leptogenesis. As has been discussed in [32], the lepton-antilepton asymmetry gets connected to both the solar and the atmospheric mass squared differences. The transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B + L$ violating (sphaleron) processes is discussed in [6].

Neutrino masses and mixings are connected with the atmospheric and solar neutrino fluxes; this is suitable to explain flavor changing neutral current processes and FCNC processes, like $\mu \longrightarrow e + \gamma$ processes. In supersymmetric theories like cMSSM, NUHM, NUGM, and NUSM where the origin of the ν masses is via the seesaw mechanism, it can be shown that the prediction for $BR(\mu \longrightarrow e, \gamma)$, $BR(\tau \longrightarrow \mu, \gamma)$, and $BR(\tau \longrightarrow e, \gamma)$ is in general larger than the experimental upper MEG bound [34, 35]. Also, some studies on decays of$b$-flavored hadrons in the context of cMSSM/mSUGRA models is being done in [36].

A small explicit breaking of $\mu - \tau$ symmetry is put by hand, by inheriting the property in Equation (13).

$$h_{22} \neq h_{33}. \quad (29)$$

This introduces CP violation in PMNS matrices and results in θ_{13} being nonzero. Although an explicit breaking of the $\mu - \tau$ symmetry is used in [11], the magnitude of the breaking needed in order to get a large CP violating phase is very minute. This tiny amount of breaking which is used here for generating nonzero CP asymmetry producing a measurable CP violating phase via Dirac neutrino Yukawa couplings used from [11] is fixed to a well specific numerical value, which in turn allows one to replicate mixing angles precisely.

$$\frac{h_{22} - h_{33}}{h_{22} + h_{33}} = 0.0045. \quad (30)$$

The mass matrices defined in Equations (7), (8), (9), (10), (11), and (12) in the model are symmetric under CP invariance if Yukawa couplings are taken to be $\mu-\tau$ symmetric. Small explicit breaking of this symmetry defined in Equation (30) is enough to produce the required CP violating phase.

The Higgs field in the 10 and 126 representations are symmetric and invariant under the generalized $\mu-\tau$ symmetry while the 120-dimensional representation changes sign. This assumption in turn allows spontaneous breaking of the $\mu-\tau$ symmetry. 120 Higgs vev only contributes to off-diagonal elements. Also, one can break the exact $\mu-\tau$ symmetry explicitly through the use of Equation (30) which also involves both diagonal and off-diagonal elements. H, F (G) are any complex symmetric (anti) matrices in general, which are a measure of the fermion Yukawa couplings to the 10, $1\overline{2}6$, and 120 Higgs field, respectively. So the 22 entries and 33 entries in symmetric H matrix can be broken down by explicitly breaking the $\mu-\tau$ symmetry by hand. A required amount of the CP violating phase δ_{CP} is generated by explicitly breaking the $\mu-\tau$ symmetry. This assumption by using Equation (30) leads to $\mathrm{Sin}^2\theta_{23}\sim 0.42-0.63$ and $\mathrm{Sin}^2\theta_{13}>0.006$. All these remarks enable one to use Dirac neutrino Yukawa coupling mass matrices reproduced by the explicit use of broken $\mu-\tau$ symmetry embedded in Equation (30).

The $\mu-\tau$ exchange symmetry in the neutrino mass matrix restricts the 2-3 and 1-3 neutrino mixing angles as $\theta_{23}=\pi/4$ and $\theta_{13}=0$. We find that the $\mu-\tau$ symmetry breaking prefers a large CP violation to realize the observed value of θ_{13} and also keeping θ_{23} nearly maximal. We also propose a concrete model to break the $\mu-\tau$ exchange symmetry spontaneously, and its breaking is mediated by a Yukawa coupling to the Higgs field transforming in 120 of SO(10). As a result of the explicit $\mu-\tau$ breaking in the neutrino mass matrix, a large Dirac CP phase is preferable. As a consequence, nonzero θ_{13} is generated opening up the possibility of having Dirac CP violation in the lepton sector. The latter may be responsible for generation of the observed baryon asymmetry of the Universe (BAU).

The deviations of $\delta\theta_{23}$ from maximal $\theta_{23}=\pi/4$ and the explanation of nonzero $\theta_{13}=0$ are major predictions, goals, and motivation for breaking the $\mu-\tau$ symmetry.

The input parameters defined here are r, s, t, p, and q; Equations (7), (8), (9), (10), (11), and (12); the real elements of the matrices G, H, and F; Equations (13), (14), and (15); and the overall scales $r_{\mathrm{R,L}}$. There can be an overall rotation R on $G,H,F:(G,H,F)R^T(G,H,F)R$. This equals to a choice of initial basis for the 16 plets of fermions. We can then set $h_{12}=0$. This is done with a specific choice $R=R_{23}^T(\pi/4)R_{12}(\theta_{12})R_{23}(\pi/4)$. Here, $R_{ij}(\theta)$ symbolises rotation in the $i\,j$ plane by an angle θ and

$$\tan 2\theta_{12}^h=\frac{2\sqrt{2}h_{12}}{h_{11}-h_{22}-h_{23}}. \quad (31)$$

This rotation equals to reformulation of elements of F and G which still retain the same form as in Equations (13), (14), and (15). With the option $h_{12}=0$, there are 15 input parameters in the case of type I seesaw mechanism. These input parameters when well organized set up 12 fermion masses and six mixing angles. The exact $\mu-\tau$ symmetric H, F and G are not able to provoke CP violation. CP violation is introduced by adding a small $\mu-\tau$ breaking difference between the 22 and 33 elements in H as seen in Equation (30).

We concentrated here in building comprehensive fits to fermion masses and mixings rather than taking into account the whole parameter space of the theory provided by the Yukawa couplings and basic parameters in the superpotential Lagrangian. Parameters in fermion mass matrices are a measure of the strengths of the light Higgs components in different SO(10) Higgs representations.

Many models make use of complex vev to obtain $Z_2^{\mu-\tau}$ breaking. In our approach, the breaking is by hand; i.e., we introduce small explicit breaking of $\mu-\tau$ symmetry in H. The models which use complex vev have 20 free parameters compared to 15 used here.

The explicit breaking of the $\mu-\tau$ symmetry is methodologically natural in the supersymmetric circumstances. However, one can achieve such breaking by introducing an additional 10 plets of the Higgs field which changes sign under the $\mu-\tau$ symmetry. Integrated benefaction of these two 10 plets would then provide an explicitly $\mu-\tau$ noninvariant H.

Also, if we abide by the best fit values of leptonic CP phase $\delta_{\mathrm{CP}}=222^\circ$ discussed in the literature [14, 37], then our scenario of explicit breaking of the $\mu-\tau$ symmetry by hand, evident from Equation (30), leads to $\delta_{\mathrm{CP}}\in 222^0$ for inverted ordering of ν masses corresponding to $\langle m_{ee}\rangle = 0.01\,\mathrm{eV}$ which exactly matches with the best fit values of $\delta_{\mathrm{CP}}=222^\circ$ with $\Delta\chi^2=6.2$ w/o SK-ATM [14].

The novelty of this work lies in the successful explicit breaking of the $\mu-\tau$ symmetry within the SO(10) framework in order to obtain a constrained picture of fermion masses. This framework provides a user-defined neutrino Yukawa coupling [11] for the Yukawa interactions associated with the broken $\mu-\tau$ symmetry model for the generation of nonzero reactor mixing angle θ_{13} and leptonic CP phase δ_{CP} in type I seesaw mechanism; in the light of leptogenesis, there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B+L$ violating (sphaleron, Sakharov conditions) processes as discussed in [6]. A small explicit breaking of the $\mu-\tau$ symmetry [11] inherits the property of generating nonzero CP violation in U_{PMNS} matrices and δ_{CP} phase and results in θ_{13} being nonzero. Here, we consider the type I seesaw as the main donor to neutrino mass. We also take into account both inverted and normal ordering of neutrino mass spectrum as well as two different types of the lightest neutrino mass $m_1(m_3 = 0.07118eV(0.0657\,\mathrm{eV}))$ to visualise the results of hierarchical ν mass spectrum. This scenario is characterized by the predictions that in case of normal ordering of ν masses, the favoured values of the δ_{CP} phase in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$, are found to lie in the range $\delta_{\mathrm{CP}}\in[304^\circ,307^\circ]$

for best fit values of $\theta_{13} = 8.41$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The favoured values of the lightest ν mass, m_1, in this case come out to be $\in[0.09,0.1]$ eV. In case of inverted hierarchy, the variation of the δ_{CP} phase is found to be very intense with best fit values of $\theta_{13} = 8.49$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]. Values of δ_{CP} phase favoured are $\delta_{CP} = 220°, 223°, 252°, 268°, 293°, 309°, 345°$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$). The allowed spectrum of the lightest ν mass is m_3, $\in[0.02,0.055]$ eV.

We also plot the allowed values of $|m_{ee}|$ eV for neutrinoless double beta decay and the Jarkslog invariant, J_{CP}, for normal ordering of ν masses. Prediction of future leptonic CP violation experiments should be able to rule out or take into account some of the results discussed in this work. If we abide by the best fit values of leptonic CP phase $\delta_{CP} = 222°$ discussed in the literature [14, 37], then our scenario, $\delta_{CP} \in 222°$, for inverted ordering of ν masses corresponding to $\langle m_{ee}\rangle = 0.01$ eV exactly matches with the best fit values of $\delta_{CP} = 222°$ with $\Delta\chi^2 = 6.2$ w/o SK-ATM [14].

The model here is motivated in the sense that it provides a specific form of Dirac neutrino Yukawa coupling matrix in the constraint of explicit breaking of the $\mu - \tau$ symmetry as is evident from the relation $(h_{22} - h_{33})/(h_{22} + h_{33}) = 0.0045$ between the h_{22} and the h_{33} elements of the H matrix. If we use this specific form of Dirac neutrino Yukawa coupling matrix to calculate baryon asymmetry of the Universe, then in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$ as a result of contribution of the type I seesaw mechanism to neutrino mass matrix, we get a range of allowed values of nonzero θ_{13} and large δ_{CPV} phase.

The motivation of this work is to relate baryogenesis through leptogenesis and the hint of CP violation in the neutrino oscillation data to a breaking of the $\mu - \tau$ symmetry. Small explicit tiny breaking of the $\mu - \tau$ symmetry allows a large Dirac CP violating phase in neutrino oscillation which in turn is characterized by awareness of measured value of nonzero θ_{13} and to provide a hint towards a better understanding of the experimentally observed near-maximal value of $\nu_\mu - \nu_\tau$ mixing angle $\theta_{23} \simeq \pi/4$. Precise breaking of the $\mu - \tau$ symmetry can be achieved by adding a 120-plet Higgs to the $10 + 1\overline{2}6$-dimensional representation of Higgs.

Here in this work, the estimated three-dimensional density parameter space of the lightest neutrino mass m_1, δ_{CP}, and reactor mixing angle θ_{13} is constrained here for the requirement of producing the observed value of baryon asymmetry of the Universe through the mechanism of leptogenesis. Carrying out numerical analysis, the allowed parameter space of m_1, δ_{CP}, and θ_{13} is found out which can produce the observed baryon to photon density ratio of the Universe, the details of which are discussed below.

3. Numerical Analysis

In this section, numerical analysis has been carried out. Firstly, the free parameter called the lightest ν mass, the ν oscillation parameters like reactor angle θ_{13}, Dirac CPV phase, δ_{CP}, Majorana phases α_{21} and α_{31} in broken $\mu - \tau$ symmetry, and type I seesaw model are scanned to search for dependance of the δ_{CP} phase on θ_{13}, m_1 (m_3), Jarkslog invariant J_{CP}, and effective mass for $0\nu\beta\beta$ decay in case of normal ordering (inverted ordering) in the context of producing correct baryon asyymetry of the Universe [13]. We use the best fit values of ν oscillation parameters. The two mass square differences Δm^2_{12} and Δm^2_{13} are embedded in neutrino mixing matrix so we are left out with lightest ν mass as the only free parameter in this model. In the charged lepton basis, we parameterize the PMNS matrix U_{PMNS}, by diagonalizing the neutrino mass matrix m_ν in terms of three mixing angles $\theta_{ij}(i, j = 1, 2, 3\,; i < j)$, one CP violating Dirac CPV phase δ_{CP}, and two Majorana phases (α_{21} and α_{31}) as follows:

$$U^* P^* m_\nu P^\dagger U^\dagger = m_\nu^{\mathrm{D}}. \tag{32}$$

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix is UP, where U is

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e_{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{33}$$

where $\theta_{12} = 33.82°, \theta_{23} = 48.3°(48.6°)$, and $\theta_{13} = 8.61°(8.65°)$ [14] in case of normal hierarchy (inverted hierarchy) are the solar, atmospheric, and reactor angles, respectively. The Majorana phases reside in P, where

$$P = \mathrm{diag}\begin{pmatrix} 1 & e^{i\alpha_{21}} & e^{i(\alpha_{31}+\delta)} \end{pmatrix}. \tag{34}$$

We have taken complex and orthogonal matrix $R = U_{PMNS}$, in terms of user-defined Dirac neutrino Yukawa couplings defined in Equation (13) in order to produce correct baryon asymmetry of the Universe.

For the normally ordered light ν masses, we have

$$M_R^{\mathrm{diag}} = \mathrm{diag}\,(M_1, M_2, M_3) = M_1\ \mathrm{diag}\left(1, \frac{\mathrm{M}_2}{M_1}, \frac{M_3}{M_1}\right) = M_1\ \mathrm{diag}\left(1, \frac{m_1}{m_2}, \frac{m_1}{m_3}\right), \tag{35}$$

with $m_1 \in [10^{-6}\,\mathrm{eV}, 10^{-1}\,\mathrm{eV}]$, $m_2^2 - m_1^2 = 7.39 \times 10^{-5}\,\mathrm{eV}^2$, and $m_3^2 - m_1^2 = 2.48 \times 10^{-3}\,\mathrm{eV}^2$ as is evident from the ν oscillation

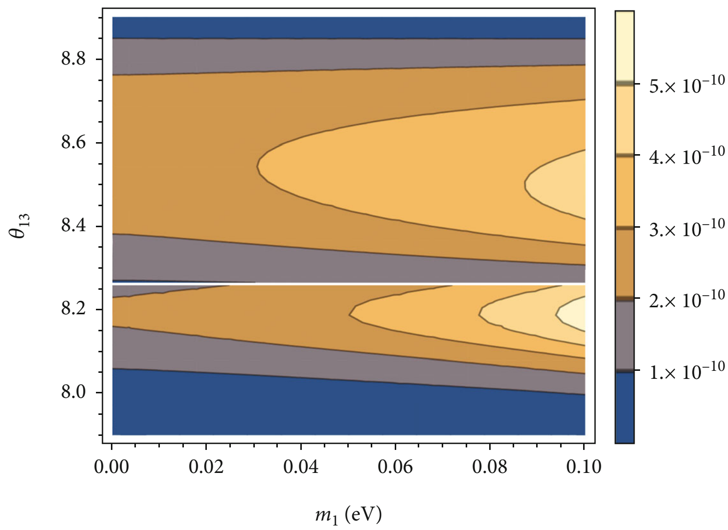


FIGURE 5: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering: contour plot for predicted favoured values of the (m_1, θ_{13}) plane for the best fit values of $\delta_{\mathrm{CP}} = 222^\circ$ of $\Delta_{\chi^2} = 6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

data [14], m_1 being the lightest of three ν masses. For the inverted ordered light ν masses, we have

$$M_R^{\mathrm{diag}} = \mathrm{diag}\,(M_1, M_2, M_3) = M_1\ \mathrm{diag}\left(1, \frac{M_2}{M_1}, \frac{M_3}{M_1}\right) = M_1\ \mathrm{diag}\left(1, \frac{m_1 * m_3}{m_2^2}, \frac{m_1}{m_2}\right), \tag{36}$$

with m_3 being the lightest of three ν masses. Here, we take $M_1 \sim 10^{12}$ GeV. For normal ordering, the choices of the lightest neutrino mass is $m_1 = 0.07118$ eV whereas for inverted ordering, the choice of the lightest neutrino mass is $m_3 = 0.0657$ eV. This sustainable allowance of $m_1(m_3) = 0.07(0.065)$ eV signifies a neutrino mass spectrum where the sum of absolute neutrino masses lies below the cosmological upper bound, $\sum_i m(\nu_i) < 0.23$ eV [12]. Next, random scan of the ν mixing matrix parameter space for NH and IH in order to produce correct the baryon asymmetry of the Universe $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$ is performed in the following 3 σ range of δ_{CP} with respect to the tabulated χ^2 map of the Super-Kamiokande analysis of the data within $\Delta\chi^2 = 6.2$ [14]:

(i) $m_1(m_3) \in [10^6\ \mathrm{eV}, 0.1\ \mathrm{eV}](10^6\ \mathrm{eV}, 0.1\ \mathrm{eV})$

(ii) $\delta_{\mathrm{CP}} \in [141, 370]$ for normal ordering

(iii) $\delta_{\mathrm{CP}} \in [205, 354]$ for inverted ordering

(iv) $\theta_{13} \in [7.9, 8.9]$ for normal ordering for tabulated $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]

(v) $\theta_{13} \in [8.0, 9.0]$ for normal ordering for tabulated $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]

(vi) $\theta_{23} \in [40.8, 51.3]$ for normal ordering for tabulated $\Delta\chi^2 = 6.2$ w/o SK-ATM [14]

While doing parameter scan, we find favoured values of the lightest ν mass and dirac CPV phase δ_{CP}, for producing correct baryon asymmetry of the Universe, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$.

The lepton flavor effects are significant if the lightest right-handed Majorana neutrino mass $M_{\nu_{\mathrm{R1}}}$ is below 10^{12} GeV. Here, $M_1 = 10^{12}$ GeV. In the type I seesaw mechanism, one can always find the right-handed neutrino mass matrix as

$$M_{\alpha\beta} = -\left(m_{\mathrm{D}}^T\right)_{\alpha I}\left(M_{IJ}^{-1}\right)\left(m_{\mathrm{D}}\right)_{J\beta}, \tag{37}$$

where $(m_{\mathrm{D}})_{\alpha I}$ is the Dirac mass matrix. We consider a Dirac neutrino mass matrix defined in Equation (13). Here, when we fix $m_1(m_3), Y_\nu, M_{\mathrm{R}}$, the remaining free parameter in the neutrino sector within our broken $\mu-\tau$ framework is the leptonic CPV phase δ_{CP}. When we vary the CPV phase δ_{CP}, we compute the favoured regions of $m_1(m_3)$. The variations of leptonic CPV phase δ_{CP} with $m_1(m_3)$, θ_{13}, J_{CP}, and m_{ee} for $0\nu\beta\beta$ as shown in the figures discussed here.

For global fit values of ν oscillation parameters, we compute the Jarlskog invariant, δ_{CP}, given by PMNS matrix elements $U_{\alpha i}$. We also compute the Jarkslog invariant for allowed values of the δ_{CP} phase, θ_{13}, and lightest ν mass explored here in this work for both normal ordering and inverted ordering.

$$J_{\mathrm{CP}} = \mathrm{Im}\left[U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*\right] = s_{23} c_{23} s_{12} c_{12} s_{13} c_{13}^2 \sin\delta_{\mathrm{CP}}. \tag{38}$$

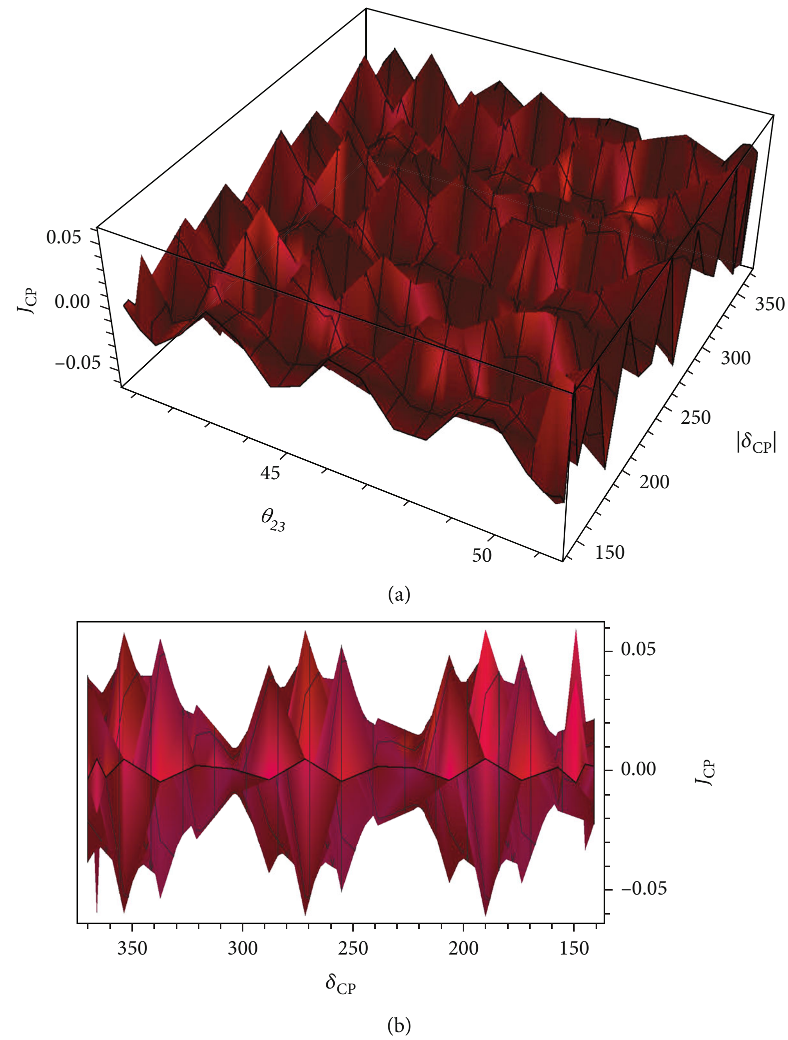


FIGURE 6: Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: (b) the preferred three-dimensional regions of the (δ_{CP}, θ_{23}, J_{CP}) plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]; (a) allowed two-dimensional space of the (δ_{CP}, J_{CP}) plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14].

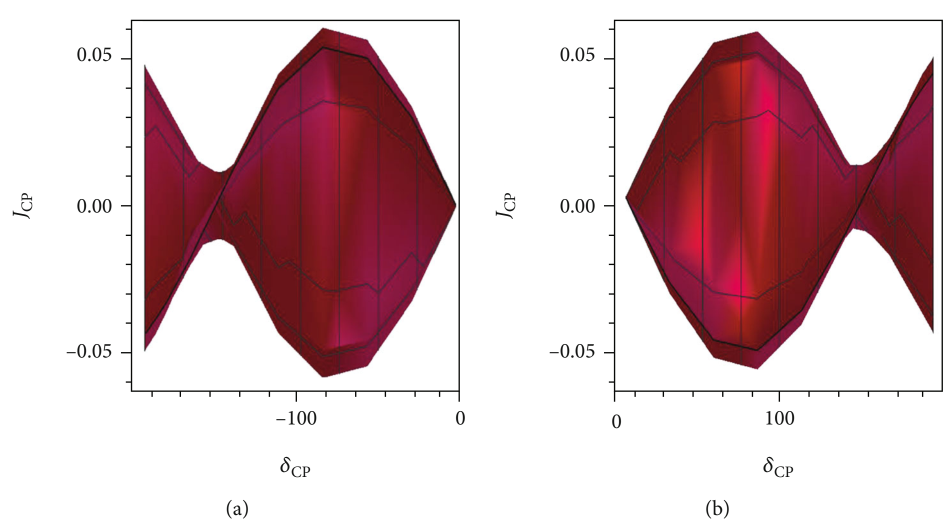


FIGURE 7: Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: (b) shows the allowed two-dimensional space of the (δ_{CP}, J_{CP}) plane for an absolute range of $\delta_{CP} \in [-180,+180]$.

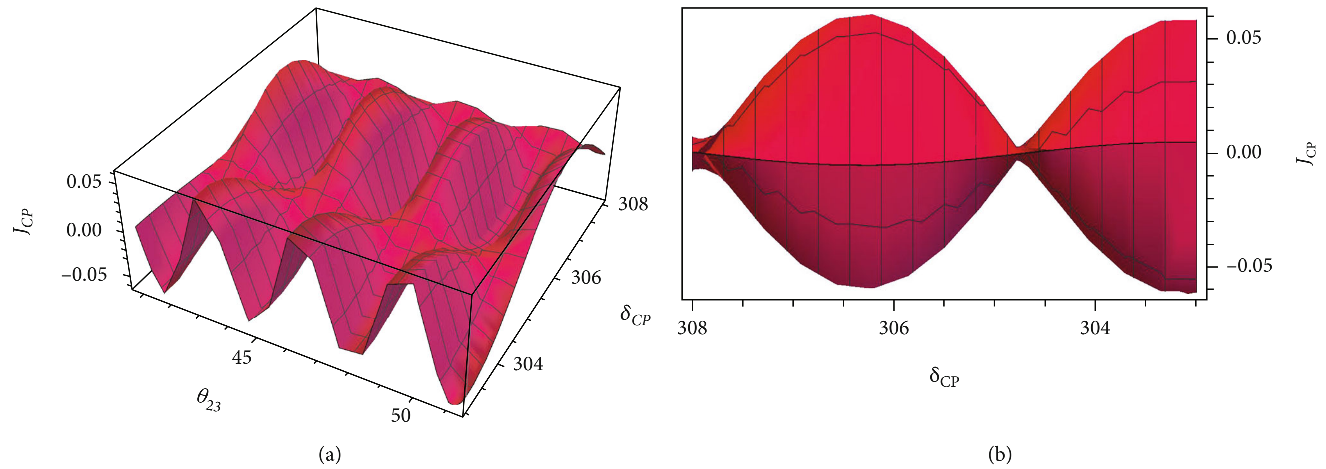


FIGURE 8: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering: (b) preferred three-dimensional regions of the ($\delta_{CP}, \theta_{23}, J_{CP}$) plane for favoured values of $\delta_{CP} \in [303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]; (a) allowed two-dimensional space of the (δ_{CP}, J_{CP}) plane for favoured values of $\delta_{CP} \in [303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$).

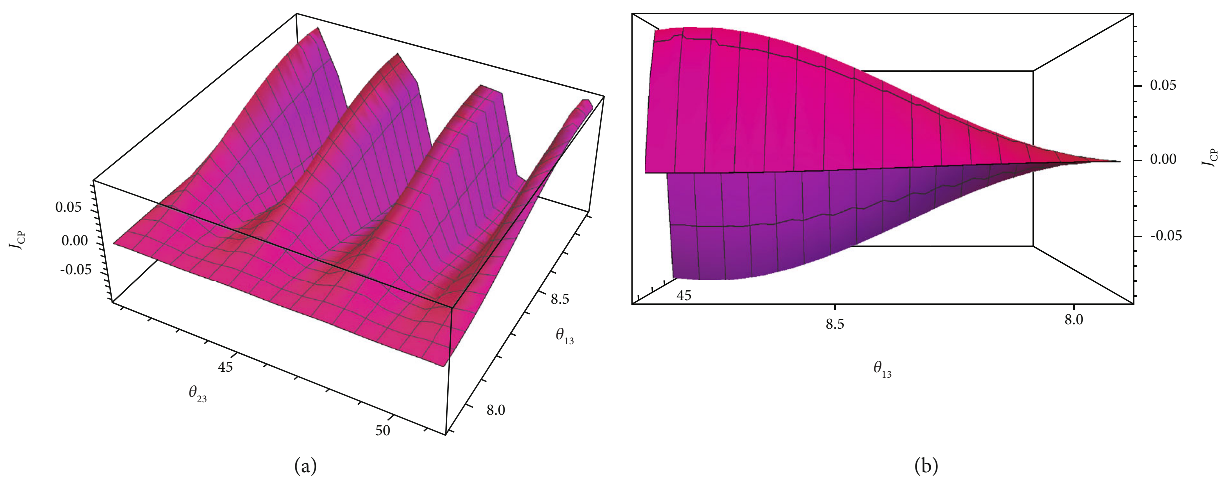


FIGURE 9: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering: (b) preferred three-dimensional regions of the ($\theta_{13}, \theta_{23}, J_{CP}$) plane for the best fit values of $\delta_{CP} = 222^\circ$ of $\Delta\chi^2 = 6.2$ w/o SK-ATM [14]; (a) allowed two-dimensional space of the (θ_{13}, J_{CP}) plane for the best fit values of $\delta_{CP} = 222^\circ$ of $\Delta\chi^2 = 6.2$ w/o SK-ATM [14].

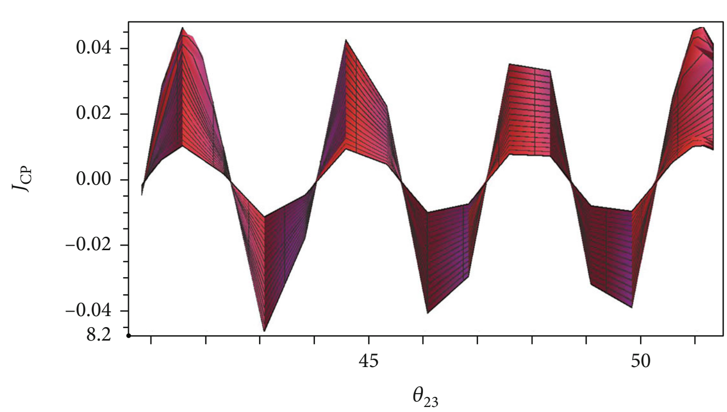
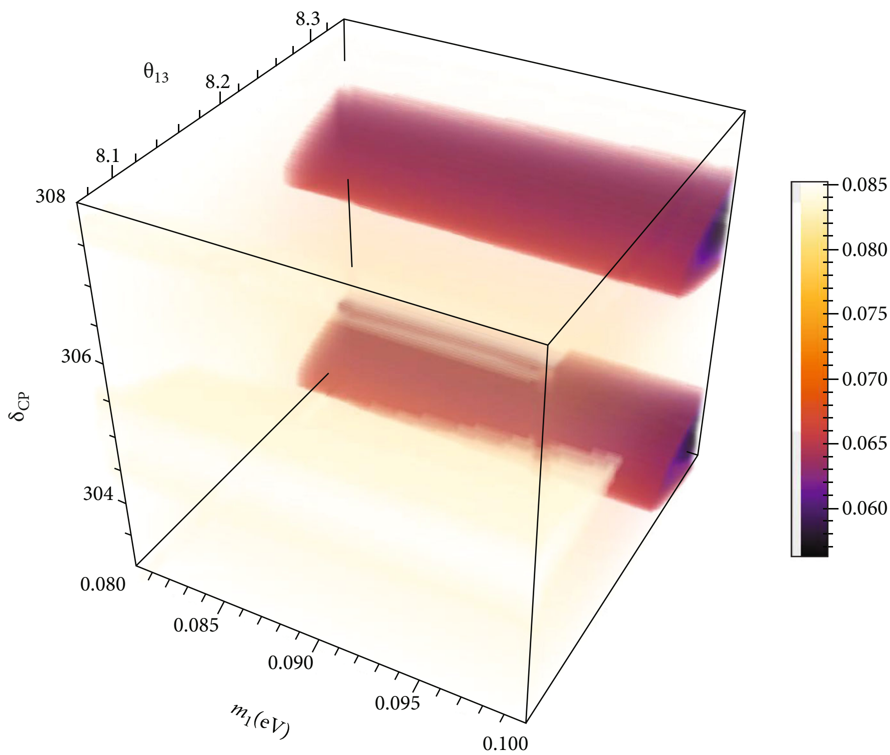
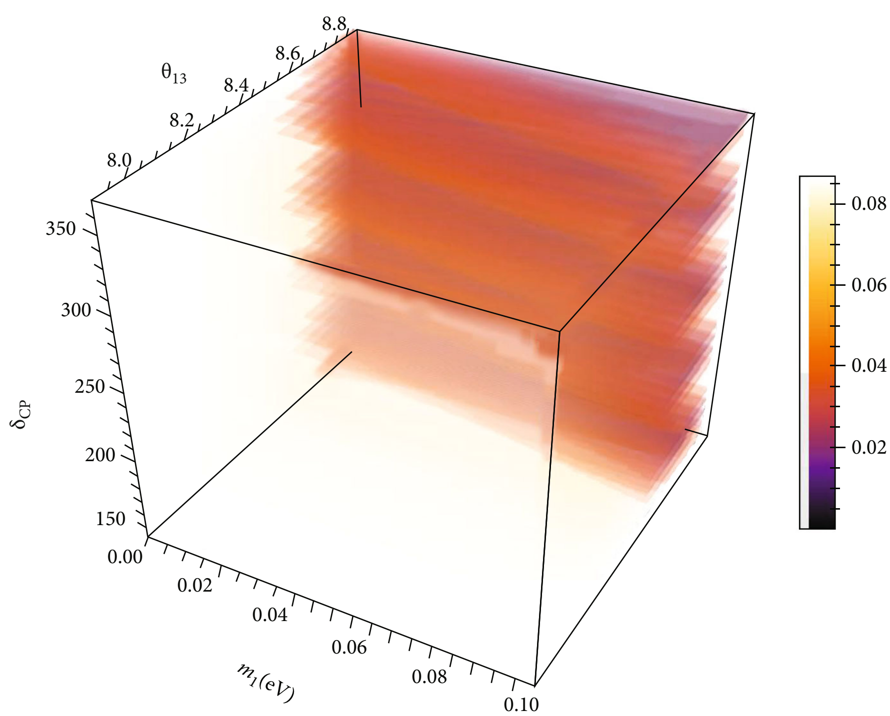


FIGURE 10: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering: variation of J_{CP} as a function of θ_{23}, $\theta_{23} \in [40.8,51.3]$, for normal ordering for tabulated $\Delta\chi^2 = 6.2$. [14].



(a)



(b)

FIGURE 11: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering: (a) depicts predicted three-dimensional space of $(m_1, \delta_{CP}, \theta_{13})$ for m_{ee} [eV], $0\nu\beta\beta$ decay for favoured values of m_1, δ_{CP}, θ_{13} (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10} < \eta < 6.6\times10^{-10}$). (b) Depicts the predicted three-dimensional space of $(m_1, \delta_{CP}, \theta_{13})$ for m_{ee} [eV], $0\nu\beta\beta$ decay for values of m_1, δ_{CP}, and θ_{13} in the given three σ range, corresponding to $\Delta\chi^2 = 6.2$ and $\Delta\chi^2 = 9.5$ w/o SK-ATM [14].

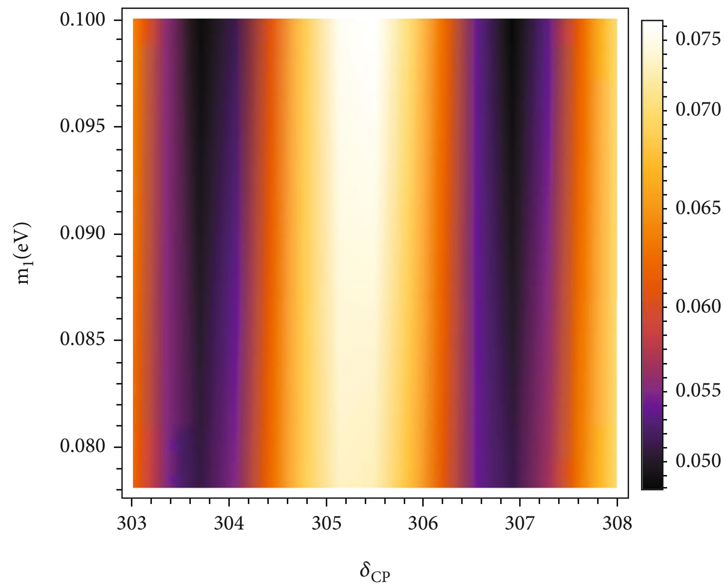


Figure 12: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering: depicts the density plot of $(m_1, \delta_{\mathrm{CP}})$ for m_{ee} [eV], $0\nu\beta\beta$ decay for favoured values of m_1, δ_{CP}, and θ_{13} (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times1$ 0^{-10}).

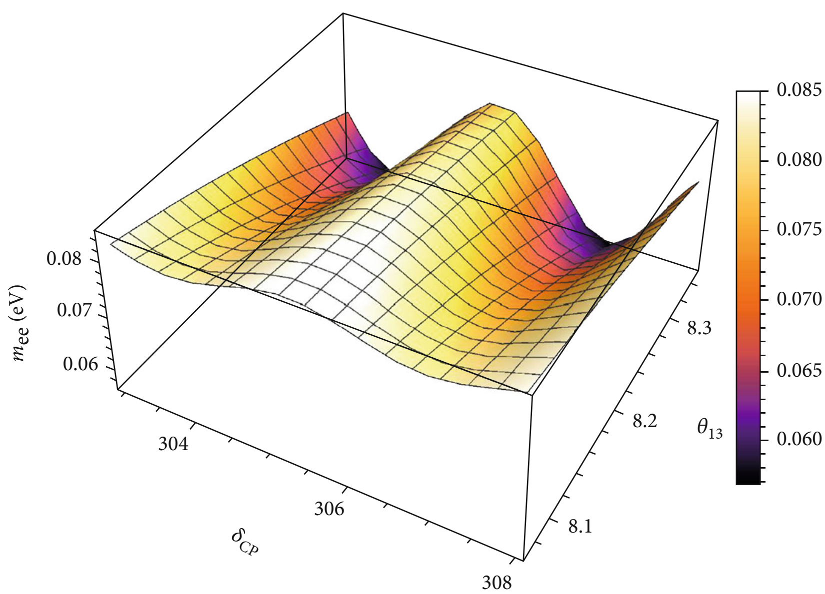


Figure 13: Predictions in the broken $\mu-\tau$ symmetry model for normal ordering. (a) Depicts the predicted three-dimensional space of $(m_{ee}, \delta_{\mathrm{CP}}, m_1)$ for m_{ee} [eV], $0\nu\beta\beta$ decay for favoured values of m_1, δ_{CP}, and θ_{13} (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$). Depicts predicted three-dimensional space of $(m_{ee}, \delta_{\mathrm{CP}}, m_1)$ for m_{ee} [eV], $0\nu\beta\beta$ decay for favoured values of m_1, $\delta_{\mathrm{CP}}, m_{ee}$ for values of δ_{CP} in the given three σ range, corresponding to $\Delta\chi^2=9.5$ w/o SK-ATM [14].

We also calculate the favourable space of the effective mass for $0\nu\beta\beta$ decay for favourable values of the δ_{CP} phase, θ_{13}, and lightest ν mass given by

$$m_{ee}=\left|m_1c_{12}^2c_{13}^2+m_2s_{12}^2c_{13}^2e^{i\alpha_{21}}+m_3s_{13}^2e^{i(\alpha_{31}-2\delta_{\mathrm{CP}})}\right|. \quad (39)$$

The colour coding in the different figures imply the values of baryon asymmetry of the Universe in the allowed range, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$.

In Figure 1, we have presented the predictions in the broken $\mu-\tau$ symmetry model for normal ordering. Panel (a) conveys the predicted favoured values of the (m_1, θ_{13}) plane for the best fit values of $\delta_{\mathrm{CP}}=222^\circ$ with $\Delta\chi^2=6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The allowed spectrum of the lightest ν mass here lies in the range, 0.09 eV–0.095 eV corresponding to favoured values of reactor angle, θ_{13}, in the interval, 8.1°

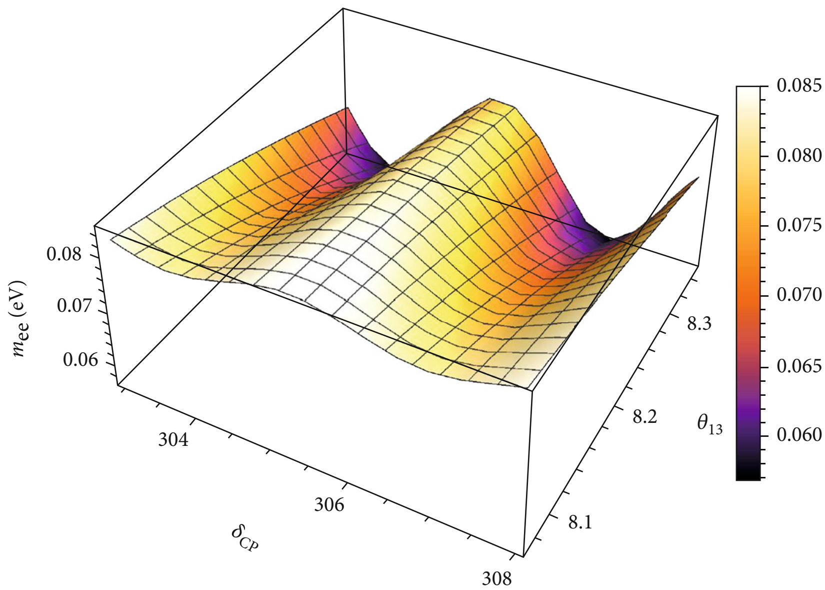


Figure 14: The predicted three-dimensional space of $(m_{ee}, \delta_{CP}, \theta_{13})$ for m_{ee} [eV], $0\nu\beta\beta$ decay for favoured values of $m_1, \delta_{CP}, \theta_{13}$ (in the light of recent ratio of the baryon to photon density bounds, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) for the lightest ν mass $m_1 = 0.07118$ eV.

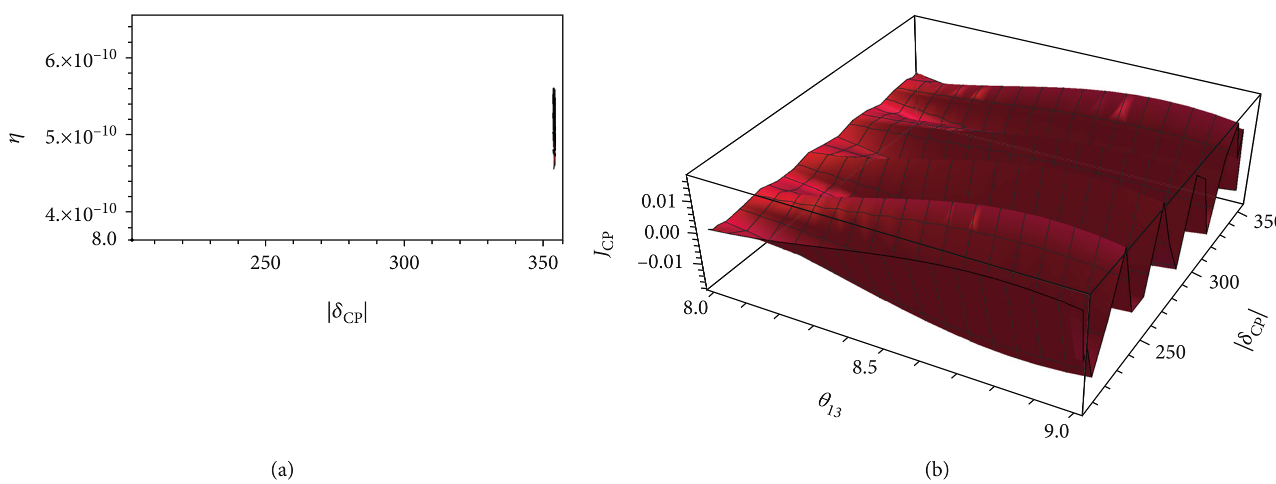
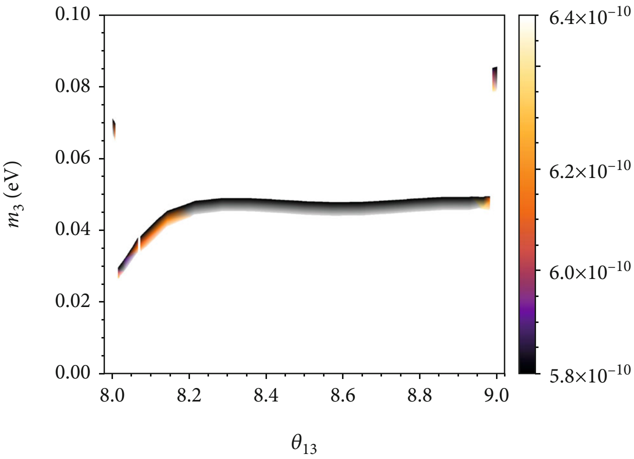


Figure 15: Predictions in the broken $\mu - \tau$ symmetry model for inverted ordering. (a) Predicted the favoured values of δ_{CP} (in the light of recent ratio of the baryon to photon density bounds, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) for the lightest ν mass $m_3 = 0.0657$ eV, as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, (b) predicted the allowed three-dimensional space of the $(\delta_{CP}, \theta_{13}, J_{CP})$ plane for allowed regions of Jarkslog invariant, J_{CP}, values for best fit value of $\theta_{23} = 48.6^\circ$ of $\Delta\chi^2 = 6.2$ [14] as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

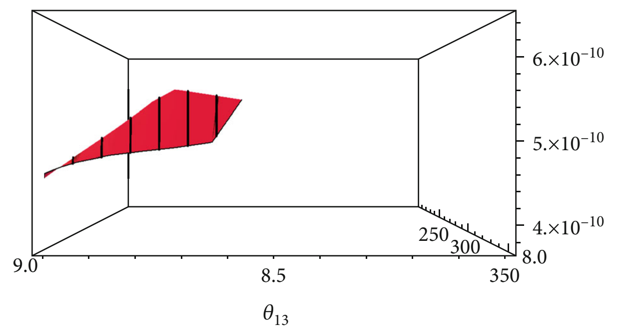
-8.35° in light of the correct baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$. Panel (b) manifests itself in the predicted favoured value (m_1, δ_{CP}) plane, for the best fit value of $\theta_{13} = 8.41$ with $\Delta\chi^2 = 9.5$ [14]. The favoured value of δ_{CP} is around 307° in the light of the correct baryon asymmetry of the Universe which can be manifested from the colour coding in the figure.

We have shown the contour plot for predicted favoured values of (m_1, θ_{13}) plane for the best fit values of $\delta_{CP} = 222^\circ$ of $\Delta_{\chi^2} = 6.2$ w/o SK-ATM [14] (as allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix in Figure 5. The colour coding in the contour plot also shows that for η to lie in the interval, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$, the allowed lightest neutrino mass m_1 must lie around 0.095 eV.

In Figure 2, we have speculated the predictions in the broken $\mu - \tau$ symmetry model for normal ordering. The (a) presents the three-dimensional plot of preferred values of the (η, m_1, δ_{CP}) plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The preferred value



(a)



(b)

Figure 16: Predictions in the broken $\mu-\tau$ symmetry model for inverted ordering. (a) The density plot of predicted favoured values of the (m_3, θ_{13}) plane for the best fit values of $\delta_{CP}=285^\circ$ of $\Delta\chi^2=6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, (b) shows the three-dimensional plot for predicted favoured values of the $(\theta_{13}, \delta_{CP}, \eta)$ plane for the lightest ν mass, $m_3=0.0657$ (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

of the leptonic CPV phase came out to be around 304°-307°. Similarly, the panel (b) communicates the three-dimensional plot of favourable values of the (η, m_1, θ_{13}) plane, for the best fit value of $\delta_{CP}=222^\circ$ of $\Delta\chi^2=6.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10}<\eta<6.6\times10^{-10}$).

In Figure 3, we have depicted the predictions in the broken $\mu-\tau$ symmetry model for normal ordering: panel (a) favours preferred values of δ_{CP} for the best fit values of $\theta_{13}=8.41$ of $\Delta\chi^2=9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds 5.8 times $10^{-10}<\eta<6.6\times10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The preferred value of the leptonic CPV phase δ_{CP} came out to be around 304°-307° as is obvious from the figure. Similarly, panel (b) shows the favoured values of the lightest neutrino mass, m_1, for the best fit value of $\delta_{CP}=222^\circ$ of $\Delta\chi^2=6.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10}<\eta<6.6\times10^{-10}$). The favoured values of m_1 is around 0.09 eV–0.1 eV.

Figure 6 reveals the predictions in the broken $\mu-\tau$ symmetry model for normal ordering: panel (b) shows the preferred three-dimensional regions of the $(\delta_{CP}, \theta_{23}, J_{CP})$ plane for the best fit values of $\theta_{13}=8.41$ of $\Delta\chi^2=9.5$ w/o SK-ATM [14]. Panel (a) presents allowed two-dimensional space of the (δ_{CP}, J_{CP}) plane for the best fit values of $\theta_{13}=8.41$ of $\Delta\chi^2=9.5$ w/o SK-ATM [14].

In Figure 7, we present the predictions in the broken $\mu-\tau$ symmetry model for normal ordering. Panel (b) favours allowed two-dimensional space of the (δ_{CP}, J_{CP}) plane for an absolute range of $\delta_{CP}\in[-180,+180]$.

In Figure 8, we have speculated the predictions in the broken $\mu-\tau$ symmetry model for normal ordering. Panel (b) conveys preferred three-dimensional regions of the $(\delta_{CP}, \theta_{23}, J_{CP})$ plane for favoured values of $\delta_{CP}\in[303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10}<\eta<6.6\times10^{-10}$) for best fit values of $\theta_{13}=8.41$ of $\Delta\chi^2=9.5$ w/o SK-ATM [14]. Panel (a) presents allowed two-dimensional space of the (δ_{CP}, J_{CP}) plane for favoured values of $\delta_{CP}\in[303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10}<\eta<6.6\times10^{-10}$) for the best fit values of $\theta_{13}=8.41$ of $\Delta\chi^2=9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10}<\eta<6.6\times10^{-10}$).

In Figure 9, we have shown predictions in the broken $\mu-\tau$ symmetry model for normal ordering. Panel (b) presents preferred three-dimensional regions of the $(\theta_{13}, \theta_{23}, J_{CP})$ plane for the best fit values of $\delta_{CP}=222^\circ$ of $\Delta\chi^2=6.2$ w/o SK-ATM [14]. The left panel communicates the allowed two-dimensional space of the (θ_{13}, J_{CP}) plane for the best fit values of $\delta_{CP}=222^\circ$ of $\Delta\chi^2=6.2$ w/o SK-ATM [14].

Figure 10 depicts the predictions in the broken $\mu-\tau$ symmetry model for normal ordering. The variation of J_{CP} as a function of θ_{23}, $\theta_{23}\in[40.8,51.3]$ for normal ordering for tabulated $\Delta\chi^2=6.2$. [14] is speculated.

In Figure 11, we have presented predictions in the broken $\mu-\tau$ symmetry model for normal ordering. Panel (a) depicts three-dimensional space of $(m_1, \delta_{CP}, \theta_{13})$ for m_{ee} [eV], $0\nu\beta\beta$ decay for favoured values of m_1, δ_{CP}, and θ_{13} (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$). Panel (b) demonstrated three-dimensional space of $(m_1, \delta_{CP}, \theta_{13})$ for m_{ee} [eV], $0\nu\beta\beta$ decay for values of m_1, δ_{CP}, and θ_{13} in the given three σ range, corresponding to $\Delta\chi^2=6.2$ and $\Delta\chi^2=9.5$ w/o SK-ATM [14].

In Figure 12, we have introduced predictions in the broken $\mu-\tau$ symmetry model for normal ordering. The figure depicts the density plot of (m_1, δ_{CP}) for m_{ee} [eV], $0\nu\beta\beta$ decay

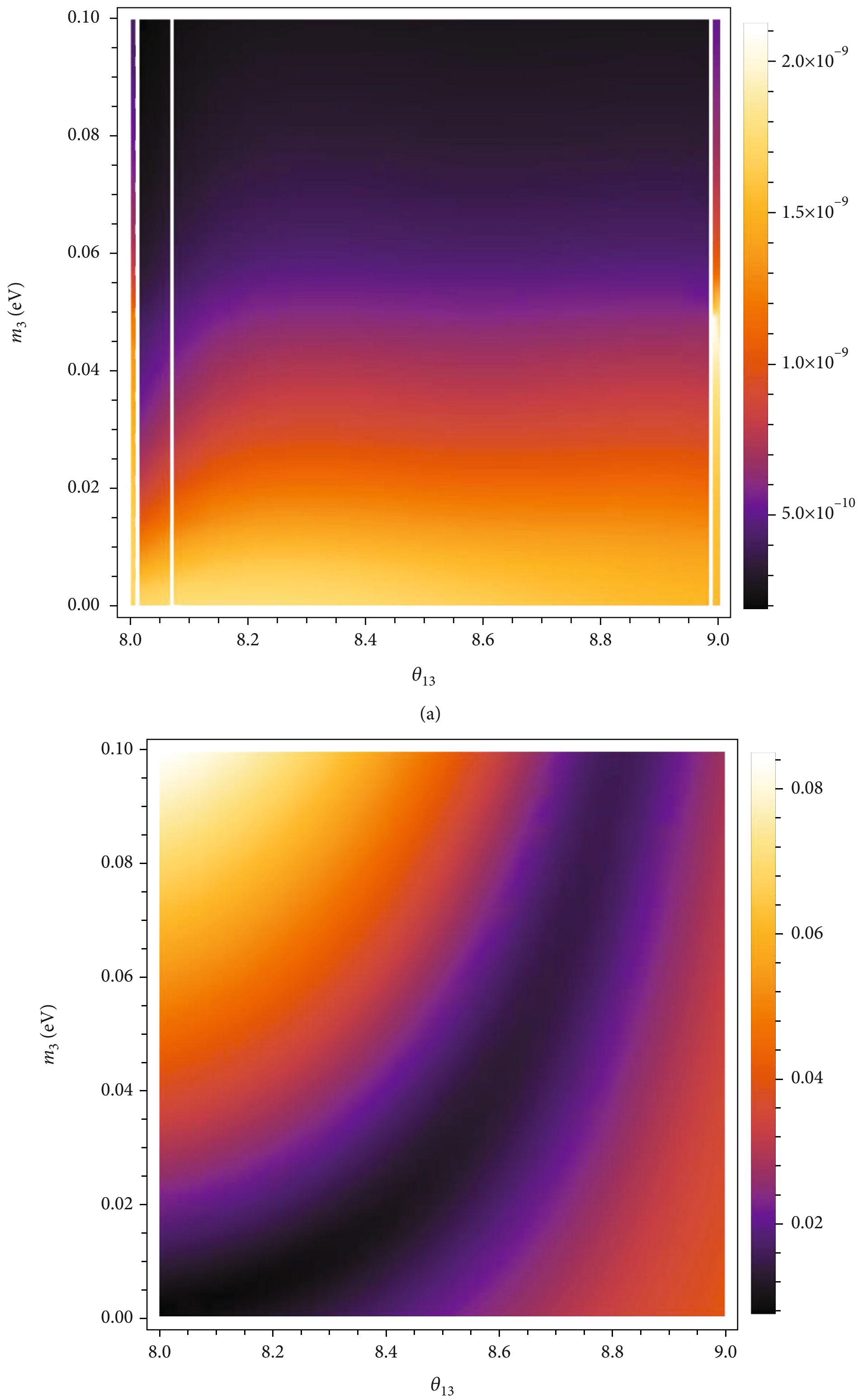


FIGURE 17: Predictions in the broken $\mu-\tau$ symmetry model for inverted ordering. (a) The density plot of predicted favoured values of the (m_3, θ_{13}) plane for the best fit values of $\delta_{CP} = 285^\circ$ of $\Delta\chi^2 = 6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, (b) shows the predicted two-dimensional space of (m_3, θ_{13}) for m_{ee} [eV], $0\nu\beta\beta$ decay, for the best fit values of $\delta_{CP} = 285^\circ$ of $\Delta\chi^2 = 6.2$ w/o SK-ATM [14].

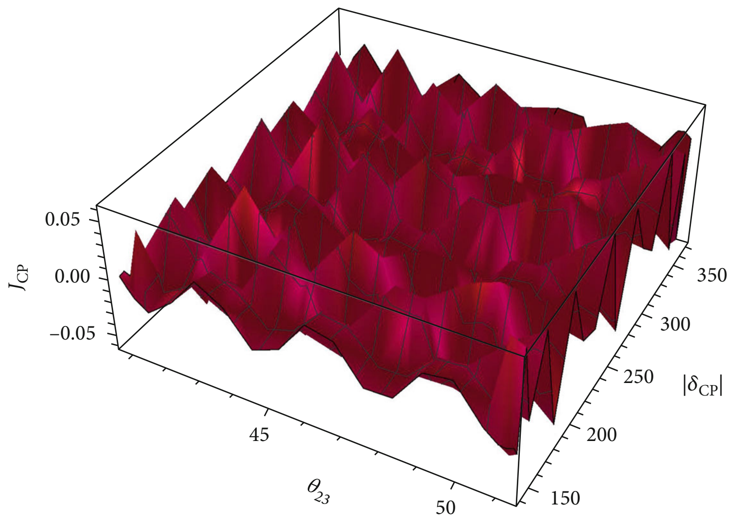


FIGURE 18: Predictions in the broken $\mu-\tau$ symmetry model for inverted ordering: predicted allowed three-dimensional space of the (delt a$_{CP}$, θ_{23}, J_{CP}) plane for allowed regions of Jarkslog invariant, J_{CP}, values for the best fit value of $\theta_{13}=8.49$ of $\Delta\chi^2=9.5$ [14] as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

for favoured values of m_1, δ_{CP}, and θ_{13} (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$)

In Figure 13, we have presented predictions in the broken $\mu-\tau$ symmetry model for normal ordering. Panel (a) depicts predicted three-dimensional space of $(m_{ee}, \delta_{CP}, m_1)$ for m_{ee} [eV], $0\nu\beta\beta$ decay for favoured values of m_1, δ_{CP}, θ_{13} (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$). Panel (b) depicts predicted three-dimensional space of $(m_{ee}, \delta_{CP}, m_1)$ for m_{ee} [eV], $0\nu\beta$ β decay for favoured values of m_1, δ_{CP}, m_{ee} for values of δ_{CP} in the given three σ range, corresponding to $\Delta\chi^2=9.5$ w/o SK-ATM [14].

In Figure 14, we have conveyed the three-dimensional space of $(m_{ee}, \delta_{CP}, \theta_{13})$ for m_{ee} [eV], $0\nu\beta\beta$ decay for favoured values of m_1, δ_{CP}, θ_{13} (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times1$ 0^{-10}) for the lightest ν mass $m_1=0.07118$ eV.

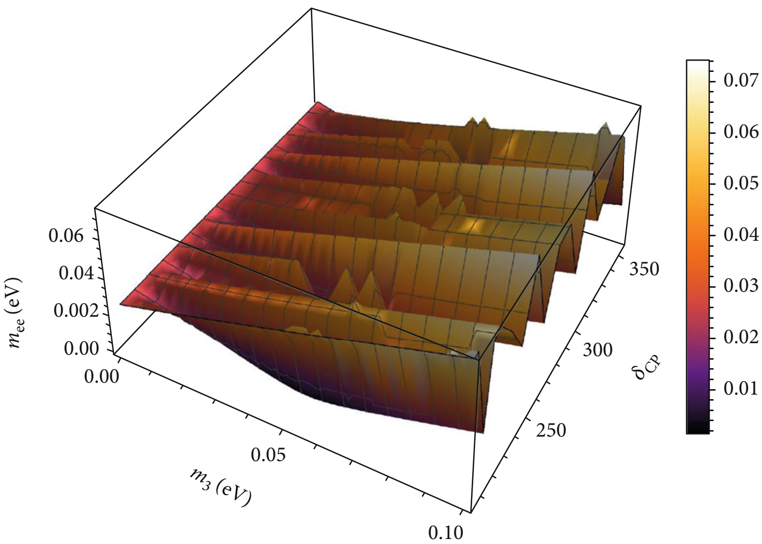
In Figure 15, we have shown the predictions in the broken $\mu-\tau$ symmetry model for inverted ordering. Panel (a) predicts favoured values of δ_{CP} (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$) for the lightest ν mass $m_3=0.0657$ eV, as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, panel (b) shows predicted allowed three-dimensional space of the $(\delta_{CP}, \theta_{13}, J_{CP})$ plane for allowed regions of Jarkslog invariant, J_{CP}, values for the best fit value of $\theta_{23}=48.6^\circ$ of $\Delta\chi^2=6.2$ [14] as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

In Figure 4, we have shown predictions in the broken $\mu-\tau$ symmetry model for inverted ordering. The three-dimensional plot for predicted favoured values of (m_3, δ_{CP}, η) plane for the best fit values of $\theta_{13}=8.49^\circ$ of $\Delta\chi^2=9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix is presented. The favoured values of the δ_{CP} phase predited here are 295° and 303° – 306° in the light of recent ratio of baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$. The value of the lightest ν mass m_3 depicted here from the figure is around 0.005 eV.

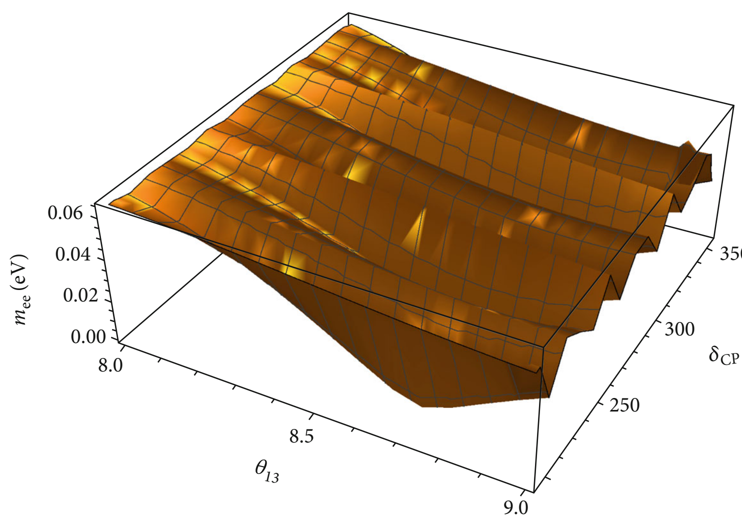
In Figure 16, panel (a) presents the density plot of predicted favoured values of the (m_3, θ_{13}) plane for the best fit values of $\delta_{CP}=285^\circ$ of $\Delta\chi^2=6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The favoured values of the lightest ν mass depicted here are $m_3\sim0.03$ eV–0.047 eV for producing correct baryon asymmetry of the Universe. Also, the favoured value reactor angle, θ_{13}, lies in the range 8°–9°. Similarly, in panel (b), we have shown the three-dimensional plot for predicted favoured values of the $(\theta_{13}, \delta_{CP}, \eta)$ plane for the lightest ν mass, $m_3=0.0657$ (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10}<\eta<6.6\times10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

We also plot the allowed values of $|m_{ee}|$ eV for neutrinoless double beta decay and the Jarkslog invariant, J_{CP}, in Figures 15 and 17–20 for inverted ordering of ν masses.

In Figure 17, we have shown the predictions in the broken $\mu-\tau$ symmetry model for inverted ordering. Panel (a) shows the density plot of predicted favoured values of the (m_3, θ_{13}) plane for the best fit values of $\delta_{CP}=285^\circ$ of $\Delta\chi^2=6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, panel (b) depicts the predicted two-dimensional space of (m_3, θ_{13}) for m_{ee} [eV], $0\nu\beta\beta$ decay, for the best fit values of $\delta_{CP}=285^\circ$ of $\Delta\chi^2=6.2$ w/o SK-ATM [14]. The favoured values of the lightest ν mass depicted here are m_3



(a)



(b)

Figure 19: Predictions in the broken $\mu - \tau$ symmetry model for inverted ordering. (a) Depicts the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, m_3)$ for values of $m_3 \in [10^{-6}, 0.1]$ eV and δ_{CP} in the given 3 σ range, corresponding to $\Delta\chi^2 = 6.2$, and the best fit values of $\theta_{13} = 8.49^\circ$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]. (b) Depicts the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, \theta_{13})$ for values of the lightest ν mass, $m_3 = 0.0657 eV$, δ_{CP}, and θ_{13} in the given 3 σ range, corresponding to $\Delta\chi^2 = 6.2$ and $\Delta\chi^2 = 9.5$ w/o SK-ATM [14].

~ 0.03 eV–0.047 eV for producing correct baryon asymmetry of the Universe. Also, the favoured value reactor angle, θ_{13}, lies in the range 8°–9°. In Figure 18, we show the predicted allowed three-dimensional space of the $(\text{delta}_{CP}, \theta_{23}, J_{CP})$ plane for allowed regions of Jarkslog invariant, J_{CP}, values for the best fit value of $\theta_{13} = 8.49$ of $\Delta\chi^2 = 9.5$ [14] as a result of contribution of the type I seesaw mechanism to neutrino mass matrix for inverted ordering. Figure 19 depicts in panel (a) the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, m_3)$ for values of $m_3 \in [10^{-6}, 0.1]$ eV and δ_{CP} in the given 3 σ range, corresponding to $\Delta\chi^2 = 6.2$, and the best fit values of $\theta_{13} = 8.49^\circ$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]. The favoured values of the lightest ν mass depicted here are $m_3 \sim 0.045$ eV–0.06 eV corresponding to a value of $m_{ee} =$ 0.01 eV [38]. The favoured values of the leptonic CPV phase, δ_{CP}, predicted here are 235°–237° consistent with $m_{ee} = 0.01$ eV. Panel (b) shows the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, \theta_{13})$ for values of the lightest ν mass, $m_3 =$ 0.0657 eV, δ_{CP}, and θ_{13} in the given 3 σ range, corresponding to $\Delta\chi^2 = 6.2$ and $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] for inverted ordering. The favoured values of the reactor angle predicted here are $\theta_{13} \sim 8.6^\circ - 8.9^\circ$ corresponding to a value of $m_{ee} =$ 0.01 eV [38]. The favoured values of the leptonic CPV phase, δ_{CP}, predicted here are 222° consistent with, limits on, $\langle m_{ee} \rangle$, $m_{ee} = 0.01$ eV [38]. In Figure 20, we have depicted the predicted density plot of (δ_{CP}, m_3) for values of $m_3 \in [10^{-6}, 0.1]$ eV and δ_{CP} in the given 3 σ range, corresponding to $\Delta\chi^2 =$ 6.2, and the best fit values of $\theta_{13} = 8.49^\circ$ corresponding to Δ

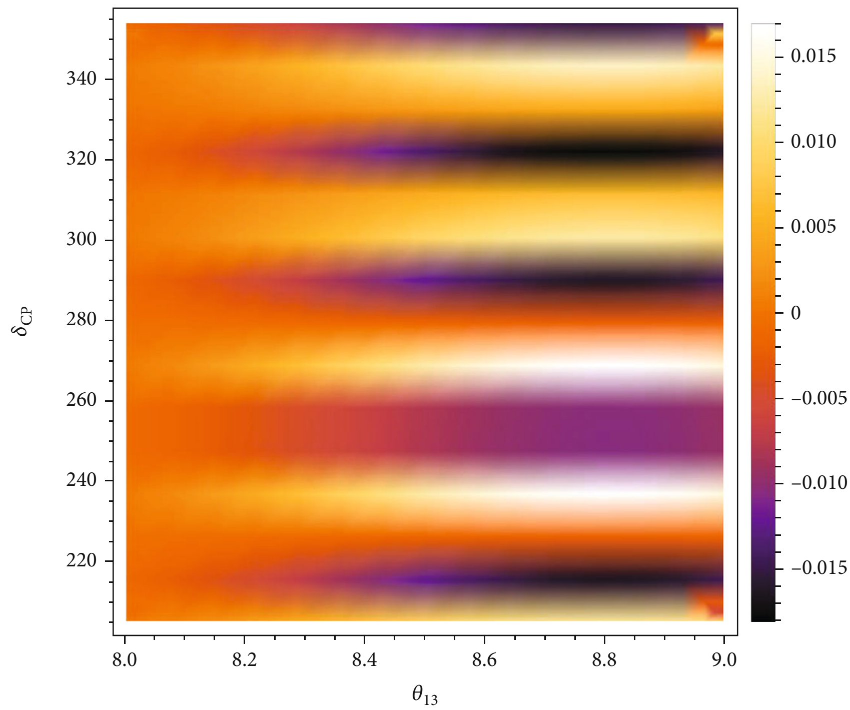


Figure 20: Predictions in the broken $\mu-\tau$ symmetry model for inverted ordering: depicts the predicted density plot of (δ_{CP}, m_3) for values of $m_3 \in [10^{-6}, 0.1]$ eV and δ_{CP} in the given 3 σ range, corresponding to $\Delta\chi^2 = 6.2$, and the best fit values of $\theta_{13} = 8.49^\circ$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14].

$\chi^2 = 9.5$ w/o SK-ATM [14] for inverted ordering of hierarchical ν masses.

4. Results and Conclusion

Fermion mass spectrum can be explained by Hermitian mass matrices derived from the renormalizable Yukawa couplings of the 16 plets of fermions with the Higgs fields transforming as 10, $1\overline{2}6$, and 120 representations of the SO(10) group. The $\mu-\tau$ symmetry upon spontaneously broken down through the 120 plets leads to nonzero reactor angle θ_{13}, which in turn induces the leptonic dirac CPV phase, δ_{CP}, in the U_{PMNS} matrix. This scenario implies a generalized CP invariance of the fermion mass matrices and vanishing CP violating phases if the Yukawa couplings are symmetric under the $\mu-\tau$ symmetry. Small explicit tiny breaking of the $\mu-\tau$ symmetry (evident from Equation (30)) allows a large Dirac CP violating phase in neutrino oscillation. Explicit breaking of the $\mu-\tau$ symmetry by hand as seen from Equation (30) provides a nice spectrum of all the fermion masses and mixing and leads to nonzero θ_{13}, which in turn implies δ_{CPV} phase and allows a large required Dirac CP violating phase in neutrino oscillation. Detailed fits to the fermion spectrum are presented in several scenarios in [11].

The model considered here is motivated in the sense since it provides a constrained fit of fermion masses and explains the largeness of the atmospheric mixing angle, θ_{23} [11]. Neutrino Yukawa coupling matrices considered here exhibit a generalized CP invariance if Yukawa couplings are $\mu-\tau$ symmetric. Small explicit breaking of this symmetry by hand as is evident from Equation (30) with the specific numerical value 0.0045 is sufficient to generate the large required CP violating phase discussed here. We have considered the type I seesaw model with very tiny explicit $\mu-\tau$ symmetry breaking. This model with very tiny explicit $\mu-\tau$ symmetry breaking is motivated in the sense as it is decribed by the predictions $\mathrm{Sin}^2\theta_{23} \sim 0.42-0.63$ and nonzero reactor angle $\mathrm{Sin}^2\theta_{13} > 0.005$ and large required CP violation in neutrino oscillations.

In Figure 1, for the best fit values of $\delta_{CP} = 222^\circ$ with $\Delta\chi^2 = 6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe), the allowed spectrum of the lightest ν mass here lies in the range 0.09 eV–0.095 eV corresponding to favoured values of reactor angle, θ_{13}, in the interval, 8.1°–8.35° in light of the correct baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$. Panel (a) manifests itself in the predicted favoured value (m_1, δ_{CP}) plane, for the best fit value of $\theta_{13} = 8.41$ with $\Delta\chi^2 = 9.5$ [14]. The favoured value of δ_{CP} is around 307° in the light of the correct baryon asymmetry of the Universe which can be manifested from the colour coding in the figure. We have shown for the best fit values of $\delta_{CP} = 222^\circ$ of $\Delta_{\chi^2} = 6.2$ w/o SK-ATM [14] (as allowed by updated values of correct baryon asymmetry of the Universe) in Figure 5, (for η to lie in the interval, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$), m_1 must lie around 0.095 eV. In Figure 2, for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$), the preferred value of the leptonic CPV

Table 1: Values of the δ_{CP} phase giving correct updated values of baryon asymmetry.

$\mu-\tau$ broken symmetry model	Calculated leptonic CP phase δ_{CP}
$m_1 \in [0.085,0.1]$ eV (NH)	$\delta_{CP} \in [304^\circ, 307^\circ]$
$m_3 \in [0.03,0.047]$, $[0.045,0.06]$ eV (IH)	$\delta_{CP} = 220^\circ, 222^\circ, 252^\circ, 268^\circ, 293^\circ, 309^\circ, 345^\circ$

phase came out to be around 304°-307°. Similarly, panel (b) communicates the three-dimensional plot of favourable values of the (η, m_1, θ_{13}) plane, for the best fit value of $\delta_{CP} = 222^\circ$ of $\Delta\chi^2 = 6.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10} < \eta < 6.6\times1$ 0^{-10}). In Figure 3, for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds 5.8 times $10^{-10} < \eta < 6.6\times10^{-10}$), the preferred value of the leptonic CPV phase δ_{CP} came out to be around 304°-307° as is obvious from the figure. Similarly, panel (b) shows the favoured values of the lightest neutrino mass, m_1, for the best fit value of $\delta_{CP} = 222^\circ$ of $\Delta\chi^2 = 6.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10} < \eta < 6.6\times1$ 0^{-10}). The favoured values of m_1 is around 0.09 eV–0.1 eV. In Figure 8, panel (a) presents the allowed two-dimensional space of the (δ_{CP}, J_{CP}) plane for favoured values of $\delta_{CP} \in [303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10} < \eta < 6.6\times10^{-10}$) for the best fit values of $\theta_{13} = 8.41$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10} < \eta < 6.6\times10^{-10}$). In Figure 4, for the best fit values of $\theta_{13} = 8.49^\circ$ of $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10} < \eta < 6.6\times10^{-10}$, the favoured values of δ_{CP} phase presented here are 295° and 303° – 306° in the light of recent ratio of baryon to photon density bounds, 5.8×1 $0^{-10} < \eta < 6.6\times10^{-10}$. The value of the lightest ν mass m_3 depicted here from the figure is around 0.005 eV. In Figure 16, for the best fit values of $\delta_{CP} = 285^\circ$ of $\Delta\chi^2 = 6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe), the favoured values of the lightest ν mass depicted here are $m_3 \sim 0.03$ eV–0.047 eV for producing correct baryon asymmetry of the Universe. Also, the favoured values reactor angle, θ_{13}, lies in the range 8°–9°. Similarly, in panel (b), we have shown the three-dimensional plot for the predicted favoured values of the $(\theta_{13}, \delta_{CP}, \eta)$ plane for the lightest ν mass, $m_3 = 0.0657$ (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10} < \eta < 6.6\times10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Figure 19 depicts in panel (a) the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, m_3)$ for values of $m_3 \in [10^{-6}, 0.1]$ eV and δ_{CP} in the given 3 σ range, corresponding to $\Delta\chi^2 = 6.2$, and the best fit values of $\theta_{13} = 8.49^\circ$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]. The favoured values of the lightest ν mass depicted here are $m_3 \sim 0.045$ eV–0.06 eV corresponding to a value of $m_{ee} = 0.01$ eV. The favoured values of the leptonic CPV phase, δ_{CP}, predicted here are 235°–237° consistent with $m_{ee} = 0.01$ eV. Panel (b) shows the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, \theta_{13})$ for values of the lightest ν mass, $m_3 = 0.0657$ eV, δ_{CP}, and θ_{13} in the given 3 σ range, corresponding to $\Delta\chi^2 = 6.2$ and $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] for inverted ordering. The favoured values of the reactor angle predicted here are $\theta_{13} \sim 8.6^\circ - 8.9^\circ$ corresponding to a value of $m_{ee} = 0.01$ eV. The favoured values of the leptonic CPV phase, δ_{CP}, predicted here are 222° consistent with, limits on, $\langle m_{ee}\rangle$, $m_{ee} = 0.01$ eV.

In this work, we learn that by using user-defined Dirac Neutrino Yukawa couplings [11] for the Yukawa interactions associated with the broken $\mu-\tau$ symmetry model for the generation of the nonzero reactor mixing angle θ_{13} and leptonic CP phase δ_{CP} in the type I seesaw mechanism in the light of leptogenesis, there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B+L$ violating (sphaleron, Sakharov conditions) processes as discussed in [6]. A small explicit breaking of $\mu-\tau$ symmetry [11] inherits the property of generating nonzero CP violation in U_{PMNS} matrices and δ_{CP} phase and results in θ_{13} being nonzero. Here, we consider the type I seesaw as the main donor to neutrino mass. We also take into account both inverted and normal ordering of neutrino mass spectrum as well as two different types of the lightest neutrino mass m_1($m_3 = 0.07118$ eV(0.0657 eV)) to visualise the results of hierarchical ν mass spectrum. In the case of normal ordering of ν masses, the dependance of the δ_{CP} phase on the lightest ν mass is predicted in Figures 1–3 (in the light of recent ratio of the baryon to photon density bounds, $5.8\times10^{-10} < \eta < 6.6\times1$ 0^{-10}). The favoured values of the δ_{CP} phase is found to lie between $\delta_{CP} \in [304^0, 307^0]$ for the best fit values of $\theta_{13} = 8.41$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10} < \eta < 6.6\times10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The favoured values of the lightest ν mass, m_1, in this case come out to be $\in[0.09,0.1]$ eV. In the case of inverted hierarchy, the variation of the δ_{CP} phase is found to be very intense with the best fit values of $\theta_{13} = 8.49$ corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14]. Values of the δ_{CP} phase favoured are $\delta_{CP} = 220^\circ, 223^\circ, 252^\circ, 268^\circ, 293^\circ, 309^\circ, 345^\circ$ (in the light of recent ratio of the baryon to photon density bounds $5.8\times10^{-10} < \eta < 6.6\times10^{-10}$) as is evident from Figure 4. The allowed spectrum of the lightest ν mass is m_3, $\in[0.02,0.055]$ eV.

We also plot the allowed values of $|m_{ee}|$ eV for neutrinoless double beta decay and the Jarkslog invariant, J_{CP}, in Figures 6–14 for normal ordering of ν masses. Prediction of future leptonic CP violation experiments should be able to rule out or take into account some of the results discussed in this work. If we abide by the best fit values of leptonic CP phase $\delta_{CP} = 222^\circ$ discussed in the literature [14, 37], then our scenario, $\delta_{CP} \in 222^\circ$, for inverted ordering of ν masses corresponding to $\langle m_{ee} \rangle = 0.01$ eV exactly matches with the best fit values of $\delta_{CP} = 222^\circ$ with $\Delta\chi^2 = 6.2$ w/o SK-ATM [14]. We show the variation of baryon asymmetry with the leptonic δ_{CP} phase in Table 1.

Future LBL experiments will hunt for the leptonic CP phase and potentially will measure it with precision. Neutrinoless double beta decay will indicate towards the Majorana CPV phase. The fundamental mysteries in the Universe are about the findings of the nature of the massive neutrinos-Dirac or Majorana. This may be sorted out by the experiments like GERDA, CUORE, KamLAND-Zen, EXO, LEGEND, and nEXO. Determination of the status of leptonic CP asymmetry (T2K, NOνA, T2HK, DUNE), determination of the type of neutrino mass ordering (T2K + NOνA, JUNO, PINGU, ORCA, T2HKK, DUNE), and determination of the order of absolute neutrino mass scale (KATRIN, cosmology) are few of the most challenging tasks today. The ideas presented in this work may definitely will rule in or rule out some of the favoured space in few of the above experiments.

Conflicts of Interest

The author declares that she has no conflicts of interest.

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